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Facultad de Ciencias Físicas y Matemáticas

Dynamos in Partially Convective M Dwarfs

Dínamos en Estrellas Enanas de Tipo M Parcialmente Convectivas

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Resumen

Los campos magnéticos desempeñan un papel crucial en la dinámica y evolución de los cuerpos astrofísicos. Esta tesis investiga los procesos de dínamo en enanas M, una clase de estrella caracterizada por su actividad magnética y un cambio en su estructura interna a las $0.35 M_{\odot}$, pasando de ser completamente convectivas a parcialmente convectivas. El estudio emplea simulaciones magnetohidrodinámicas tridimensionales utilizando el modelo *star-in-a-box* para explorar los efectos de la variación en las tasas de rotación y las condiciones de contorno sobre los campos magnéticos estelares. Se analizaron cinco conjuntos de simulaciones con períodos de rotación que van desde 9.6 hasta 96 días; dentro de cada conjunto se evaluaron dos o tres configuraciones diferentes. Estas condiciones se definieron mediante la implementación de distintos perfiles de difusividad: un perfil plano (sin saltos), un perfil con un único salto en el núcleo radiativo, y un perfil con dos saltos, uno en el núcleo y otro fuera de la estrella. Los resultados revelaron diversas soluciones para los campos magnéticos a gran escala: algunos casos mostraron ciclos bien definidos, similares a los observados en el Sol, mientras que otros presentaron dínamos cuasiestáticos. Además, en ciertos casos, se observaron campos magnéticos más irregulares, con actividad esporádica e inversiones de polaridad. En general, se encontró que los perfiles con un único salto en la difusividad magnética presentaban mayores similitudes con el dínamo solar, a diferencia de aquellos con dos saltos, los cuales carecían de ciclos definidos y cambios de polaridad.

El siguiente paso sería investigar estas mismas condiciones bajo otras tasas de rotación y analizar los efectos de un aumento en la resolución. Hasta ahora, se ha demostrado que incluso un pequeño cambio en las condiciones de borde puede influir significativamente en el comportamiento del dínamo. Por tanto, es fundamental determinar si estas variaciones se deben directamente a la implementación de condiciones de borde externas o si intervienen otros parámetros, como la resolución de las simulaciones. Además, sería interesante explorar qué ocurre al considerar tasas de rotación mayores.

Keywords – Enanas M, campo magnético estelar, dínamo, magnetohidrodinámica (MHD), estrellas parcialmente convectivas

Abstract

Magnetic fields have been shown to play a crucial role in the dynamics and evolution of astrophysical bodies. This thesis investigates dynamo processes in M dwarfs, a stellar type characterized by magnetic activity and a transition in internal structure at 0.35 solar masses from fully convective to partially convective. The study utilizes three-dimensional magnetohydrodynamical (MHD) simulations within the framework of the star-in-a-box model to explore the effects of varying rotation rates and magnetic boundary conditions on stellar magnetic fields. To this end, five sets of simulations with rotation periods ranging from 9.6 to 96 days are considered, and within each set, two or three different setups are explored. The latter is achieved by implementing different diffusivity profiles, corresponding to a flat profile (no jumps), a profile in the radiative core (one jump) and profiles in the core and outside the star (two jumps). The analysis of these simulations yielded a variety of solutions for the large-scale magnetic field, including cycles that exhibited characteristics analogous to those observed in the Sun, as well as quasi-static dynamos. Additionally, there were instances of magnetic fields exhibiting greater irregularity, accompanied by sporadic activity and occasional polarity reversals. It was found that, in general, those with one jump in magnetic diffusivity had more similarities to the solar dynamo; this is in contrast to those simulations where two jumps were implemented, which showed an absence of cycles and polarity changes.

The subsequent step will be to investigate these same conditions with different rotation rates and to analyze the effects of increased resolution. So far, it has been shown that even a small change in the boundary conditions can significantly influence the behavior of the dynamo. It is therefore essential to determine whether these variations are directly due to the implementation of external boundary conditions or whether other parameters, such as the resolution of the simulations, are involved. Furthermore, it would be interesting to explore what happens when considering higher rotation rates.

Keywords – M Dwarfs, stellar magnetic field, dynamo, magnetohydrodynamics (MHD), partially convective stars

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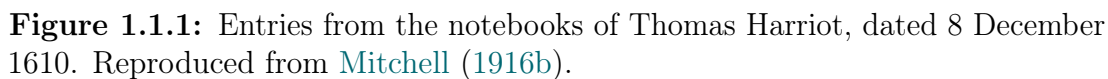
Chapter 1

Introduction

1.1 Context

It is known that magnetic activity is present in many structures of our universe. In particular, we can observe a wide range of stellar types with surface activity. However, the most studied case, is that of the Sun. Today, we know that one of the manifestations of the Sun's magnetic field are sunspots, but there is some dispute about who was the first to observe, discover and study them. The earliest known detailed record of solar observations dates from the 1600s. Galileo claimed to have discovered sunspots in the summer of 1610, however there is no evidence that Galileo recorded or studied sunspots until April, 1612 ([Mitchell, 1916a](#)). From 1609 to 1613 Thomas Harriot made numerous telescopic observations of the solar system, where a page from his notebook, dated on 8 December 1610, describes sunspot activity and includes several drawings (see [Figure 1.1.1](#)). Harriot left nearly 200 drawings of sunspots from the period of 1610 to 1612. Although he shared his sunspot observations with a group of correspondents in England, he did not publish them ([Mitchell, 1916b](#)).

In 1611, Johannes Fabricius and his father David Fabricius independently discovered sunspots, and Johannes published their observations in a pamphlet titled *On the Spots Observed in the Sun and their apparent rotation with the Sun* in the same year. In the treatise, Johannes discussed the observations made by himself and his father, without giving times or dates or providing illustrations



Christoph Scheiner began a serious study of sunspots in October of 1611, and his first publication was *Three Letters on Solar Spots*, which are letters that he wrote to his friend Mark Welser. In the first letter Scheiner commented the discovery of spots on the Sun, and since this thing was hardly to be believed, he did different tests to convince himself that the spots were not caused by some fault of the eyes or the tube, or of the atmosphere. So he observed with eight different telescopes and all showed the same spots in the same arrangement. From the lack of parallax it was concluded that the spots were not in the atmosphere of the Earth, and since they were evidently real, they must be in the Sun or around it. He assumed that the Sun does not rotate, because if it did, the spots should reappear due to the movement of the spots, but since this apparently did not happen, the Sun does not rotate. Therefore the conclusion was that the spots are not in the Sun, but are bodies or satellites which are between us and the Sun, and hence circulate around it. This last idea was discussed in the second letter, where he used the

¹https://chandra.harvard.edu/edu/formal/icecore/The_Historical_Sunspot_Record.pdf

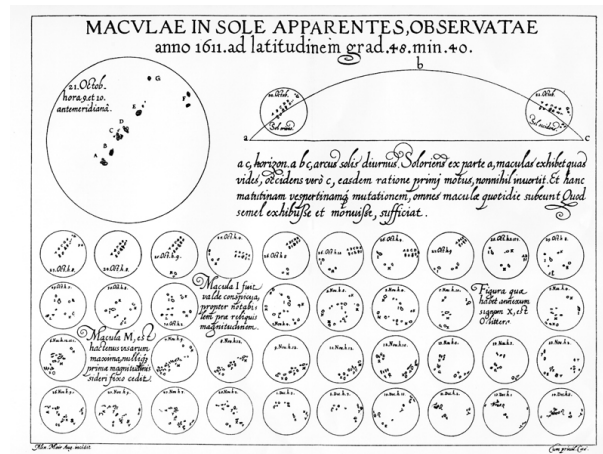


Figure 1.1.2: A illustration from Christoph Scheiner's book 'Rosa Ursina' published between 1626 and 1630, and among the first to show sketches of sunspots.

analogy of the spots circulating around the Sun to explain that Venus was orbiting the Sun. For the third letter Scheiner asserts that the spots are not in the Sun and require about fifteen days to transit the disk, and hence should reappear after an equal interval, but since they fail to do this they cannot be in the Sun. So he argued that the spots must be near the Sun, and states that the spots are large enough or they would be swallowed up by the radiation, also they are very opaque and thick, comparing them to Jupiter's satellites. The letter concluded that there were several planets between the Sun and Mercury, and that we would therefore see them pass when they were between the Earth and the Sun (Mitchell, 1916c). These letters initiated a dispute regarding the nature of sunspots. Galileo and Scheiner held opposing views on this topic. Galileo asserted that the spots are located at the surface of the Sun, or, if not, the distance is so small that it is imperceptible. The spots are not stars or bodies of long duration, as some disappear and new ones emerge, and they are irregular and changeable. After a few letters, Galileo's views prevailed. Cristoph Scheiner continued to observe and study sunspots, publishing *Rosa Ursina* in 1630. In his book *Rosa Ursina*, Scheiner published his observations and conclusions regarding sunspots (see Figure 1.1.2). Scheiner concurred with Galileo, accepting that sunspots are located on the solar surface. This work became one of the most important for more than a century as sunspot activity declined dramatically. Scheiner's work remained unchallenged until well into the 18th century (Mitchell, 1916d).

Michael Schwabe started to perform observations of sunspots without interruption

in 1826, and found that the number of groups is subject to a periodic recurrence (Schwabe, 1856). Between 1854 and 1855, the spots were confined to an equatorial belt, and in no case did they extend beyond the limits of 20° N or S latitude. Shortly afterwards, this cycle appears to have ceased, and two new belts of spots commenced abruptly on both hemispheres between 20 and 40 degrees (Carrington, 1858). In 1877, Maunder (1904) commenced data collection, during which he measured the areas of sunspots and faculae with a position micrometer. The results of his work in 1904-1905 enabled the first butterfly diagram to be created, which showed the sunspot centres in latitude for each rotation of the Sun. In the event that a sunspot centre was observed on one or more days during a given rotation at a given latitude, a line was drawn over that degree. Over time, this resulted in the creation of black vertical lines, with white gaps indicating the absence of sunspots as shown in the Figure 1.1.3.

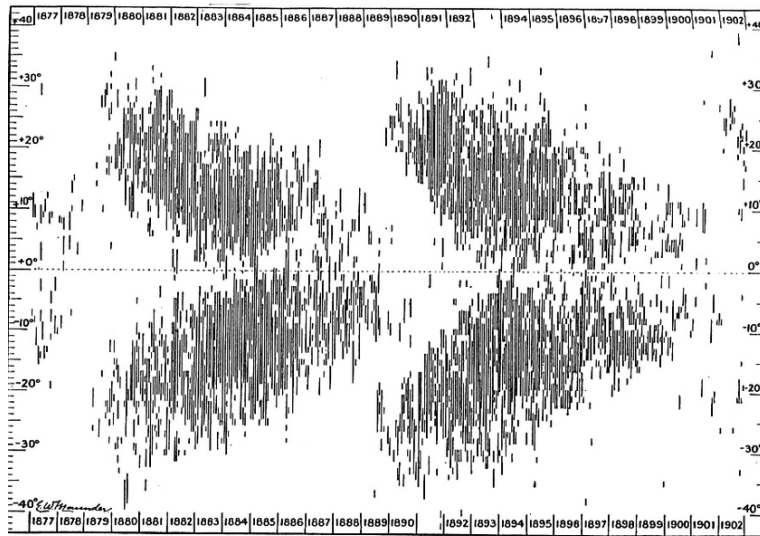


FIG. 8.—DISTRIBUTION OF SPOT-CENTRES IN LATITUDE, ROTATION BY ROTATION, 1877-1902.

Figure 1.1.3: Butterfly diagram published by Maunder (1904) for the distribution of spot-centres in latitude, rotation by rotation, between 1877 to 1902.

The investigation made by Hale (1908) was the first to discuss possible magnetic fields in sunspots through the study of their spectrum, which differs from the ordinary solar spectrum in several particulars. Today it is known that the Sun has a magnetic field that manifests itself in sunspots through Zeeman splitting of spectral lines (Brandenburg, 2007). Sunspots have an 11-year cycle, therefore, after the sunspot maximum, a main dipole field of reversed polarity is produced. The process repeats itself, so that the same configuration is reproduced after a

complete 22-year magnetic cycle (Babcock, 1961). The cycle is believed to be sustained by a dynamo.

Dynamo theory describes the process through which a rotating, convecting, and electrically conducting fluid acts to generate and maintain a magnetic field. This theory is used to explain the presence of long-lived magnetic fields in astrophysical bodies (e.g. Brandenburg and Subramanian 2005). The theory uses the magnetohydrodynamical (MHD) equations to investigate how the fluid can continuously regenerate magnetic fields. In the solar dynamo the conductive fluid is the ionized gas in the convective zone and in the tachocline, which is the transition region between the radiative and convective zones (Brandenburg, 1998). This will be discussed in more detail in Chapter 2.

1.2 Stellar magnetic fields

We can observe a wide range of stellar types with surface activity. In stars such as the Sun, which are situated in the main sequence, very few detection of magnetic fields have been reported. For instance, Marcy (1984) presents a methodology for the detection and measurement of magnetic fields in a number of G and K main sequence stars, employing Zeeman broadening observations. Some of his results are presented in the Figure 1.2.1.

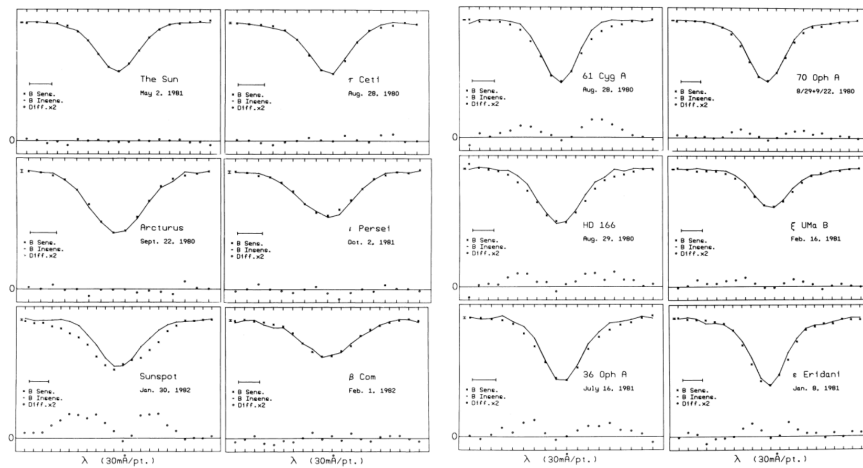


Figure 1.2.1: Observed Zeeman-sensitive (crosses) and Zeeman-insensitive (connected line) profiles for representative stars. Profiles with no evidence of Zeeman broadening, except for the Sun. Profiles where Zeeman broadening is evident.

Baliunas et al. (1995) presented chromospheric emission records and general conclusions on the variations of surface magnetic activity over time scales longer than 1 yr but shorter than a few decades. This is achieved through the monitoring of Ca II, H and K emission cores for a total of 111 stars of spectral types F2-M2 on or near the main sequence. The results of five years (five series, 23 nights) of spectropolarimetric observations of active stars are reported in the paper Donati et al. (1997). A total of 225 circularly (and four linearly) polarised spectra of 28 objects (21 active stars and seven calibration standards) were recorded using the Zeeman-Doppler imaging (ZDI) technique. This technique is employed to extract very small polarisation signatures, which are referred to as "least-squares deconvolution" (LSD). LSD shares certain similarities with other cross-correlation techniques in that it simultaneously uses information from most available spectral lines in the recorded wavelength domain. By employing this technique, the researchers were able to achieve enhanced sensitivity, which was a crucial factor in enabling the detection of the Zeeman signature. This is illustrated in Figure 1.2.2.

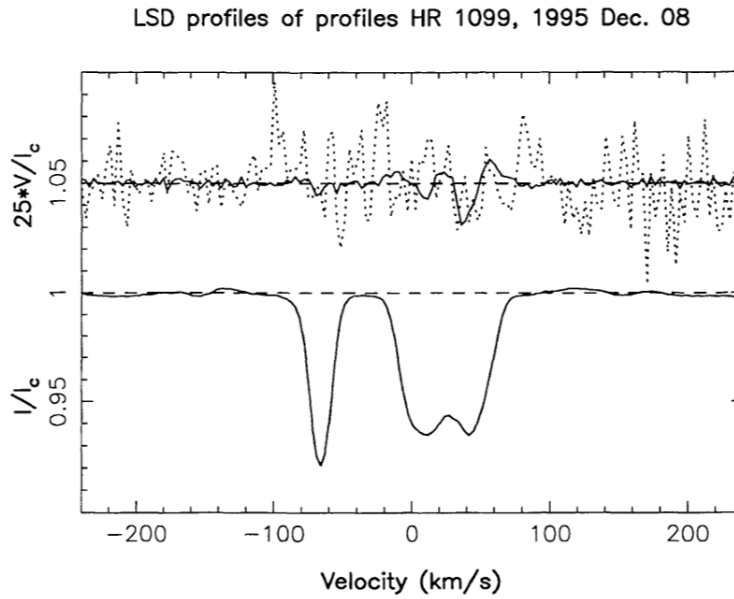


Figure 1.2.2: LSD of unpolarized (bottom curve, full line) and circularly polarized (top curve, full line) profiles of HR 1099 on 1995 December 8th. Compared with the old three-line analysis of Donati et al. (1992), whose application using the same data yields the dotted curve. Note that the polarization profiles are expanded by a factor of 25 and shifted upwards by 1.05 for display purposes (dashes lines).

In order to study the relationship between rotation, via the Rossby number ($Ro = P_{\text{rot}}/\tau$), where τ is the mass-dependent convective turnover time, and

stellar activity in the form of the X-ray luminosity ratio ($R_X = L_X/L_{\text{bol}}$), a sample of 824 solar-type and late-type stars is presented (see Figure 1.2.3), where the $R_X - \text{Ro}$ plane is subdivided into distinct regimes of coronal X-ray emission, namely unsaturated, saturated and supersaturated (Wright et al., 2011). In the unsaturated regime, the relationship between Ro and R_X probes the stellar dynamo efficiency. The results indicate that the one-slope fit for a sample of 36 stars is significantly steeper ($\beta = -2.70 \pm 0.13$) than the previously assumed value of $\beta = -2$. This finding lends weight to the theory of an interface dynamo (Parker, 1993). In the case of solar-type stars, the fractional differential rotation in the unsaturated regime has been found to scale with the angular rotation rate to the power of two-thirds, $\Delta\Omega/\Omega \propto \Omega^{0.63}$, (Montesinos et al., 2001). On the other hand, in the saturated regime, the mean saturation level was found to be $\log R_X = -3.13 \pm 0.22$. No significant correlation was found between the rotation period and Rossby number parameters and the activity level.

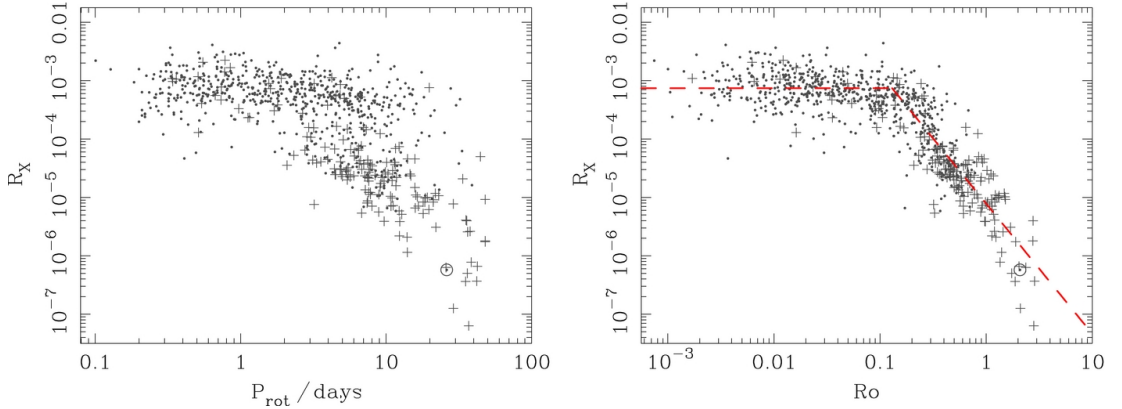


Figure 1.2.3: X-ray to bolometric luminosity ratio plotted against rotation period (left panel) and the Rossby number, $\text{Ro} = P_{\text{rot}}/\tau$ (right panel), for all stars in the sample with X-ray luminosities and photometric rotation periods. Stars known to be binaries are shown as plus symbols, and the Sun is indicated with a solar symbol. The best-fitting saturated and non-saturated activity-rotation relations are shown as a dashed red line in the right-hand panel.

For low-mass stars (at the bottom of the main sequence), we can identify M dwarfs, which represent the most common type of star in the galaxy (Winters et al., 2019). These stars have masses between 0.08 and $0.55 M_{\odot}$, and their temperatures range from 2500 to 4000 K (Pecaut and Mamajek, 2013), and they are the type of stars that we are interested in. In contrast to other stars, the properties of M dwarfs exhibit a considerable degree of variation between early and late types, spanning

from M0 to M9. This is due to the fact that at masses below $0.35 M_{\odot}$, the star is fully convective, whereas for larger mass, the star is partially convective (e.g. [Chabrier and Baraffe 1997](#)). M dwarfs present a lot of evidence of surface activity: flares, photometric rotational variability, enhanced chromospheric and coronal emission in X-rays, UV, and radio. Furthermore, in fully convective M dwarfs, a relationship between coronal activity and rotation has been observed ([Wright et al., 2018](#)). However, this relationship has been studied in considerable detail in partially convective stars, in which an interface dynamo may operate. This leaves aside fully convective stars, which lack the necessary shear layer between the radiative core and the convective envelope, and which have not been the subject of similar studies. [Wright et al. \(2018\)](#) presented X-ray observations for 19 slowly rotating fully convective stars, confirming the existence of fully convective stars in the unsaturated X-ray regime. These observations revealed that these objects follow the same rotation-activity relation as partially convective stars (see Figure 1.2.4). This finding suggests that both types of stars operate very similar dynamos that rely on the interplay of rotation and turbulent convection.

In [Reiners et al. \(2022\)](#) the mean surface magnetic fields of 292 M dwarfs were measured and presented in comparison with radiative transfer calculations. The results demonstrate a correlation between the mean field strength $\langle B \rangle$, and the Rossby number Ro , which exhibited similarities to the well-documented rotation-activity relation. For slowly rotating stars, the magnetic flux (Φ_B) was found to be proportional to the rotation period P . In contrast, for rapidly rotating stars, the mean surface fields were observed to reach a level consistent with the available kinetic energy. Furthermore, they identified a close correlation between the non-thermal coronal X-ray emission, the chromospheric $H\alpha$ and $CaII$ H&K emission, and the magnetic flux. In conclusion, these relations demonstrate, on the basis of empirical evidence, that the rotation-activity relation can be traced back to a dependence of the magnetic dynamo on rotation.

Furthermore, it is pertinent to mention that [Morin et al. \(2010\)](#) conducted a preliminary spectropolarimetric survey of a limited number of active M dwarfs, where two distinct categories of magnetic topologies were identified: firstly, strong axisymmetric dipole fields and, secondly, weak fields that typically exhibit a significant non-axisymmetric component and, on occasion, a significant toroidal component. [Hawley et al. \(2014\)](#) made the analysis of the presence of $H\alpha$ through

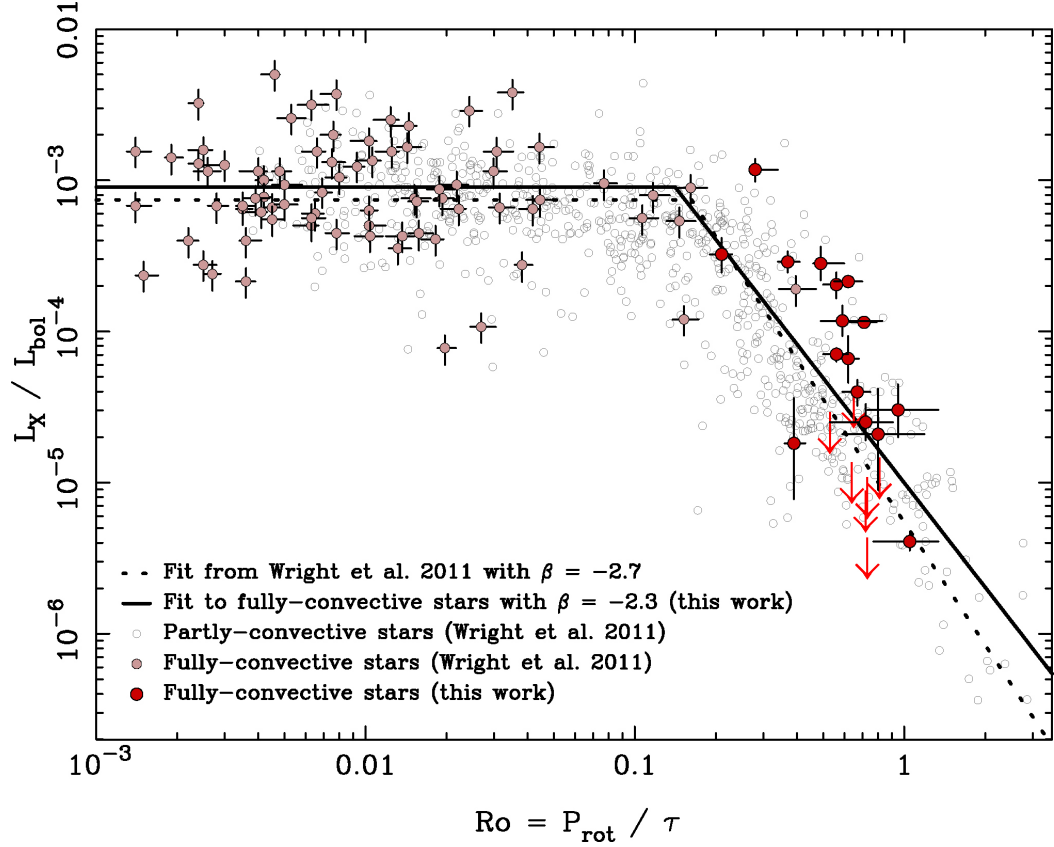


Figure 1.2.4: X-ray to bolometric luminosity ratio, L_X/L_{bol} , plotted against the Rossby number, $\text{Ro} = P_{\text{rot}}/\tau$, for the fully convective stars observed as part of the work by Wright et al. (2018) (large red points), the fully convective stars included in the sample of Wright et al. (2011) (medium, light red points), and the remaining partly convective stars from that sample (grey empty circles). Error bars are shown for all fully convective stars. Upper (3σ) limits are shown for the undetected fully convective stars observed as part of this work as red arrows. The best-fitting activity-rotation relations found for fully convective stars in this work ($\beta = 2.3$ and $\text{Ro}_{\text{sat}} = 0.14$, solid line) and from Wright et al. (2011) ($\beta = 2.7$ and $\text{Ro}_{\text{sat}} = 0.16$, dotted line) are shown.

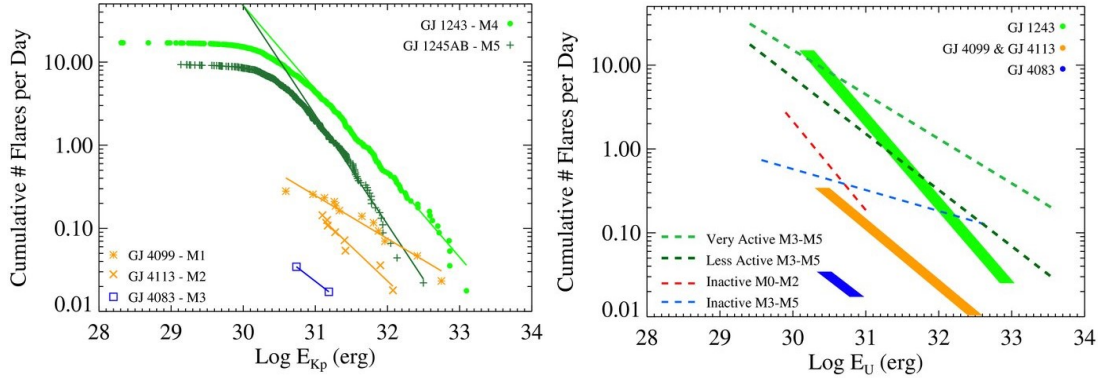


Figure 1.2.5: Left panel shows the cumulative flare frequency distributions, together with least-squares power-law fits, for five M dwarfs stars (active and inactive) from the Kepler sample, using the native Kepler flare energies. The right panel uses the Kepler to U-band energy to compare the Kepler results with ground-based data. The mean power-law relations for nearby M dwarfs are shown with dashed lines. (See [Hawley et al. 2014](#) for detail).

observations of M dwarfs with Kepler in a *short-cadence mode*, which is a data acquisition mode used by the Kepler space telescope that allows for the observation of multiple targets in a short time frame, with a distinction made between active and inactive stars. This is due to the fact that M dwarfs exhibit a wide range of flare frequencies and energies. Active stars are of mid-M spectral type, have numerous flares, and exhibit well-defined rotational modulation due to starspots. In contrast, inactive stars are of early M-type, exhibit less starspots signatures, and have fewer flares. Figure 1.2.5 illustrates this. Other features will be reviewed in Section 2.5.

Other types of stars that exhibit magnetic activity are T Tauri stars, which display indications of the existence of magnetic fields with surface strengths of several kilogauss (e.g. [Guenther and Emerson 1996](#); [Guenther et al. 1999](#)), in G-K giants (red giants) magnetic fields have been detected and the results seem to indicate that the magnetic fields of these giants are produced by a dynamo ([Aurière et al., 2015](#)). For stars more massive than $1.5 M_{\odot}$ (upper-main-sequence stars) and therefore have radiative envelope and a convective core, have been detected the presence of strong magnetic fields ($\sim 1\text{kG}$) in some A- and B-type stars ([Babcock, 1947](#)), where their origin is not yet understood (e.g. [Moss 2001](#); [Bagnulo et al. 2006](#)).

1.3 Previous works with simulations

In addition to observations, a useful tool for the study of these phenomena are MHD simulations, which help us to understand a little more in depth what happens in the interior of the stars. Three-dimensional numerical simulations of stars have been developed with the objective of understanding the magnetic fields, dynamos and convection as a function of stellar parameters, such as mass, age, rotation and dimensionless numbers that describe the physics in stellar interiors. The first simulations were focused on reproduce the Sun's magnetic cycle. For example in [Glatzmaier \(1985\)](#) a spherical shell of ionised gas constrained by solar gravity, luminosity, composition and mean rotation rate was modelled in order to examine the properties of a non-stationary dynamo for an anelastic gas with significant density stratification. The results demonstrated that the simulated angular velocity reached its maximum value in the equatorial region, which is in accordance with the Doppler measurements of the solar surface rotational velocity. Moreover, the angular velocity was observed to decrease with depth below the surface, as inferred from the division of the rotational frequency of the solar oscillations. On the other hand, some simulations are used to try to understand how the solar dynamo works, as in the case of [Brandenburg \(2005\)](#) where direct simulations of the dynamo action in a slab with turbulence and shear are presented to discuss the filling factor and the tilt angles of the bipolar regions in such a model. In order to evaluate the arguments for and against of a solar dynamo situated in the tachocline, alternative concepts for dynamos operating within the convection zone or potentially even in the near-surface shear layer are also presented.

Other studies have concentrated on fully convective stars, such as that presented in [Browning \(2008\)](#), which presented three-dimensional nonlinear magnetohydrodynamic simulations of the interiors of fully convective M dwarfs, considering a mass of $0.3M_{\odot}$, with a spherical computational domain extending from 0.08 to 0.96 times the stellar radius. This study found that fully convective stars can generate kG-strength magnetic fields without the aid of a tachocline. Figure 1.3.1 shows how the intensity of the mean fields changes with time, with some regions exhibiting exceptionally strong field structures, while others remain comparatively quiescent.

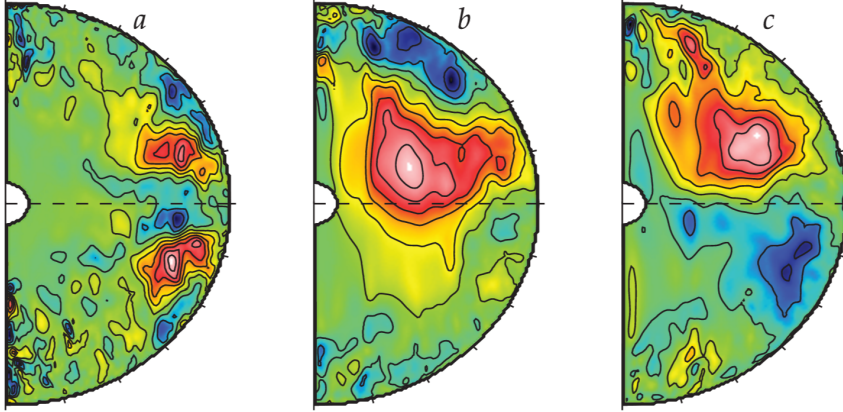


Figure 1.3.1: Azimuthally averaged B_ϕ as contour plots in radius and latitude at three instants in the evolution of case Cm of [Browning \(2008\)](#). The three renderings sample (a) a time prior to the saturation of the volume-averaged magnetic energy density, and times roughly (b) 8500 and (c) 11,000 days later. Polarity is indicated by the color map, with reddish tones positive (prograde polarity) and bluish tones negative (retrograde).

[Brown et al. \(2020\)](#) performed simulations of fully convective M dwarfs in spherical coordinates including the center of the star, finding a hemispheric dynamo solution for every computed case, and at all times the global poloidal fields have a dipolar component, with strong contributions for higher order modes. [Dobler et al. \(2006\)](#) presented a series of MHD simulations that were conducted to investigate the behaviour of fully convective, rotating spheres with volume heating concentrated near the centre and cooling at the surface. The simulations were performed using a Cartesian grid; this means that the sphere is embedded in a cubic box (see Figure 1.3.2). The utilization of such a geometry obviates the singularity of coordinates at the centre when employing spherical coordinates, and circumvents the necessity for costly transformations between spherical harmonics. Furthermore, this approach has been demonstrated to be advantageous in terms of computational simplicity and the efficiency of numerical parallelization. The magnetic field generated by the dynamics was found to saturate in the equipartition field strength near the surface. In the interior of the star, the field is dominated by small-scale structures, whereas outside the sphere, the field is dominated by the global scale. Azimuthal averaging of the field reveals the presence of a large-scale field of smaller amplitude also in the interior of the star. The internal angular velocity exhibits a tendency to be constant along the cylinders, with a pattern of faster rotation at the poles and slower rotation at the equator, which is in opposition to the typical solar

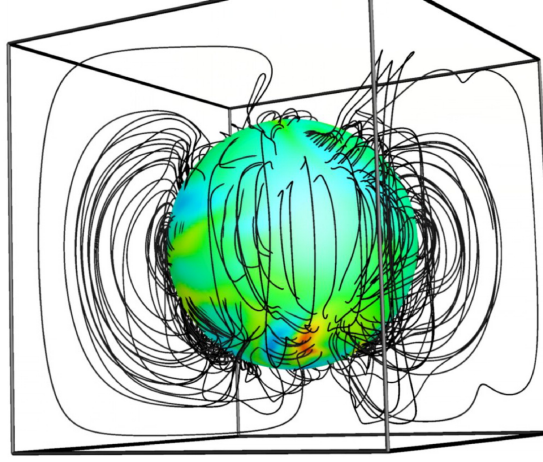


Figure 1.3.2: Three-dimensional visualization of the magnetic field for run 2c of Dobler et al. (2006) at $t = 2600 [t]$ (saturated phase). Magnetic field lines are shown, together with the surface of the sphere.

rotation pattern.

Käpylä (2021) described the star-in-a-box model, in which a spherical star is immersed inside a Cartesian cube. This model was based on the setup described by Dobler et al. (2006) and was used to investigate convection, differential rotation, and dynamos as functions of rotation in fully convective stars. Three-dimensional hydrodynamic and MHD numerical simulations were done to achieve this goal. He concluded that the model is consistent with simulations of partially convective late-type stars in spherical shells. This similarity between partially and fully convective stars suggests that the processes generating differential rotation and large-scale magnetism are insensitive to the geometry of the star. In their study, Ortiz-Rodríguez et al. (2023) studied the magnetic and flow properties of fully convective M dwarfs as a function of rotation period, magnetic Reynolds number and Prandtl number. They employed stellar parameters of a main-sequence M5 dwarf with a mass of $0.21 M_{\odot}$ at three rotation rates corresponding to rotation periods (P_{rot}) of 43, 61, and 90 days (sets of simulations A, B and C of the Figure 1.3.3 respectively), and varied the magnetic Prandtl number in the range from 0.1 to 10. The non-ideal MHD equations were solved using the `Pencil Code`, in accordance with the model described by Käpylä (2021). The results are presented in Figure 1.3.3, where the value of Pr_M increases from left to right for all the sets of simulations. It is evident that the large-scale magnetic field behaviour is influenced by rotation and magnetic Prandtl number (for further details see

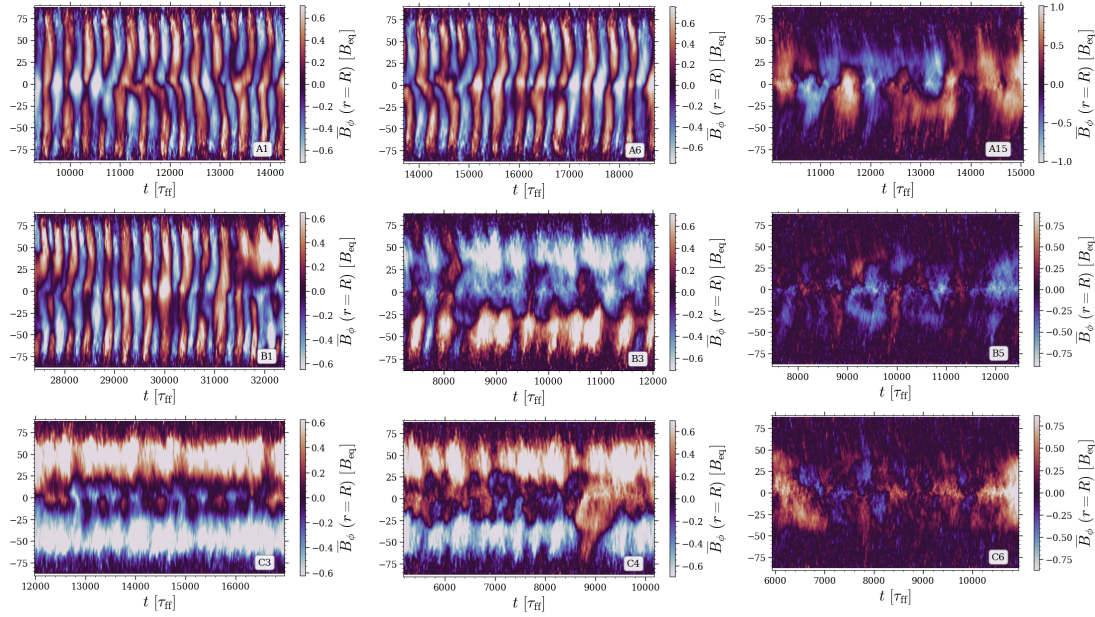


Figure 1.3.3: Azimuthally averaged toroidal magnetic field near the surface of the star as a function of time from [Ortiz-Rodríguez et al. \(2023\)](#). The names of the simulations are indicated at the bottom right of each panel.

[Ortiz-Rodríguez et al. 2023](#)).

In the case of partially convective stars, such as solar-like stars, [Brown et al. \(2011\)](#) conducted simulations of dynamo action in rapidly rotating Suns using the three-dimensional magnetohydrodynamic anelastic spherical harmonic (ASH) code to investigate dynamo action occurring within the convective envelope of a solar-type star rotating at five times the solar rotation rate. The results demonstrated that dynamo action generates globally organised magnetic fields within the middle of the convection zone. The dynamo exhibits cyclic activity and undergoes quasi-periodic polarity reversals, where both the global-scale poloidal and toroidal fields change in sense on a roughly 1500-day timescale. These patterns of magnetic activity arise spontaneously from turbulent flow, and thus, the competing processes of magnetic field creation and destruction within the simulations are evaluated in detail to ascertain their contribution to the global-scale reversals (see Figure 1.3.4).

The study of [Warnecke et al. \(2016\)](#) examines the impact of a stably stratified coronal envelope on rotating turbulent convection, differential rotation and large-scale dynamo action in spherical wedge models. To this end, the magnetohydrodynamic equations were solved in a two-layer model comprising

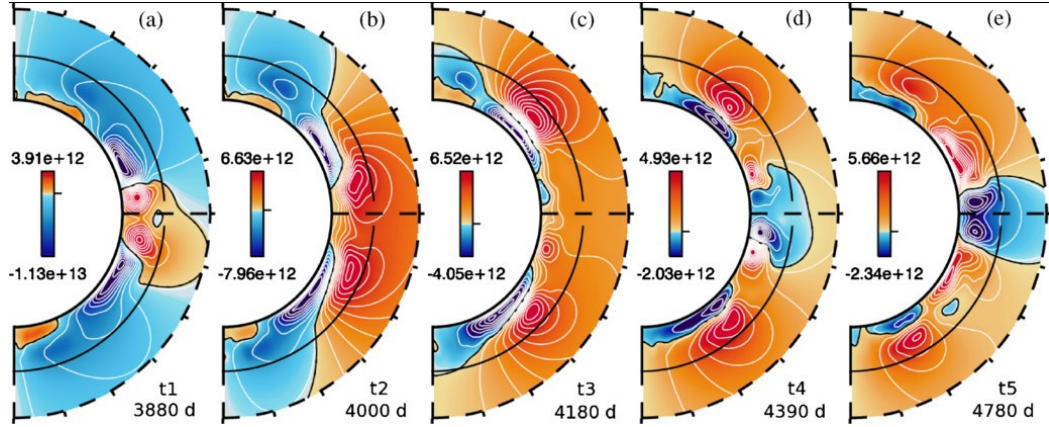


Figure 1.3.4: Evolution of poloidal field during a polarity reversal in case D5 of [Brown et al. \(2011\)](#). Shown are the vector potential of the mean poloidal field $\langle A_\phi \rangle$ in case D5 at times t1-t5 (with corresponding days noted). Colors denote amplitude and polarity (red clockwise; blue, counterclockwise); units are G. The stellar surface is indicated with a solid line, and the poloidal field is extended with a potential field extrapolation to $1.3R_\odot$ (dashed surface).

an unstable stratification below the surface, representing the convection zone, and a stably stratified coronal envelope above. It was found that the presence of a free surface resulted in a change to the evolution of the magnetic field, with the toroidal field becoming concentrated closer to the surface (see Figure 1.3.5). Nevertheless, the migration direction of the mean magnetic field can be explained by the Parker-Yoshimura rule ([Parker, 1955](#); [Yoshimura, 1975](#)) in all simulations, which is in agreement with previous findings.

Both dynamo theory and observations of large-scale stellar magnetic fields suggest a shift from quasi-axisymmetric configurations at solar rotational velocities to non-axisymmetric configurations for fast rotation. To understand this transition through numerical simulations, three-dimensional simulations of turbulent magnetohydrodynamic convection in spherical shells were performed, where rotation rates between 1 and 31 times the solar value was considered ([Viviani et al., 2018](#)). They found that a transition from axi- to non-axisymmetric solutions occurs at approximately 1.8 times the solar rotation rate. This transition coincides with a change in the rotation profile, whereby it transitions from an anti-solar to a solar-like differential rotation, with a faster equator and slower poles. In the case of slow rotation, the axisymmetric mode is dominant, whereas in the opposite case of fast rotation, the non-axisymmetric mode is dominant. The axisymmetric mode gives rise to latitudinal dynamo waves with polarity reversals, whereas

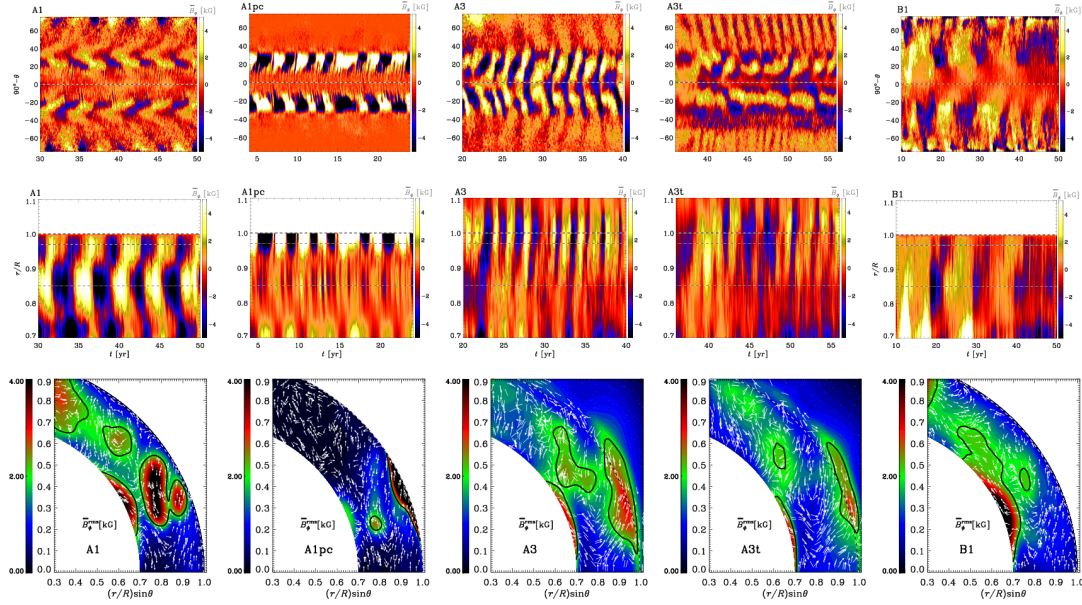


Figure 1.3.5: Mean toroidal magnetic field B_ϕ evolution as time-latitude (butterfly) diagram plotted at a radius $r = 0.98 R$ (top row) and time-radius diagram plotted at a 25° latitude (middle row) in kG during a 20 yr interval in the saturated stage for Runs A1, A1pc, A3, A3t, and B1 of [Warnecke et al. \(2016\)](#).

the non-axisymmetric mode frequently exhibits a gradual shift in the rotational reference frame and a cyclical variation in the strength of the active longitudes between the different hemispheres. The majority of dynamo solutions obtained in this study demonstrate cyclic variability resulting from the influence of latitudinal or azimuthal dynamo waves. In this regard, an activity-period diagram reveals that the cycle lengths normalised by the rotation period fall into two distinct categories, contingent on the rotation rate or the level of magnetic activity. In 2020 ([Navarrete et al., 2020](#)), two non-ideal compressible MHD simulations of the magnetic dynamo in a solar-mass star were presented, one at three times the solar rotation rate (*slow rotator*) and the other at 20 times the solar rotation rate (*fast rotator*). This was done to account for the high rotation rates observed in close binary systems. The computational domain employed is a spherical wedge, lacking the poles. This enables higher spatial resolution, though it necessitates the exclusion of connecting fluxes through the poles and the introduction of artificial boundaries at high latitudes. For the slow rotator, quasi-periodic behaviour was observed in both the magnetic field and the stellar quadrupole moment. In contrast, the behaviour of the magnetic field and quadrupole moment is more complex for the fast rotator ([Navarrete et al., 2020](#)). Two years later, [Navarrete](#)

[et al. \(2022\)](#) conducted a simulation of the magnetic field of the star in the rapidly rotating regime, with rotation periods of 8.3, 1.2, and 0.8 days. The objective was to study the connection between the quadrupole moment and the magnetic field using spherical harmonic decomposition. In the simulation with a rotation period of 1.2 days, a more complex magnetic fields were observed, in which the large-scale non-axisymmetric field was dominant throughout the simulation, while the axisymmetric component was predominantly hemispherical. This resulted in variations in the density field that followed the asymmetry of the magnetic field with respect to the equator. In contrast, two magnetic fields exhibit reduced temporal variability and enhanced symmetry with respect to the equator. Consequently, the associated density variations are diminished, resulting in only minor fluctuations in the quadrupole moment.

In a recent paper [Käpylä \(2022\)](#) applied a star-in-a-box model to solar-like stars with different rotation rates. The simulations result in the production of solar-like differential rotation, characterised by a fast equator and slow poles, and magnetic activity that bears resemblance to that observed in the Sun, exhibiting equatorward migrating activity at the surface. Moreover, the ratio of rotation to cycle period remains nearly constant, as the rotation period decreases in the limited sample under consideration. While the exact excitation mechanism of the dynamos in the simulations is not yet clear, it was demonstrated that it is plausible that the greater freedom that the magnetic field has due to the inclusion of the radiative core and regions exterior to the star are important in shaping the dynamo. Figure 1.3.6 illustrates the concentration of strong magnetic fields near the surface of the star, both in the region exterior to the tangent cylinder and in the interface between the radiative and convective zones (CZ) within the tangent cylinder. Additionally, strong magnetic fields have been observed within the CZ at higher latitudes.

1.4 This work

The aim of this work is to test a new computational model for the dynamo of partially convective M dwarfs. This model consists of three-dimensional magnetohydrodynamical (MHD) numerical simulations using the star-in-a-box model developed by [Käpylä \(2021\)](#), in which a spherical star is immersed inside a

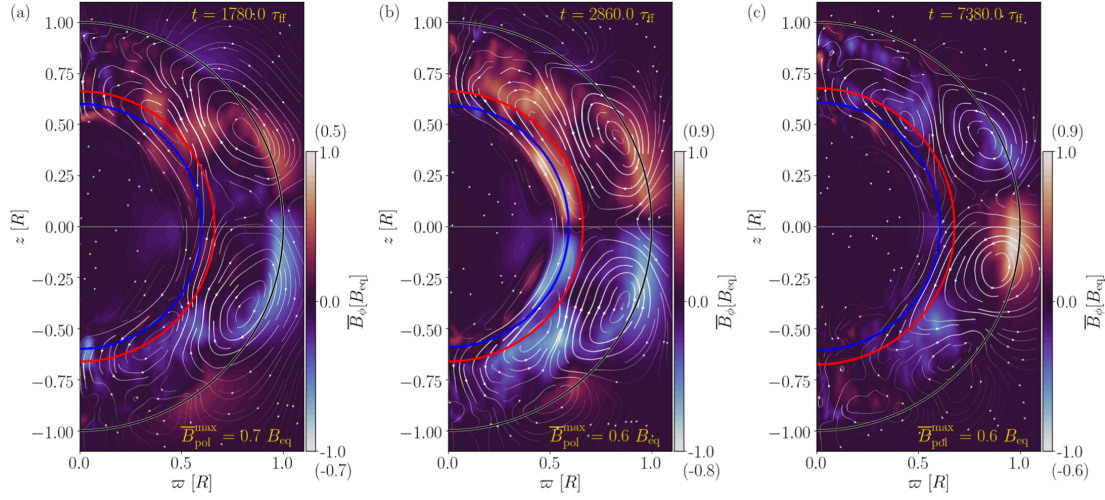


Figure 1.3.6: Azimuthally averaged magnetic fields in units of B_{eq} in cylindrical coordinates (φ, z) for runs A, C, and D as functions of time simulated by Käpylä (2022). The color contours show the toroidal fields, and the arrows indicate the poloidal fields.

Cartesian cube. In this model the non-ideal equations of MHD are solved by means of the Pencil Code (Pencil Code Collaboration et al., 2021). The use of a Cartesian box is beneficial because it includes the entire star, allowing areas outside the star to be changed at our convenience to see how this affects magnetic fields and fluxes, which can be advantageous compared to spherical shell models. We will change the magnetic boundary condition to see how this affects the dynamo, and also study different rotation rates.

This thesis is presented as follows: in Chapter 2 the most important concepts about magnetohydrodynamics and the dynamo mechanism are given, together with the flux properties that are important for large-scale magnetic fields, and the main characteristics of M dwarfs. In Chapter 3 presents the computational methods used, as well as the codes, the model used to perform the simulations and some important parameters. The results are presented in Chapters 4. Finally, the main conclusions are presented in Chapter 5.

Chapter 2

Convection and dynamos

This chapter gives a brief overview of stellar physics, including energy transport mechanisms, dynamo theory, and an explanation of magnetohydrodynamics. A brief description of dimensionless parameters that are useful for characterising systems in order to understand how magnetic fields evolve in stars. Large-scale flows in stars, which are important for large-scale magnetic fields, are also discussed.

2.1 Stellar structure

Stellar structure models provide a detailed account of the internal structure of a star and enable the forecasting of characteristics such as luminosity, color, and future evolution. The internal structures of different classes and ages of stars exhibit variation, which is reflective of their elemental compositions and energy transport mechanisms (see Figure 2.1.1). To illustrate, in stars such as the Sun, the radiated energy is generated in the hot, dense core through nuclear fusion. The transfer of energy to the stellar surface can occur via a number of different means, depending on the local physical situation. The dominant mechanisms of transfer are radiation, conduction, and convection. The process of energy transport by convection is distinguished from other processes by the fact that it drives large-scale mass movements. This renders convection a particularly intriguing phenomenon, especially in the astrophysical context, where the convective fluid is typically fully ionised. The following subsections will provide a more detailed examination of the radiation (Sect. 2.1.1) and convection (Sect. 2.1.2) mechanisms that are involved in the specific type of star that is the main focus of this study.

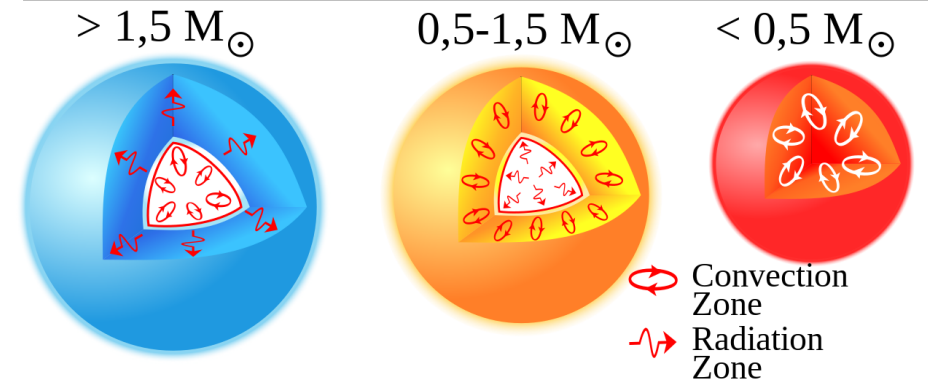


Figure 2.1.1: The different structures of high-mass, intermediate-mass and low-mass stars.

2.1.1 Radiation

It has been demonstrated that in an isolated region of the universe, matter and radiation gravitate towards a state of thermodynamic equilibrium. This occurs within the interior of a star when photons, carrying energy from the core, are perpetually scattered, absorbed and re-emitted in a random pattern by particles such as electrons. This allows the radiation field to become isotropic, and the energy distribution of the photons can be characterised by the Planck function. It can therefore be posited that within the interior of the star, the radiation can be considered to be approximately that of a black body. So it can be concluded that a state of local thermodynamic equilibrium (LTE) is reached within a star. This implies that within a region that is considerably smaller than the dimensions of a star ($\ll R_*$), yet larger than the typical distance between particle interactions (including both gas particles and photons), and there exists a well-defined local temperature that characterises the statistical distributions of particles.

For example, let us estimate the mean free path ℓ_{ph} of a photon at an "average" point inside a star like the Sun:

$$\ell_{\text{ph}} = \frac{1}{\kappa \rho}, \quad (2.1.1)$$

where κ is a mean absorption coefficient, i.e., a radiative cross section per unit mass averaged over frequency, and ρ is the density of the star. Considering $\bar{\rho}_{\odot} = 3 M_{\odot}/4\pi R_{\odot}^3 = 1.4 \text{ g cm}^{-3}$ and $\kappa \approx 0.4 \text{ cm}^2 \text{g}^{-1}$, values given in [Kippenhahn](#)

et al. (2013), the mean free path for the Sun is $\ell_{\text{ph}} \approx 2\text{cm} \ll R_{\odot}$, which means that stellar matter is very opaque to radiation.

If we estimate the temperature difference over a distance ℓ_{ph} , i.e., between emission and absorption, for the Sun, we get

$$\Delta T \approx \frac{dT}{dr} \ell_{\text{ph}} \approx \frac{T_c}{R} \ell_{\text{ph}} \approx \frac{10^7}{10^{11}} \approx 10^{-4} \text{K}. \quad (2.1.2)$$

It can be seen that the relative anisotropy of the radiation at a point with a temperature of $T = 10^7$ K, which is a typical interior temperature of a star, and therefore $\Delta T/T \sim 10^{-11}$, which is a tiny fraction, but is capable of transporting the entirety of the Sun's energy flux (Pols, 2011).

These estimates allow us to treat radiation as a diffusion process, which yields an enormous simplification of the formalism. A more rigorous derivation can be found in e.g. Weiss et al. (2004). Thus the radiative energy flux is given by

$$\mathbf{F}_{\text{rad}} = -K_{\text{rad}} \nabla T, \quad (2.1.3)$$

where K_{rad} is the radiative heat conductivity, which represents the coefficient of conduction for this radiative transport, and is defined as:

$$K_{\text{rad}} = \frac{4acT^3}{3\kappa\rho}, \quad (2.1.4)$$

here $a = \frac{8\pi^5 k^4}{15h^3 c^3}$ is the radiation density constant and c is the speed of light. Owing to the spherical symmetry of the problem, $\mathbf{F}_{\text{rad}}$ has only a radial component $F_{\text{rad}}(r) = |\mathbf{F}_{\text{rad}}| = F_{\text{rad}}$, and thus $\nabla T = (\partial T / \partial r) \hat{\mathbf{r}}$. Then, we can express the energy flux in terms of the local luminosity $l = 4\pi r^2 F_{\text{rad}}$, obtaining:

$$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho l}{16\pi a c r^2 T^3}. \quad (2.1.5)$$

Considering $dm = 4\pi r^2 \rho dr$, we can rewrite the expression above as

$$\frac{\partial T}{\partial m} = -\frac{3\kappa l}{64\pi^2 a c T^3 r^4}, \quad (2.1.6)$$

which is the basic equation for radiative transport of energy, and becomes invalid when one approaches the surface of the star, i.e., $\ell_{\text{ph}} \geq R_*$. The equation 2.1.6 can

be rewritten in a form which will be convenient. Assuming hydrostatic equilibrium, we divide 2.1.6 by $\partial P/\partial m = -Gm/4\pi r^4$ and obtain

$$\frac{\partial T/\partial m}{\partial P/\partial m} = \frac{3}{16\pi acG} \frac{\kappa l}{mT^3}. \quad (2.1.7)$$

From this expression (2.1.7) we understand that in a star in hydrostatic equilibrium and transporting energy by radiation, $(dT/dP)_{\text{rad}}$ is a gradient describing the temperature variation with depth, represented by the pressure, which increases inwards. If we use the customary abbreviation

$$\nabla_{\text{rad}} := \left(\frac{d \ln T}{d \ln P} \right)_{\text{rad}}, \quad (2.1.8)$$

(2.1.7) can be written in the form

$$\nabla_{\text{rad}} = \frac{3\kappa}{16\pi acG} \frac{lP}{mT^4}. \quad (2.1.9)$$

In addition, ∇_{rad} can be used in more general cases, such as other modes of energy transport, e.g., convective deviations from hydrostatic equilibrium, to provide a criterion for determining when the star (or a particular region of the star) is in radiative equilibrium.

2.1.2 Convection

The process of convective energy transport is defined as the exchange of energy between regions of varying temperatures in a system that is characterised by dynamic instability. Such energy exchange occurs through fluid motion, whereby warmer mass elements (*bubbles* or *convective elements*) move upwards, while cooler ones move downwards. The transfer of energy from the moving mass elements to the surrounding medium occurs as a result of this convective process. Due to the high density of stellar interiors, convective transport can be very efficient, and in stars with convective envelopes like the Sun, convection can be observed in the photosphere. It is crucial to acknowledge that convection can be conceptualised as a *dynamical instability*, based on the premise that the moving mass elements have insufficient time to exchange substantial quantities of heat with their surrounding environment, resulting in an adiabatic movement.

The following describes an analytical method for identifying convective instability in an atmosphere or a region of a star. It also provides an overview of the mixing length concept, which is commonly employed in models of stellar structure and evolution to describe convection.

2.1.2.1 Mixing length theory

Equation 2.1.9 provides the gradient ∇_{rad} that would be maintained in a star if the entirety of the luminosity l were to be transported by radiation. In the case of convection contributing to the energy transport, the actual gradient ∇ will be different. An estimation of how much energy can be transported by convection (F_{conv}) and, related to this, what is the actual temperature gradient ∇ inside a convective region have to be provided.

It is first necessary to introduce the concept of *mixing length*, which was originally developed to describe incompressible terrestrial flows (e.g. Taylor 1915; Prandtl 1925; Schmidt 1926). The model was initially employed in the context of stellar convection by Biermann (1932) and Cowling (1935). In the mixing length theory, convective motions are approximated by blobs of gas that travel up or down over a radial distance ℓ_m (the mixing length). Subsequently, the blobs dissolve into their surroundings and lose their identity.

In accordance with the formulation proposed by Vitense (1953), the mixing length is proportional to the pressure scale height, which is the radial distance over which the pressure changes by an e-folding factor, as indicated by:

$$\ell_m = \alpha_{\text{MLT}} H_P, \quad (2.1.10)$$

where $H_P = -1/(d \ln P/dr)$ is the pressure scale height, and α_{MLT} is a constant at the order of unity. By employing the approximation (2.1.10), a simple expression can be derived for the convective velocity and temperature fluctuations, which can then be used to calculate the convective energy flux, (see e.g. Schleicher and Mennickent, 2017),

$$F_{\text{conv}} = \rho c_P T \left(\frac{\ell_m}{H_P} \right)^2 \sqrt{\frac{1}{2} g H_P (\nabla - \nabla_{\text{ad}})^{3/2}}, \quad (2.1.11)$$

where ρ is the density, c_P is the heat capacity at constant pressure, T is the

temperature, g is the gravitational acceleration, and ∇_{ad} represents the *adiabatic temperature variation* in a specific gas element undergoing a change in pressure, that is

$$\nabla_{\text{ad}} := \left(\frac{\partial \ln T}{\partial \ln P} \right)_s, \quad (2.1.12)$$

where the subscript s indicates that the definition is valid for constant entropy. It should be noted that $(\nabla - \nabla_{\text{ad}})$ represents the phenomenon of *superadiabaticity*, defined as the degree to which the actual temperature gradient ∇ exceeds the adiabatic value (Pols, 2011). Once the flux is known, it is possible to solve for the convective temperature gradient (see e.g. Stix, 2002).

2.1.2.2 Schwarzschild criterion

If we consider the condition for *stability* from Kippenhahn et al. (2013)

$$\left(\frac{d \ln T}{d \ln P} \right)_s < \left(\frac{d \ln T}{d \ln P} \right)_e + \frac{\varphi}{\delta} \left(\frac{d \ln \mu}{d \ln P} \right)_s, \quad (2.1.13)$$

where $\delta := - \left(\frac{\partial \ln \rho}{\partial \ln T} \right)$, and $\varphi := \left(\frac{\partial \ln \rho}{\partial \ln \mu} \right)$.

We can define three new derivatives, similar to the previously defined quantities ∇_{rad} and ∇_{ad} ,

$$\nabla := \left(\frac{d \ln T}{d \ln P} \right)_s, \quad \nabla_e := \left(\frac{d \ln T}{d \ln P} \right)_e, \quad \nabla_\mu := \left(\frac{d \ln \mu}{d \ln P} \right)_s. \quad (2.1.14)$$

The subscripts s indicate that the derivatives are to be taken in the surrounding material, which represent the variation of T and μ with depth. ∇_e describes the variation of T in a convective element (or *bubble*) during its motion, where the position of the element is measured by P . In this sense, ∇_{ad} and ∇_e are similar, since they describe the temperature variation of a gas subjected to pressure variations; on the other hand, ∇_{rad} and ∇ describe the spatial variation of T and μ in the surroundings.

Using the definitions (2.1.14) the condition (2.1.13) for stability becomes

$$\nabla < \nabla_e + \frac{\varphi}{\delta} \nabla_\mu. \quad (2.1.15)$$

If we consider a layer that effectively transports all the energy by radiation, we

can state that the gradient ∇ is equal to ∇_{rad} . Furthermore, if we assume that the elements change adiabatically, i.e. $\nabla_e = \nabla_{\text{ad}}$, we can conclude that the radiation layer will be stable if

$$\nabla_{\text{rad}} < \nabla_{\text{ad}} + \frac{\varphi}{\delta} \nabla_{\mu}, \quad (2.1.16)$$

a form known as the *Ledoux criterion* for dynamical stability. In the case of the region in question exhibits a homogeneous chemical composition, i.e., $\nabla_{\mu} = 0$, thereby allowing for the application of the simplified *Schwarzschild criterion* (Schwarzschild, 1906) for dynamical stability,

$$\nabla_{\text{rad}} < \nabla_{\text{ad}}, \quad (2.1.17)$$

where ∇_{rad} is the radiative temperature gradient (2.1.9), and ∇_{ad} represents the adiabatic temperature variation (2.1.12). Accordingly, in this criterion (2.1.17) if the left-hand side is larger than the right, i.e.

$$\nabla_{\text{rad}} = \frac{3}{16\pi acG} \frac{\kappa l}{m} \frac{P}{T^4} > \nabla_{\text{ad}}, \quad (2.1.18)$$

the layer is dynamically unstable, which means that convection occurs. This instability can be caused either by a large flux of matter or by the matter with high degree of opacity. Thus, small perturbations will grow to a finite amplitude until the whole region boils, with convective motions carrying some of the flux. In the case that these criteria indicate stability, it can be concluded that convective motions will not occur. Consequently, the entire flux will be transported by radiation, whereby the actual gradient at the specified location is equal to the radiative gradient ($\nabla = \nabla_{\text{rad}}$).

For stars on the main sequence there is convection in the following cases (see fig. 2.1.1):

- ▷ $M > 1.4M_{\odot}$: Convective core and radiative envelope.
- ▷ $M < 1.4M_{\odot}$: Convective envelope and radiative core.
- ▷ $M < 0.3M_{\odot}$: Fully convective.

It should be noted that (2.1.17) is a local criterion. It is a highly practical approach, as it allows for the evaluation of a given location using only the local values of P , T and ρ , without the need to consider other regions of the star.

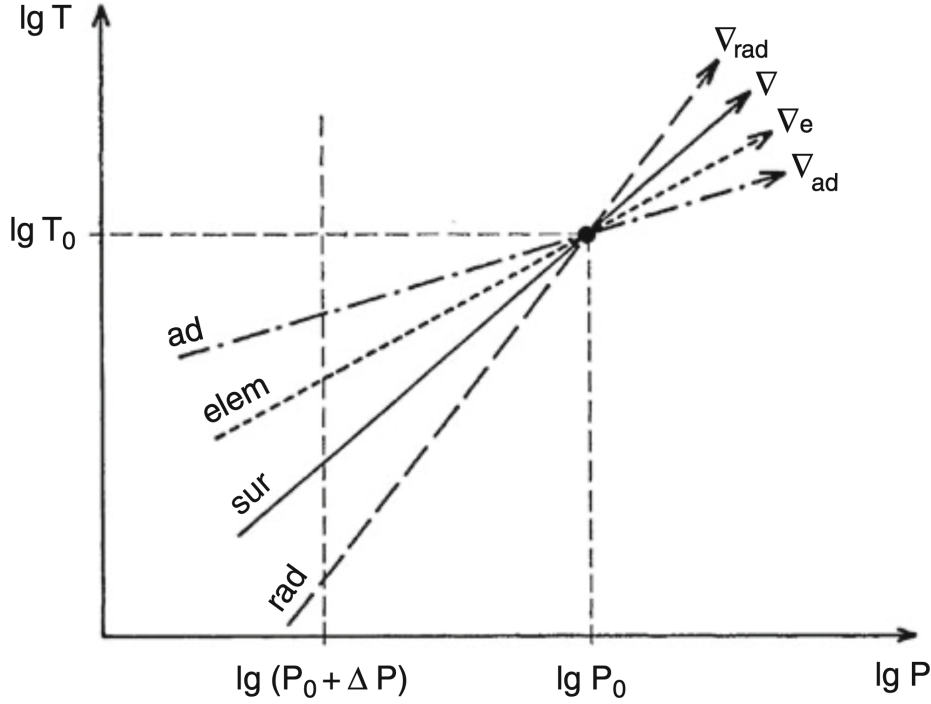


Figure 2.1.2: Temperature-pressure diagram with a schematic sketch of the different gradients $\nabla (\equiv \partial \ln T / \partial \ln P)$ in a convective layer. Starting at a common point with P_0 and T_0 , the different types of changes (adiabatic, in rising element in the surroundings, for radiative stratification) lead to different temperatures at a slightly higher point with $P_0 + \Delta P$ ($< P_0$, since P decreases outwards).

An examination of the diagram 2.1.2 reveals a plot of $\ln T$ against $\ln P$ (decreasing outwards) for an unstable layer that violates the Schwarzschild criterion. A qualitative relationship between the different gradients can be derived immediately. The gradients ∇_{ad} and ∇_e represent the slope following an adiabatic change and the change in a rising element, respectively, while the stratifications in the surroundings of a radiative layer are shown by lines with slopes ∇ and ∇_{rad} , respectively.

In this analysis, we assume the presence of convection in a chemically homogeneous layer, where $\nabla_\mu = 0$. It is necessary to violate the criterion (2.1.17), i.e., $\nabla > \nabla_e$. If a portion of the flux is transported by convection, we have that $\nabla < \nabla_{\text{rad}}$, as a portion of the total flux is allocated for radiative transfer. If the element rises from the point P_0, T_0 , i.e., it moves downwards to the left in the diagram with slope ∇_e , then, since $\nabla > \nabla_e$, the element will have an increasing excess temperature over its new surroundings, which temperature changes with ∇ . Thus, the element will radiate energy to its surroundings, which means that the element cools more

than adiabatically, i.e., $\nabla_e > \nabla_{\text{ad}}$. The combination of these inequalities yields the following relation:

$$\nabla_{\text{rad}} > \nabla > \nabla_e > \nabla_{\text{ad}}, \quad (2.1.19)$$

which is illustrated in Fig. 2.1.2.

2.2 Magneto-hydrodynamics (MHD)

In many astrophysical settings, including stars and galaxies, the gas is partially or fully ionized and capable of carrying electric currents that generate magnetic fields. In the momentum equation for the gas, the related Lorentz force acting on the ionized gas (also known as plasma) can no longer be neglected. The study of the interaction between a fluid plasma and a magnetic field is known as magnetohydrodynamics (MHD). Maxwell's equations of electrodynamics and fluid equations, which include the Lorentz forces resulting from electromagnetic fields, are combined in MHD. First, we go over Maxwell's equations, which describe how the magnetic field evolves.

2.2.1 Induction Equation

Maxwell's equations in SI units are written as

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \nabla \cdot \mathbf{B} = 0, \quad (2.2.1)$$

$$\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mu_0 \mathbf{J}, \quad \nabla \cdot \mathbf{E} = \rho_e / \epsilon_0, \quad (2.2.2)$$

where $\mathbf{B}$ is the magnetic flux density (or simply the magnetic field), $\mathbf{E}$ is the electric field, $\mathbf{J}$ is the current density, c is the speed of light, ρ_e is the charge density, and $\epsilon_0 = 1/(\mu_0 c^2)$ is the permittivity of free space.

It is convenient to define $\mathbf{B} = \nabla \times \mathbf{A}$ and substitute Eq. 2.2.1 for the *uncurled* equation for the magnetic vector potential $\mathbf{A}$,

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla \phi, \quad (2.2.3)$$

where ϕ is the scalar potential. This is to ensure that $\nabla \cdot \mathbf{B} = 0$ is satisfied at all

times.

Using the standard Ohm's law in a fixed frame of reference,

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}), \quad (2.2.4)$$

where σ is the electric conductivity, and introducing the magnetic diffusivity $\eta = (\mu_0 \sigma)^{-1}$, so we can eliminate $\mathbf{J}$ from Eq. 2.2.2, thus we have

$$\left(\frac{1}{\eta} + \frac{1}{c^2} \frac{\partial}{\partial t} \right) \mathbf{E} = - \left(\frac{1}{\eta} \mathbf{u} - \nabla \right) \times \mathbf{B}. \quad (2.2.5)$$

This formulation demonstrates that if the relevant time scale over which the electric field varies is greater than the Faraday time $\tau_{\text{Faraday}} = \eta/c^2$, the time derivative term, also known as the Faraday displacement current, can be neglected. Ampere's law $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ can be used in place of Eq. 2.2.2 for the applications covered in this thesis since we can safely ignore the displacement current and eliminate $\mathbf{E}$. In addition, we will consider units where $\mu_0 = 1$, so we will simply write:

$$\mathbf{J} = \nabla \times \mathbf{B}. \quad (2.2.6)$$

Substituting Ohm's law (Eq. 2.2.4) into Faraday's law of induction (Eq. 2.2.1) and eliminating $\mathbf{J}$ using Ampere's law (Eq. 2.2.6):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}). \quad (2.2.7)$$

which is known as the induction equation, which is the evolution equation for $\mathbf{B}$. If η is constant we can use the expression $\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$, and furthermore considering the solenoidal restriction of the magnetic field ($\nabla \cdot \mathbf{B} = 0$), the induction equation can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}. \quad (2.2.8)$$

As can be observed, the first term in Eq. 2.2.8 is an advection term, which quantifies the alteration of the magnetic field as a consequence of fluid flow. The latter term is a diffusive term, which describes the diffusion of the magnetic field.

The advection term can be ignored if the fluid is at rest ($\mathbf{u} = 0$), and equation

2.2.8 simplifies to:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}, \quad (2.2.9)$$

which is a diffusion equation. Every fluid needs $\mathbf{u} \neq 0$ to magnify a magnetic field, which is a direct evidence that the magnetic field decays in the absence of motion. In contrast, the fluid will become a perfect conductor ($\sigma \rightarrow \infty$) and we will have the ideal MHD scenario if $\eta = 0$.

2.2.2 Continuity and Navier-Stokes equations

The fluid evolution (including diffusivities) with compressible flows ($\nabla \cdot \mathbf{u} \neq 0$) and the evolution of magnetic fields are described by the non-ideal MHD equations that will be examined in this thesis. The continuity equation, which describes the conservation of mass, is as follows:

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{u}, \quad (2.2.10)$$

where $D/Dt = \partial/\partial t + \mathbf{u} \cdot \nabla$ is the advective derivative, ρ is the density of the fluid, and $\mathbf{u}$ is the flow velocity. Given a non-inertial co-rotating frame of reference with angular velocity $\boldsymbol{\Omega}$, the conservation of momentum is described by the Navier-Stokes equation, which is as follows:

$$\frac{D\mathbf{u}}{Dt} = \mathbf{g} - \frac{1}{\rho} \nabla p + \mathbf{F}_{\text{visc}} + \mathbf{F}_{\text{cent}} + \mathbf{F}_{\text{Cor}} + \mathbf{F}_{\text{Lor}}, \quad (2.2.11)$$

where p is the pressure, $\mathbf{g}$ is the gravitational acceleration, $\mathbf{F}_{\text{visc}} = \frac{1}{\rho} \nabla \cdot (2\nu\rho\mathbf{S})$ is the viscous force, $\mathbf{F}_{\text{cent}} = -\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$ is the centrifugal force, $\mathbf{F}_{\text{Cor}} = -2\boldsymbol{\Omega} \times \mathbf{u}$ is the Coriolis force, and $\mathbf{F}_{\text{Lor}} = \frac{1}{\rho} \mathbf{J} \times \mathbf{B}$ is the Lorentz force, where $\boldsymbol{\Omega}$ is the rotation rate, $\mathbf{r}$ is the distance from the rotation axis, ν is kinematic viscosity, $\mathbf{S}$ is the rate-of-strain-tensor, $\mathbf{J}$ is the current density, and $\mathbf{B}$ is the magnetic field.

The equation for conservation of energy is given by

$$\rho T \frac{Ds}{Dt} = -\nabla \cdot \mathbf{F}_{\text{rad}} + 2\nu \mathbf{S}^2 + \eta \nu_0 \mathbf{J}^2 + \Gamma, \quad (2.2.12)$$

where s is the specific entropy, $\mathbf{F} = -K \nabla T$ is the radiative flux, where K is the heat conductivity and Γ correspond to other sources of energy.

Finally, the induction equation 2.2.8, which will be rewritten in the most common

version for completeness (with constant η), is the equation we must add for the evolution of the magnetic field:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}). \quad (2.2.13)$$

The solenoidality requirement for the magnetic field is added to the set of MHD equations:

$$\nabla \cdot \mathbf{B} = 0. \quad (2.2.14)$$

An equation of state is needed to be consistent thermodynamically. For an ideal gas it is given by

$$p = (c_p - c_v)\rho T = \mathcal{R}\rho T, \quad (2.2.15)$$

where $\mathcal{R}$ is the universal gas constant, ρ is the density, and T is the temperature of the gas.

2.2.3 Dimensionless parameters

A variety of physical and astrophysical systems can be quantitatively characterized through the use of dimensionless numbers in fluids and plasmas. In the context of the MHD equations, these parameters frequently involve a comparison of the ratio between two terms. Some of them are as follows:

- The Reynolds number (Re) is the ratio of the inertial to viscous forces from the momentum equation, which determines if a flow is laminar ($\text{Re} \lesssim 1$) or turbulent ($\text{Re} \gg 1$) ([Choudhuri, 1998](#)). It is given by:

$$\text{Re} = \frac{vl}{\nu},$$

where u is the typical amplitude of the velocity, l is the characteristic length scale of the motion, and ν is the kinematic viscosity.

- The magnetic Reynolds number (Re_M) is the ratio of advection to diffusion terms in the induction equation, i.e.,

$$\text{Re}_M = \frac{ul}{\eta}.$$

The magnetic field obeys a diffusion equation if the second term in the induction equation dominates, i.e., $\text{Re}_M \ll 1$. Ohmic dissipation is how the

magnetic energy is thermalized. The magnetic field is locked in the flow and the first part in the induction equation takes dominance if $\text{Re}_M \gg 1$. In this instance, the magnetic field is elastic and oscillates at a frequency proportional to the Alfvén velocity ([Davidson and Belova, 2002](#)).

- The Prandtl number (Pr) is the ratio between the kinematic viscosity (or momentum diffusivity) to thermal diffusivity χ :

$$\text{Pr} = \frac{\nu}{\chi}.$$

Consequently, it provides the relationship between a fluid's thermal transport capability and momentum transport. To put it another way, fluid conduction rather than fluid momentum is more likely to be the cause of heat transfer if $\text{Pr} \ll 1$. This number can be interpreted as an intrinsic attribute of the plasma because it has nothing to do with the flows.

- The magnetic Prandtl number (Pr_M) characterizes the ratio of momentum diffusivity (ν) to magnetic diffusivity (η):

$$\text{Pr}_M = \frac{\text{Re}_M}{\text{Re}} = \frac{\nu}{\eta}.$$

It is also an intrinsic property of the plasma.

- The Coriolis number (Co) is formally defined as the ratio between the Coriolis force and the acceleration of the fluid, and it is the inverse of the Rossby number ([Rossby, 1939](#)):

$$\text{Ro}^{-1} = \text{Co} = \frac{lf}{u}$$

where $f = 2\Omega$ is the Coriolis frequency, given by the angular velocity Ω .

2.3 Dynamo theory

As mentioned above, a dynamo maintains a magnetic field in a conductive fluid. This process transforms kinetic energy into magnetic energy and must be efficient enough to compensate for Ohmic dissipation losses. Small-scale dynamos can be found, corresponding to those that operate on scales comparable to or smaller than the typical scale of convective eddies. On the other hand, there are large-scale

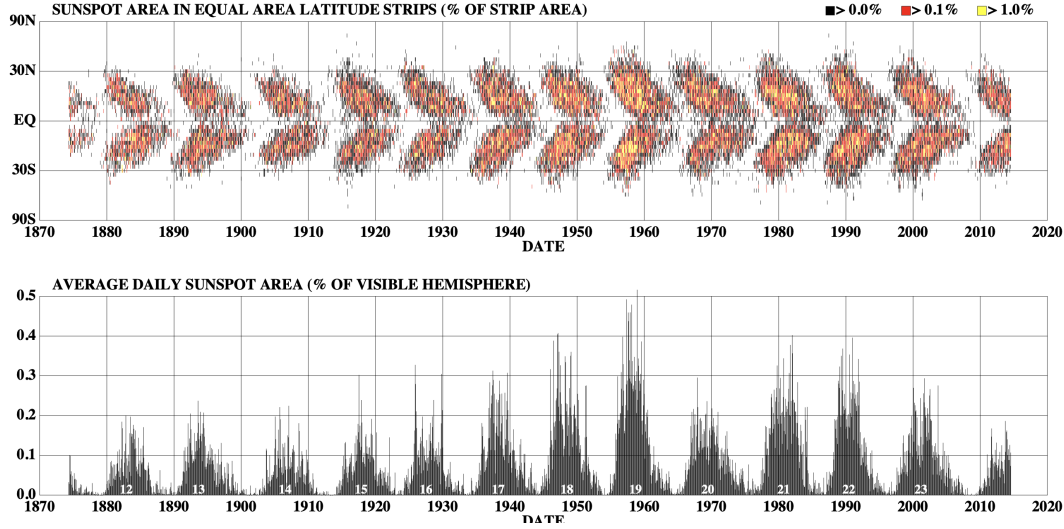


Figure 2.3.1: The relative area in equal area latitude strips is illustrated with a color code in the upper panel. The average daily sunspot area for each solar rotation since May 1874 as a function of time in the lower panel ([Hathaway, 2015](#)).

dynamos, which are responsible for maintaining the large-scale magnetic fields in stars (the focus of this thesis). An overview of the solar dynamo theory is given below, along with a review of what happens in M dwarfs.

2.3.1 Solar dynamo

The solar magnetic field is structured and evolves on an astonishing variety of spatial and temporal scales. It has been established that sunspots are surface manifestations of emerging magnetic fields produced in the Sun’s interior. Moreover, temporal observations indicate the presence of a significant large-scale order. In particular, at the beginning of the cycle, the number of sunspots on the solar disc exhibits an increase and subsequent decrease over a timescale of approximately 11 years. Additionally, sunspots occur at mid-latitudes at the beginning of this cycle around $\sim 30^\circ$, and progressively approach the equator as the cycle progresses, as seen in Fig. 2.3.1, where sunspots form in two bands, one in each hemisphere, which start at about 35° from the equator at the start of a cycle and migrate toward the equator as the cycle progresses.

The spots exhibit a consistent magnetic polarity in each hemisphere, which is reversed between hemispheres. These striking hemispheric regularities, known as *Hale’s polarity law* ([Hale et al., 1919](#)), reflect the presence of an internal magnetic field that is spatially well organized on the scale of the Sun as a whole and

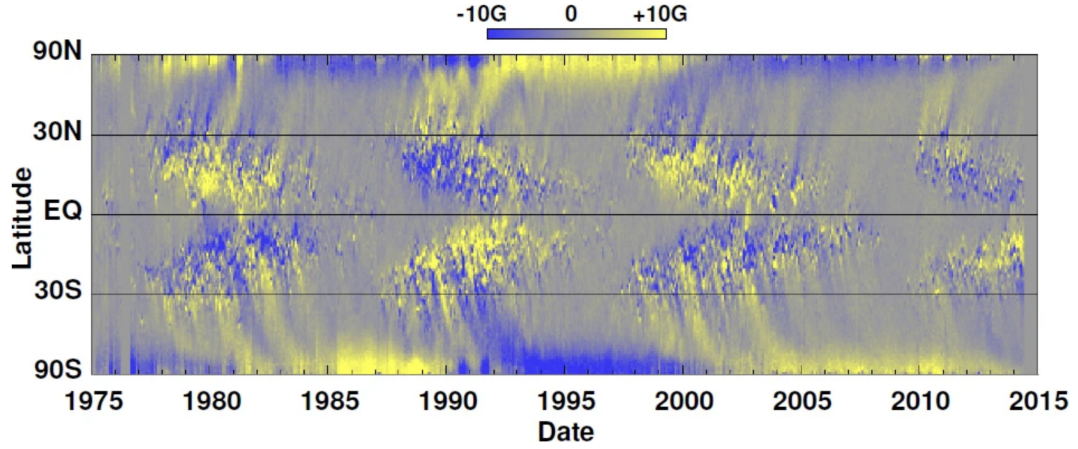


Figure 2.3.2: Longitudinally averaged magnetic field at the surface of the Sun for the four most recent sunspots cycles (Hathaway, 2015).

antisymmetric around the solar equator, which is shown in Figure 2.3.2. The reversal of the relative magnetic polarities of sunspot pairs from one cycle to the next provides evidence that the underlying magnetic cycle has a period of approximately 22 years, which is twice that of the sunspot cycle (Babcock, 1961).

The challenge facing dynamo theory is therefore to provide an explanation for the existence and observed evolution of the solar magnetic field. This is achieved through the utilization of diverse models, which serve as a theoretical construct to facilitate comprehension of intricate physical systems that are too complex to be fully understood through direct inference from observed data. The dynamo is responsible for maintaining the magnetic field on a large scale, and for transforming the poloidal magnetic field into a toroidal magnetic field and vice versa. The principal models that explain this phenomenon are outlined below.

- ◇ **Ω effect** The Ω effect represents a significant inductive process in any shearing or differentially rotating system. It involves the winding up of a magnetic field component parallel to the direction of the velocity gradient into a stronger magnetic field component parallel to the direction of the velocity field itself (Cowling, 1933). This effect is illustrated in Fig. 2.3.3 in spherical geometry. In order to provide a general overview of the restrictions encountered by dynamos, it is useful to consider an axisymmetric system in cylindrical geometry (R, φ, z) , assuming that $\mathbf{B}$ is an axisymmetric vector field,

$$\mathbf{B} = \nabla \times (\chi \hat{\varphi} / R) + R \psi \hat{\varphi}, \quad (2.3.1)$$

where $\chi(R, z, t)$ is a poloidal flux function and $R\psi$ is the toroidal magnetic field, and $\hat{\varphi}$ is the unit vector in φ -direction. Similarly, assume that $\mathbf{U}$ is axisymmetric with respect to the same axis of symmetry as $\mathbf{B}$, we can write

$$\mathbf{U} = \mathbf{U}_{\text{pol}} + R\Omega\hat{\varphi}, \quad (2.3.2)$$

where $\mathbf{U}_{\text{pol}}(R, z, t)$ is an axisymmetric poloidal velocity field and $\Omega(R, z, t)\hat{\varphi}$ describes an axisymmetric toroidal differential rotation. Furthermore, the induction equation's poloidal and toroidal components are as follows:

$$\frac{\partial \chi}{\partial t} + \mathbf{U}_{\text{pol}} \cdot \nabla \chi = \eta \left(\nabla^2 \chi - \frac{2}{R} \frac{\partial \chi}{\partial R} \right), \quad (2.3.3)$$

$$\frac{\partial \psi}{\partial t} + \mathbf{U}_{\text{pol}} \cdot \nabla \psi = \mathbf{B}_{\text{pol}} \cdot \nabla \Omega + \eta \left(\nabla^2 \psi + \frac{2}{R} \frac{\partial \psi}{\partial R} \right). \quad (2.3.4)$$

The Ω effect, or the stretching of poloidal fields into toroidal fields by differential rotation, is described by $\mathbf{B}_{\text{pol}} \cdot \nabla \Omega$ in equation 2.3.4. It is not possible to create a toroidal field from a poloidal field, as equation 2.3.3 demonstrates no relation between ψ and χ . In essence, the Cowling (1933) theory states that dynamo action cannot sustain a purely axisymmetric magnetic field. Therefore, non-axisymmetric mechanisms are required to explain the maintenance of the poloidal field. This is known as *Cowling's theorem*, which essentially proposes that dynamo action is incapable of sustaining a completely axisymmetric magnetic field. Consequently, non-axisymmetric mechanisms are necessary to explain the maintenance of the poloidal field.

◇ α effect

The precise physical process responsible for the regeneration of the poloidal field remains uncertain. One of the proposed mechanisms is turbulence, which was initially postulated by Parker (1955). In this mechanism, turbulence possesses helicity when rotation is present, thereby twisting the toroidal field to produce a poloidal component that is perpendicular to the field. Furthermore, the *mean-field dynamo theory*, a mathematical theory of large-scale dynamos that is outlined below, was employed by Steenbeck et al. (1966) to demonstrate the α -effect mathematically rigorously.

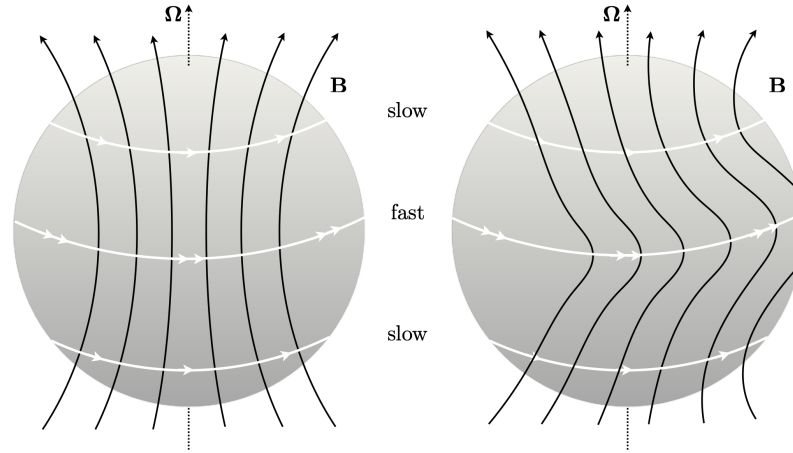


Figure 2.3.3: Perspective drawing of the Ω effect in a spherical fluid system with latitudinal differential rotation. In the areas of fastest rotation, the latitudinal shear transforms an initial axisymmetric poloidal field into a stronger axisymmetric toroidal field. Figure from the review by [Rincon \(2019\)](#).

◇ Flux transport dynamos

The tachocline, the shear layer situated at the bottom of the convection zone, may play a crucial role in maintaining the Sun’s large-scale magnetic field, as postulated in *flux transport dynamo models*. These models were developed based on the *Babcock-Leighton mechanism*, which was initially proposed by [Babcock \(1961\)](#) and [Leighton \(1969\)](#). The toroidal magnetic field is produced by shearing in the tachocline, whereas the poloidal magnetic field is produced by buoyant, unstable flux tubes that rise from the tachocline and tilt as a result of the Coriolis force. In order for the cycle to resume, it is necessary for the poloidal field to be transported to the poles and subsequently returned to the tachocline via the meridional circulation, as outlined in section 2.4.2 Nevertheless, the tachocline is not a significant factor in *distributed dynamo models* (e.g. [Käpylä et al. 2006](#)).

◇ Mean-field dynamo theory

The concept of mean-field dynamo theory, also known as mean-field electrodynamics ([Krause and Raedler, 1980a](#)), uses ensembles or averages to divide the magnetic and velocity field into a mean and a fluctuating part while taking into account a two-scale decomposition of the dynamics. In

other words, the magnetic and velocity fields can be expressed as follows:

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}, \quad (2.3.5)$$

$$\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}, \quad (2.3.6)$$

where $\overline{\mathbf{U}}$ and $\overline{\mathbf{B}}$ represent the large-scale (mean) parts of the velocity and magnetic fields, while $\mathbf{u}$ and $\mathbf{b}$ are the fluctuating parts. According to the Reynolds rules, these averages satisfy

$$\overline{\mathbf{U}_1 + \mathbf{U}_2} = \overline{\mathbf{U}_1} + \overline{\mathbf{U}_2}, \quad (2.3.7)$$

$$\overline{\overline{\mathbf{U}}} = \overline{\mathbf{U}}, \quad (2.3.8)$$

$$\overline{\overline{\mathbf{U} \mathbf{u}}} = 0, \quad (2.3.9)$$

$$\overline{\overline{\mathbf{U}_1 \mathbf{U}_2}} = \overline{\mathbf{U}_1} \overline{\mathbf{U}_2}, \quad (2.3.10)$$

$$\frac{\partial \overline{\mathbf{U}}}{\partial t} = \frac{\partial \overline{\mathbf{U}}}{\partial t}, \quad (2.3.11)$$

$$\frac{\partial \overline{\mathbf{U}}}{\partial x_i} = \frac{\partial \overline{\mathbf{U}}}{\partial x_i}. \quad (2.3.12)$$

In the case of a significantly greater averaging length in comparison to the eddy scale, the resulting averages can be considered reliable. If we apply these averages to the induction equation, we get

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{u} \times \mathbf{b}} - \eta \mu_0 \mathbf{J}), \quad (2.3.13)$$

where $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ is the *mean electromotive force* (EMF). The solution to the closure issue, which is the process of finding an expression for $\overline{\mathcal{E}}$, is to write $\overline{\mathcal{E}}$ in terms of $\overline{\mathbf{B}}$ and its gradients. Equation 2.3.13 requires an expression of $\overline{\mathcal{E}}$ in terms of the large-scale quantities, which is

$$\overline{\mathcal{E}}_i(\mathbf{x}, t) = a_{ij} \overline{B}_j + b_{ijk} \frac{\partial \overline{B}_k}{\partial x_j}, \quad (2.3.14)$$

where a_{ij} and b_{ijk} are second and third-order tensors that are solely dependent on the properties of the flow.

2.4 Flow Properties

The dynamo activity observed in the convection zones of stars is subject to influence from a number of large-scale fluxes and flow features, including differential rotation, meridional circulation, and kinetic helicity. These will be discussed in detail below.

2.4.1 Differential rotation

In many dynamo models, including the majority of the solar ones, shear flows are important. The non-uniform angular velocity of the convection zone, is known as differential rotation (DR). In the mean-field theory of stellar rotation ([Ruediger, 1989](#)), the off-diagonal components of the Reynolds stress tensor, $Q_{ij} \equiv \overline{u_i u_j}$, which characterize the turbulent fluxes of angular momentum in terms of correlations of fluctuating velocity components, and the meridional flow are responsible for the angular momentum transport within the convection zone. In addition, the Coriolis force affects turbulence, giving rise to the Reynolds stress, resulting in the redistribution of the angular momentum. This leads to alterations in the behavior of the global rotation, which in turn results in differential rotation.

Figure 2.4.1 shows the internal rotation of the Sun as obtained from the helioseismic inversions. The internal rotation of the Sun is characterized by a positive radial gradient of Ω at low latitudes and a negative one at latitudes $\Theta \gtrsim 40^\circ$. While $\frac{\partial \Omega}{\partial r} \approx 0$ in the bulk of the convection zone, the tachocline, a shallow layer at the bottom or immediately below it, is where the majority of the radial shear is concentrated. The distinct angular momentum transmission mechanisms and efficiency in the two provide a qualitative explanation for the abrupt transition from rigid rotation of the core to latitudinally and radially variable rotation in the convection zone. The solid body rotation in the radiative core observed nowadays may be caused by a weak poloidal relic magnetic field (e.g. [Mestel and Weiss 1987](#); [Kitchatinov and Rüdiger 2006](#)) or weak turbulence caused by magnetorotational instability ([Arlt et al., 2003](#)), which has had enough time to smooth out all gradients of Ω during the Sun's lifetime, according to the long diffusion time scale in the radiative core. However, the turbulent diffusion time scale in the convection zone is significantly shorter (one or a few years), indicating the need for a process that continuously generates differential rotation.

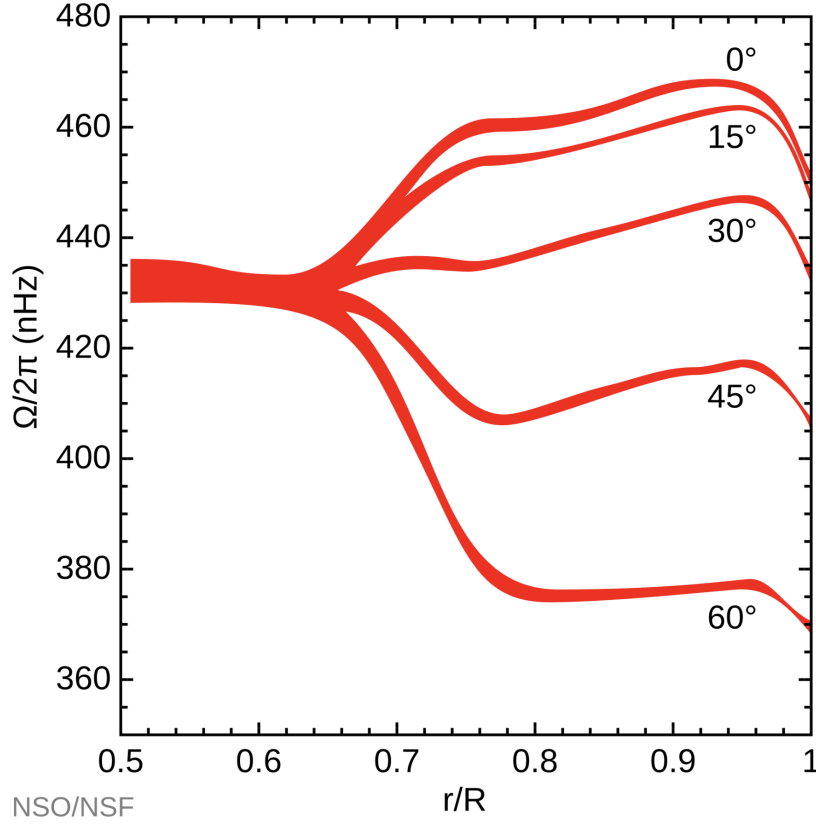


Figure 2.4.1: Rotation rate of the sun at different latitudes as a function of depth. The error bars are shown by the thickness of the curves. Credits: GONG/NSO/AURA/NSF.

2.4.2 Meridional circulation

Meridional circulation is used to describe a large-scale flow that transports angular momentum and contributes to energy transport inside stars. This process can be defined as the axisymmetric motions in the meridional (r, θ) plane in spherical coordinates. Theoretically, the azimuthal component of the vorticity equation, derived by calculating the curl of the Navier-Stokes equation, offers the most straightforward means of distinguishing between the various processes responsible for the generation of meridional flow (Küker and Stix, 2001)

$$\frac{\partial \omega_\phi}{\partial t} = r \sin \theta \frac{\partial \Omega^2}{\partial z} - \left[\nabla \times \frac{1}{\rho} \nabla(\rho \mathcal{Q}) \right]_\phi + \frac{1}{\rho^2} (\nabla \rho \times \nabla p)_\phi, \quad (2.4.1)$$

where $\frac{\partial}{\partial z} = \cos \theta \frac{\partial}{\partial r} - \frac{\sin \theta}{r} \frac{\partial}{\partial \theta}$ is the derivative along the rotation axis. Three distinct effects correspond to the three terms on the right-hand side (rhs) of Eq. 2.4.1, can essentially produce the meridional flow. The first of these is known as

the *Eddington-Sweet circulation*, which consists of meridional flows that can be induced by the presence of radial and/or latitudinal differential rotation alone. This is due to the conservation of the angular momentum, which is represented by the first term in the rhs (Kippenhahn 1963; Biermann 1951).

Secondly, Reynold stresses have a direct impact on the meridional flow through the component $Q_{r\theta}$, which is present in the meridional components of the Reynolds equation, and an indirect effect through the generation of differential rotation. However, due to our current limitations in understanding the component $Q_{r\theta}$, it has not played a significant role in stellar rotation models.

The final component on the rhs of Eq. 2.4.1, which indicates that the contours of constant pressure and density do not overlap, induces baroclinic flow, which is likely the most significant source of meridional motions. This term denotes a temperature differential between the equator and the pole, which gives rise to a flow known as the thermal wind, thereby modifying the angular momentum balance (see e.g. Brandenburg et al. 1990; Kitchatinov and Ruediger 1995; Rüdiger et al. 2005).

2.4.3 Kinetic helicity

The measurement of the twisting and connecting of vortex lines in fluids is referred to as kinetic helicity. The phenomenon is observed in rotating fluids, such as stars. As postulated by Krause and Raedler (1980b), the α -effect is contingent upon the presence of an underlying velocity field characterized by a non-zero kinetic helicity and the absence of reflectional symmetry. The kinetic helicity is given by

$$\mathcal{H} = \int_V \vec{\omega} \cdot \vec{u} \, dV, \quad (2.4.2)$$

where ω is the vorticity. The sign of $\mathcal{H}$ is significant for stars. For instance, a negative kinetic helicity in the northern hemisphere, with $\frac{\partial \Omega}{\partial r} > 0$, is projected to result in a poleward-propagating dynamo wave. Following Parker (1955) and Yoshimura (1975), this is referred to as the *Parker-Yoshimura rule*.

2.5 M dwarfs

As mentioned above, M dwarfs, also known as red dwarfs, are the most abundant type of star in the Galaxy and are low-mass late-type main-sequence stars (Winters et al., 2019). Evidence of significant surface magnetic activity has been observed (Johns-Krull and Valenti, 1996). The effective temperatures of these stars range from 2450 to 3850 K , with masses between 0.08 and 0.55 $M_{\odot}$. They exhibit significantly lower luminosity than the Sun. A notable attribute of these stars is the alteration in their stellar structure above 0.35 $M_{\odot}$, as postulated by Chabrier and Baraffe (1997). Like the Sun, they are partially convective above this mass, but fully convective below it (see Fig. 2.1.1).

2.5.1 Fully and partially convective M dwarfs

Fully convective M dwarfs represent a particularly intriguing subject for dynamo investigations due to their lack of a tachocline. This is due to a lack of consensus among researchers regarding the role of the tachocline in dynamo operation in solar-like stars. Some argue that it is a crucial component (Charbonneau, 2014), while others disagree (Brandenburg, 2005). Wright and Drake (2016) propose that the X-ray emission of late-type partially and fully convective stars exhibits a comparable trend with the Rossby number (Ro), increasing as Ro decreases until $Ro \sim 0.1$, at which point the X-ray brightness reaches a saturation point. This suggests that the tachocline is irrelevant and that dynamos functioning in partially and fully convective stars are comparable.

Comparing the magnetic fields of M dwarfs obtained using the two direct diagnostic techniques based on the Zeeman effect is instructive. The average large-scale magnetic field $\langle \mathbf{B}_V \rangle$ derived using ZDI from polarimetric data is probably going to be less than the total magnetic field $\langle \mathbf{B} \rangle$ deduced from the Zeeman broadening and magnetic intensification of lines in intensity spectrum. The intricacy of the underlying M dwarfs magnetic topology is characterized by the degree of divergence between the two magnetic observations. Such preliminary comparisons (Morin et al. 2008, 2010; Reiners and Basri 2009) were based on coarse $\langle \mathbf{B} \rangle$ obtained with approximate methods and contained only a few stars. Kochukhov and Shulyak (2019) expanded their evaluation to include a large number of additional low-mass stars, utilizing the larger sample of M dwarfs with precise total field strength

measurements. An updated plot from Kochukhov and Shulyak (2019) present in Kochukhov (2021) is shown in Fig. 2.5.1. The stars that have both the $\langle \mathbf{B} \rangle$ and $\langle \mathbf{B}_V \rangle$ measurements are 23. An axisymmetric poloidal dipolar field dominates the global magnetic configurations of ten of them. The remaining 13 stars display a variety of large-scale field types, such as oblique poloidal fields, mixes of poloidal and toroidal components, etc.

For partially convective stars, the ratio of the global to total mean field, $\langle \mathbf{B}_V \rangle / \langle \mathbf{B} \rangle$, does not exceed 9% when plotted as a function of stellar mass (Fig. 2.5.1a). This ratio seems to be dependent on the global magnetic field topology for fully convective M dwarfs. A higher percentage of the overall field strength recovered by ZDI is typically found in stars with axisymmetric dipolar fields, $\langle \mathbf{B}_V \rangle / \langle \mathbf{B} \rangle = 11 - 21\%$, and up to 26% for GJ 51. However, $\langle \mathbf{B}_V \rangle / \langle \mathbf{B} \rangle < 7\%$ are found in stars with various kinds of large-scale fields.

The overall magnetic field strength is correlated with the ratio $\langle \mathbf{B}_V \rangle / \langle \mathbf{B} \rangle$ (Fig. 2.5.1b). It is possible to draw the inference that a star is more likely to have a strong, dipole-dominated global field if its average field is stronger. These tendencies do have exceptions. Despite having a distinctly non-axisymmetric field, the star GJ 873 (EV Lac) has a high $\langle \mathbf{B}_V \rangle / \langle \mathbf{B} \rangle$ ratio. Additionally, its overall field seems to be excessively strong for the rotation period of 4.37 days. The non-axisymmetric arrangement of the GJ 65 system's major component (BL Cet) also results in an exceptionally powerful total field. Despite having a low $\langle \mathbf{B}_V \rangle / \langle \mathbf{B} \rangle$ ratio and a very weak total field, two stars, GJ 388 (AD Leo) and GJ 569A (CE Boo), appear to have axisymmetric, dipole-dominated global fields.

2.5.2 Observational evidence

Understanding the surface magnetic fields of low mass magnetic stars has become more clear because of recent advancements in computer techniques for modeling their spectra and the availability of high-quality spectropolarimetric measurements of these objects. We now know that multi-kG average (total) fields are present in active, rapidly rotating M dwarfs. These stars have the strongest fields, up to 6 - 7 kG. This measurement, which correlates to the disk-integrated magnetic field modulus, indicates that the field strength within localized surface magnetic features probably exceeds $\sim 10\text{kG}$. There is no discernible qualitative change in this field characteristic at the transition from partially to fully convective

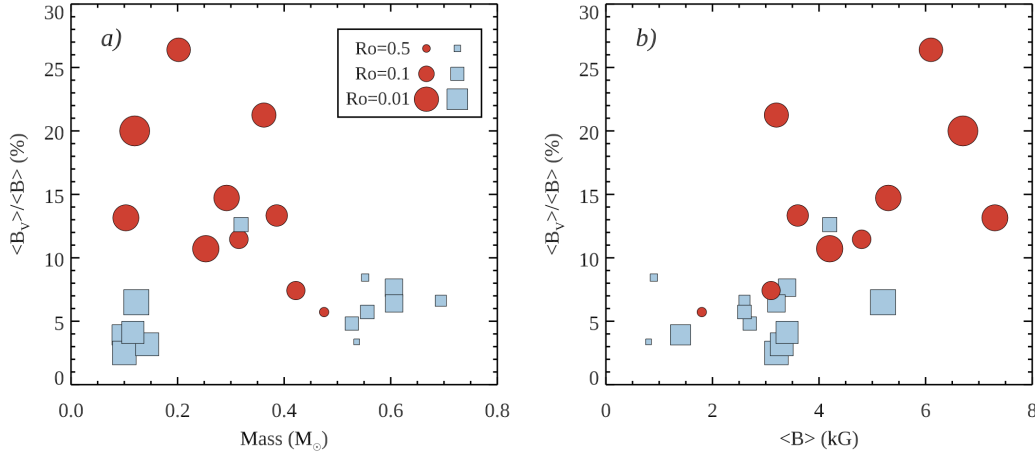


Figure 2.5.1: Ratio of the average global magnetic field strength $\langle B_V \rangle$ derived by ZDI studies to the total field strength $\langle B \rangle$ obtained from Stokes I as a function of stellar mass (a) and total field strength (b). The red circles show stars with predominantly dipolar, axisymmetric global fields. The blue squares correspond to stars with multipolar or non-axisymmetric dipolar fields. The symbol size reflects the Rossby number, as indicated by the legend in panel (a) (Kochukhov, 2021).

stars, however it seems that the average field strength generally increases towards later spectral types. M dwarfs also have a wide variety of large-scale magnetic field geometries. Complex, weak global fields, frequently with a large toroidal component, are characteristic of early-M dwarfs. Mid-M dwarfs often have stronger poloidal global fields with an axisymmetric dipolar component. At the edge of full convection, these fields appear. With both early-M and mid-M field types found in stars with comparable mass and rotation rate, late-M dwarfs display an enigmatic global field dichotomy. The global magnetic fields of M dwarfs rarely have an average intensity of 1 kG. Even in stars with relatively straightforward dipolar large scale magnetic topologies, a notable discrepancy with significantly larger total fields suggests that M dwarfs' magnetic fields have a complicated, small-scale nature. According to a qualitative model that aligns with all of the findings that are currently available, the M dwarf field structure is a carpet of strong, small-scale fields that are superimposed on top of a weaker large-scale component (as shown in Fig. 2.5.2, for example). It is unclear whether this global magnetic component is a distinct physical entity, possibly related to a different dynamo process, or if it is linked to one of the elements (such as strong-field spots) of the continuous power spectrum that describes the magnetic structures at the small scale. The former conjecture is supported by the diversity of the correlations

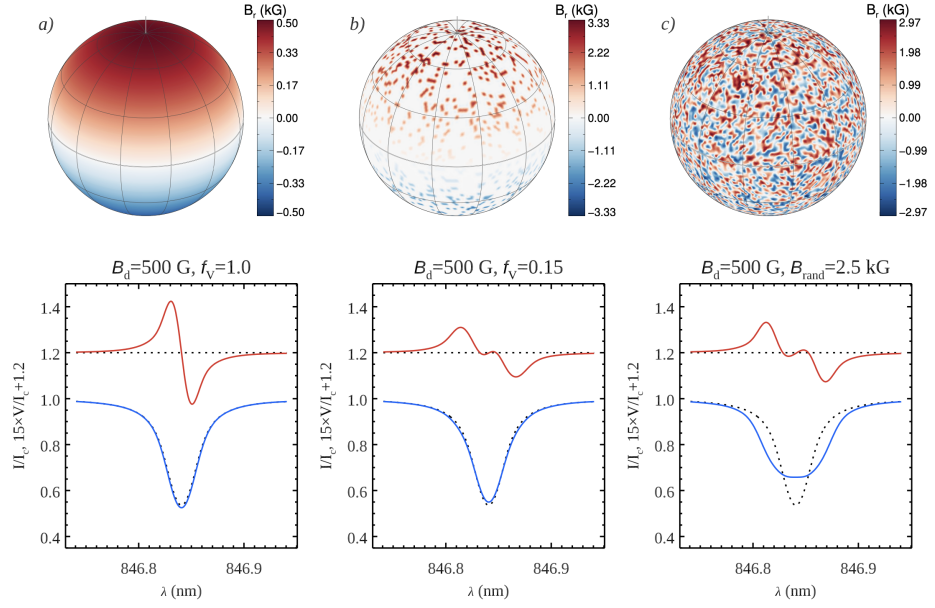


Figure 2.5.2: Synthetic disk-integrated Stokes I and V profiles of the $\text{Fe I} = 846.84$ nm line computed with different treatment of the global magnetic field geometry. (a) A standard axisymmetric dipolar field with a polar strength of 0.5 kG. (b) 15% of the stellar surface is occupied by the same dipolar field. (c) Superposition of 0.5 kG dipolar field and 2.5 kG randomly oriented field. The side bar displays the field strength in kG, while the spherical color maps in the top row show the radial field distributions. The corresponding model Stokes I (blue solid line) and Stokes V (red solid line) profiles are plotted in the lower row. Calculations without a magnetic field are indicated by the black dotted lines (Kochukhov, 2021).

between the features of large-scale fields and total fields seen for active M dwarfs in the previous subsection (sect. 2.5.1).

Chapter 3

Computational methods

This chapter provides an overview of the numerical models employed in this project, commencing with the MESA code, which was utilized to study stellar structure, and the PENCIL CODE, which generated the MHD simulations. The output files, configurations, simulation approach, physical units and diagnostic quantities are described in brief.

3.1 MESA code

The open-source tool known as Modules for Experiments in Stellar Astrophysics¹ (MESA) is employed in a diverse range of computational stellar astrophysics applications. The MESA `star` module is a one-dimensional stellar evolution module that integrates multiple physical and numerical modules to simulate a wide variety of stellar evolution scenarios, including those pertaining to low-mass to high-mass stars and advanced evolutionary phases. The fully coupled structure and composition equations are solved simultaneously. Additionally, it is capable of supporting OpenMP-based shared memory parallelism and makes use of adaptive mesh refinement and complex time step controls.

In particular, the online version of MESA-Web², created by Rich Townsend and based on the original MESA code, was employed in this study to obtain the parameters of a partially convective M dwarf. These will subsequently be used to perform the conversion from code to physical units.

¹<https://github.com/MESAHub/mesa>

²<http://user.astro.wisc.edu/~townsend/static.php?ref=mesa-web>

3.1.1 Input parameters

The requisite parameters for the simulation are to be entered on the submission page. The specified parameters are classified into the following categories:

- ◇ *Initial Properties:* In this case, the initial mass of the star must be specified in solar masses. The rotation rate of the star is defined in terms of $\Omega_{\text{ZAMS}}/\Omega_{\text{crit}}$, which is the ratio of the angular velocity of the star in the zero-age main-sequence (ZAMS) to the critical angular velocity, and the metallicity, which is defined as the mass fraction of metals.
- ◇ *Nuclear Reactions:* The nuclear processes taking place in the stellar core are defined. This can be employed to inhibit the release of energy and/or the consumption of fuel via nucleosynthesis. Additionally, the nuclear reactions network can be defined (e.g., general H/He burning, general H/He/C/O burning, and others). Furthermore, the nuclear reaction can be customized.
- ◇ *Mixing:* This includes overshooting, thermohaline, and stellar convection factors derived from the mixing length formulation.
- ◇ *Winds:* Application of wind mass loss in the red giant branch.
- ◇ *Numerical Resolution:* To specify the mass and temporal resolution of the global simulation.
- ◇ *Custom Stopping Condition:* The quantity to be monitored will determine the point at which the calculation should be terminated.
- ◇ *General:* In this instance, the user is able to specify the version of MESA that is to be employed, in addition to the frequency of the output files.

3.1.2 Output from MESA-Web

Once the MESA-Web calculation has been completed, a notification will be sent via email, containing a link to a compressed file. Upon downloading and unpacking the files, a folder containing a variety of MESA-Web-generated files will be created. The files contain a multimedia file in MP4 format, which presents a variety of plots derived from the simulation. The data represented in each frame of the movie is based on the information from every fifth MESA time step.

As a function of time, the history file, `trimmed_history.data`, provides

comprehensive details on the complete stellar model. The file is constituted by a series of rows, each corresponding to a distinct time-step. In addition, a few header lines provide global data.

The internal structure of the stellar model at a specific time step is detailed in the profile files, designated as `profileN.dat` (where N is an integer increasing from 1). The `profiles.index` file outlines the correlation between profile number N and time step. A profile file comprises several header lines providing global data, followed by a series of rows representing individual interior zones (arranged from the outermost zone to the innermost).

3.1.3 M dwarf star model

A main-sequence M dwarf star was modelled using a stellar mass of $0.45 M_{\odot}$ and a metallicity of 0.02. A rotating model was used with $\Omega_{\text{ZAMS}}/\Omega_{\text{crit}} = 0.5$. The nuclear reaction network was set to *basic*, i.e., for general H/He burning, which included the isotopes ^1H , ^3He , ^4He , ^{12}C , ^{14}N , ^{16}O , ^{20}Ne , and ^{24}Mg . The formula proposed by Cox and Giuli (1968) was employed, with α_{MLT} assumed to be equal to 2. This defines the mixing length as $l_m = \alpha_{\text{MLT}} H_P$, where H_P represents the local pressure scale height.

In the absence of a three-dimensional hydrodynamical treatment of convection, a parametric model must be employed to account for hydrodynamical mixing instabilities at convective boundaries, otherwise known as overshoot mixing, which is defined as

$$D_{\text{OV}} = D_{\text{conv},0} \exp\left(-\frac{2z}{fH_p}\right), \quad (3.1.1)$$

where f is an adjustable parameter, z is the distance in the radiative layer away from the overshoot zone, and $D_{\text{conv},0}$ is the MLT derived diffusion coefficient at a user-defined location close to the Schwarzschild boundary (Herwig, 2000). The semi-convection process is based on the equation proposed by Paxton et al. (2013), where $\alpha_{\text{sc}} = 0.01$. In accordance with the findings of Kippenhahn et al. (1980), thermohaline mixing can be considered a diffusion process, with the diffusion coefficient determined by equation (14) of Paxton et al. (2013) and an efficiency of $\alpha_{\text{th}} = 1.0$. In our case we used $f = 0.0001$. In addition, in the process of searching for main-sequence parameters, it is not necessary to implement Red Giant Branch winds.

The stellar parameters to be used from this model are the mass $M_\star$, luminosity $L_\star$, the central mass density $\rho_c^\star$ and the radius $R_\star$ of the star.

3.2 The Pencil Code

The `PENCIL CODE`³, a high-order finite-difference code for solving differential equations with primary applications in compressible astrophysical magnetohydrodynamics, was employed to execute the simulations ([Pencil Code Collaboration et al., 2021](#)). The code is capable of operating effectively on massively parallel machines with distributed memory, utilizing the Message Passing Interface (MPI). The non-ideal MHD equations, which are outlined in section 2.2 of the model description, are the equations that `PENCIL CODE` is designed to solve.

3.2.1 Files in the run directory

It is imperative to define the requisite directories and establish the optimal configurations in accordance with the requirements of the task at hand. The script `pc_setupsrc`, which is located in the `pencil-code/bin` folder, is responsible for performing this task. The files designated as `start.in`, `run.in`, `phiaver.in`, `print.in`, `run.csh` and `start.csh` are located in the directory designated as `run`. The subdirectory `src` contains two local files, which are `Makefile.local` and `cparam.local`. It is necessary to configure these files in accordance with the physics of the simulation to be carried out. Furthermore, the output data must be stored in a subdirectory designated as `data`, which the user is responsible for creating.

In the time series and averages over ϕ , the files with the extension `.in` define the startup, runtime, and output parameters, respectively. The files ending with `.csh` serve as links to the directory `pencil-code/bin` to initialize the code. The selection of individual physical and technical modules can be achieved through the utilization of the `Makefile.local`, as described in section 3.2.2. Furthermore, the grid size and processor layout (`nprocx`, `nprocy` and `nprocz`) can be modified through the `cparam.local` file. The sub-directory `data` has the following output files:

³<https://github.com/pencil-code/pencil-code/>

- ◇ `dim.dat`: The global array dimensions.
- ◇ `legend.dat`: The header line specifying the names of the diagnostic variables in `time_series.dat`.
- ◇ `time_series.dat`: Time series of diagnostic variables.
- ◇ `tsnap.dat`, `tvid.dat`: Time when the next snapshot `VARN` or animation slice should be taken.
- ◇ `params.log`: Keeps a log of all parameters.
- ◇ `param.nml`: Set of startup parameters.
- ◇ `param2.nml`: Set of runtime parameters.
- ◇ `index.pro`: Can be used as include file in IDL and contains the column in which certain variables appear in the diagnostics file.
- ◇ `sn_series.dat`: Time series of SN explosions locations and diagnostics.
- ◇ `proc0`, `proc1`, ...: These are the directories containing data from the individual processors. Each of the directories can contain the following files:
 - `var.dat`: binary file containing the latest snapshot;
 - `VARN`: binary file containing individual snapshot number *N*;
 - `dim.dat`: ASCII file containing the array dimensions as seen by the given processor;
 - `time.dat`: ASCII file containing the time corresponding to `var.dat`;
 - `grid.dat`: binary file containing the part of the grid seen by the given processor;
 - `seed.dat`: random seed for the next time step;
 - `X.xy`, `X.xz`, `X.yz`: two-dimensional slices of variable *X*, where *X* stands for the corresponding variable. The current list includes:

`bx.xy bx.xz by.xy by.xz bz.xy bz.xz divu.xy lnrho.xz`

`ss.xz ux.xy ux.xz uz.xy uz.xz`

3.2.2 Modularity

The file `Makefile.local` is present in each run directory. This file enables the user to select specific modules, to invoke entropy, magnetic fields, hydrodynamics, force, and other modules. For example, the settings for the simulations with the *star-in-a-box* setup, which is described in 3.3, are:

```

MPICOMM    =    mpicomm
IO          =    io_dist
FILE_IO     =    file_io_f95

HYDRO       =    hydro
DENSITY     =    density
ENTROPY     =    entropy
MAGNETIC    =    magnetic
RADIATION   =    noradiation
PSCALAR     =    nopscalar
EOS         =    eos_idealgas

GRAVITY     =    gravity_r
FORCING     =    noforcing
SHEAR       =    noshear
TIMEAVG     =    notimeavg

REAL_PRECISION = double

```

The first three modules are of a technical nature. The input/output is distributed across processors, and the Message Passing Interface (MPI) is enabled for use with multiple processors. The subsequent modules pertain to the domain of physics. These include the hydro module, which addresses velocity, the density module, the entropy module, which is concerned with the resolution of the entropy equation, and the magnetic fields. The additional passive scalar and radiation modules are disabled. The gravity module is also activated, while the forcing and shear modules are deactivated. The EOS (equation of state) applies the ideal gas equation. In this instance, the precision is specified as double precision (8-byte floating point numbers).

3.2.3 Output files

The time series is a text file that compiles the evolution of the diagnostic quantities indicated in `print.in`. It represents the code's output. To examine the simulation's time evolution, it is helpful to consider the following variables:

1. the number `it` of the current time step;
2. the time, `t`;
3. the time step, `dt`;
4. the rms velocity, `urms` = $\sqrt{\mathbf{u}^2}$;
5. the maximum velocity, `umax` = $\max |\mathbf{u}|$;
6. the rms magnetic field, `brms` = $\sqrt{\mathbf{B}^2}$;
7. the mean density, `rhom` = ρ ;
8. the kinetic helicity, `oum` = $\boldsymbol{\omega} \cdot \mathbf{u}$;
9. the mean thermal (internal) energy, `ethm` = ρe ;
10. the mean entropy, `ssm` = s/c_p .

The primary variables (log density, velocity, magnetic vector potential, and entropy) are incorporated into snapshot files. In accordance with the user's specifications, including modifications to the resolution and input variables, these files can be employed to initiate a simulation from a designated point in time. Two categories of snapshots are written: permanent and current snapshots. Both types of snapshot are stored within the `data/procN` directory.

Slice files, which can be defined in `run.in`, provide variables in a specific plane. The program `src/read_all_videofiles.x` is employed for the purpose of reading and combining the video files, while the slices are written by each processor that coincides with the slice and can be located in `data/proc*/`.

It is possible to utilize one or two dimensions for the purposes of calculating the averages. Other files that can be found are `xyaver.in`, `xzaver.in`, and `yzaver.in`, which contain the data, with the averaging directions indicated by the first two letters. However, for our analysis we are only concerned with the `phiaver.in`.

3.3 Set up of the star-in-a-box model

We use the star-in-a-box model by Käpylä (2021), which is based on the setup of Dobler et al. (2006). This model allows dynamo simulations of fully convective stars. Therefore, new set ups were implemented that allowed us to simulate partially convective stars, which will be discussed in more detail below (Käpylä, 2022).

In this instance, a sphere of radius R is situated within a cubic box of length $l = 2.2R$. The magnetohydrodynamic equations of induction, continuity, motion and energy conservation are then solved as follows:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}, \quad (3.3.1)$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{u}, \quad (3.3.2)$$

$$\frac{D \mathbf{u}}{Dt} = -\nabla \Phi - \frac{1}{\rho} (\nabla p - \nabla \cdot 2\nu \rho \mathbf{S} + \mathbf{J} \times \mathbf{B}) - 2\boldsymbol{\Omega} \times \mathbf{u} + \mathbf{f}_d, \quad (3.3.3)$$

$$\rho T \frac{Ds}{Dt} = -\nabla \cdot (\mathbf{F}_{\text{rad}} + \mathbf{F}_{\text{SGS}}) + \mathcal{H} - \mathcal{C} + 2\nu \mathbf{S}^2 + \mu_0 \eta \mathbf{J}^2, \quad (3.3.4)$$

where the magnetic vector potential is represented by $\mathbf{A}$, the velocity field is denoted by $\mathbf{u}$, the magnetic field is given by $\mathbf{B} = \nabla \times \mathbf{A}$, the magnetic permeability of the vacuum is expressed as μ_0 , the magnetic diffusivity is represented by η , and the convective derivative is defined as $D/Dt = \partial/\partial t + \mathbf{u} \cdot \nabla$. The temperature is denoted by T , the pressure by p , the kinematic viscosity by ν , the specific entropy by s , the current density by $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ and, the angular velocity is given by $\boldsymbol{\Omega} = (0, 0, \Omega_0)$ and, $\mathbf{f}_d$ is a damping function. $\mathbf{F}_{\text{rad}}$ and $\mathbf{F}_{\text{SGS}}$ are the radiative and subgrid-scale (SGS) entropy fluxes, and $\mathcal{H}$ and $\mathcal{C}$ describe heating and cooling, respectively. The gas obeys the ideal gas equation of state, which is given by $p = \mathcal{R} \rho T$, where $\mathcal{R}$ is the gas constant, which is equal to $c_p - c_v$, and the heat capacities at constant pressure and volume are represented by c_p and c_v , respectively. The gravitational potential, designated as Φ , which is associated with the star's isentropic hydrostatic condition, is determined by

$$\Phi(r) = -\frac{GM}{R} \frac{a_0 + a_2 r'^2 + a_3 r'^3}{1 + b_2 r'^2 + b_3 r'^3 + a_3 r'^4}, \quad (3.3.5)$$

where G is the gravitational constant, M is the mass of the star, and $r = r/R$. The

coefficients for a M5 dwarf are given by $a_0 = 2.3401$, $a_2 = 0.44219$, $a_3 = 2.5952$, $b_2 = 1.5986$, $b_3 = 0.20851$. The gravitational potential is equivalent to a main-sequence M5 star's polytrope with an index of $n = 1.5$ (see Appendix A of [Dobler et al., 2006](#)).

$\mathbf{S}$ is the traceless rate-of-strain tensor,

$$\mathbf{S}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{1}{3} \delta_{ij} \nabla \cdot \mathbf{u}, \quad (3.3.6)$$

where δ_{ij} is the Kronecker delta. Flows in the exterior to the star are damped through the term $\mathbf{f}_d$, which is given by

$$\mathbf{f}_d = -\frac{\mathbf{u}}{\tau_{\text{damp}}} f_e(r), \quad (3.3.7)$$

where $\tau_{\text{damp}} \approx 0.2\tau_{ff}$ is a damping timescale, where $\tau_{ff} = \sqrt{R^3/GM}$ is the free-fall time, G is the gravitational constant, and M is the mass of the star. $f_e(r)$ is given by

$$f_e(r) = \frac{1}{2} \left(1 + \tanh \frac{r - r_{\text{damp}}}{\Omega_{\text{damp}}} \right), \quad (3.3.8)$$

where $r_{\text{damp}} = 1.03R$ is the damping radius and $\Omega_{\text{damp}} = 0.03R$ is its the width.

The radiative flux is given by $\mathbf{F}_{\text{rad}} = -K\nabla T$, where

$$K(\rho, T) = K_0(\rho/\rho_0)^{a-1}(T/T_0)^{b+3}, \quad (3.3.9)$$

with $a = -1$ and $b = 7/2$ corresponds to the Kramers opacity law (e.g. [Brandenburg et al. 2000](#)). Furthermore, subgrid-scale (SGS) entropy flux is incorporated using $\mathbf{F}_{\text{SGS}} = \chi_{\text{SGS}}\rho\nabla s'$, where $s' = s - \langle s \rangle_t$ are the entropy fluctuations, and $\langle s \rangle_t$ is a running temporal mean of the specific entropy. A normalized Gaussian profile provided by the parametrization of the star's core nuclear energy generation by the expression given by

$$\mathcal{H}(r) = \frac{L_{\text{sim}}}{(2\pi w_L^2)^{3/2}} \exp\left(-\frac{r^2}{2w_L^2}\right), \quad (3.3.10)$$

which describe additional heating, L_{sim} is the luminosity and $w_L = 0.162R$ is the width of the Gaussian. The cooling term $\mathcal{C}$ models radiative losses above the

stellar surface, and is given by

$$\mathcal{C}(\mathbf{x}) = \rho c_P \frac{T(\mathbf{x}) - T_{\text{surf}}}{\tau_{\text{cool}}} f_e(r), \quad (3.3.11)$$

where $T_{\text{surf}} = T(R)$ is the temperature on the surface of the star, $\tau_{\text{cool}} = \tau_{\text{damp}}$ is a cooling timescale, and $f_e(r)$ is given by equation 3.3.8, with the same parameters.

The set-up of the simulations is identical to those in Käpylä (2022), with the amplitude of the radiative conductivity K_0 increased in the thermodynamically saturated state, to have a star with a radiative core and a convective envelope, which covers about 0.6 and 0.4 of the stellar radius, respectively.

3.3.1 Diffusion coefficients

To prevent the diffusion of magnetic fields and flows into the core, the diffusion coefficients η and ν have radial profiles that make their values in the radiative core 10^2 smaller than in the convective zone (CZ). Nevertheless, this phenomenon occurs during the initial transient phase, which precedes the attainment of a statistically stable state for the flow. It should be noted that this occurs in a considerable number of runs. This is due to the fact that the initial state of the simulations is an isentropic, which results in the star being initially fully convective.

One disadvantage of this implementation is that the low diffusivities in the radiative core are challenging to resolve with the current resolution. To address this issue, the dynamical equations can be modified to include additional sixth-order hyperdiffusivity.

It cannot be guaranteed that the results will converge with those obtained using conventional second-order dissipation; hyperdiffusivity is intended only as a numerical tool to dissipate energy at small scales (see Haugen and Brandenburg 2004 for a discussion). Indeed, simulations of closed systems where magnetic helicity is conserved have been demonstrated to result in notable alterations to the large-scale magnetism, leading to prolonged saturation periods and enhanced saturation amplitudes (Brandenburg and Sarson, 2002). In this context, the hyperviscosity force represents the additional term required in the momentum equation 3.3.3. The definition was derived through the application of the resolution-

independent mesh hyper-Reynolds number method, as outlined in (Lyra et al., 2017), Appendix A. This suggests that the relationship between the second-order coefficient and the Reynolds number is linear with respect to resolution. In order to maintain the hyper-Reynolds number constant with resolution, it is necessary to rewrite this in terms of a coefficient.

The number of hyper-Reynolds number is

$$\text{Re}_{\text{grid}} = \frac{u_{\text{rms}}}{\nu_n^{\text{mesh}} k_{\text{Ny}}^{2n-1}}, \quad (3.3.12)$$

where $k_{\text{Ny}} = \pi/\Delta q$ is the Nyquist wavenumber, and Δq is the line element. The $n = 3$ hyperdiffusion coefficient should thus be

$$\nu_3^{\text{mesh}} = \frac{u_{\text{rms}} \Delta q^5}{\text{Re}_{\text{grid}}^{(3)} \pi^5}. \quad (3.3.13)$$

So, the hyperviscous force is giving by

$$f_{\text{visc}}^{(\text{hyper})} = \nu_3^{\text{mesh}} \frac{\Delta^6 \nu}{\Delta q^6} = \frac{u_{\text{rms}}}{\text{Re}_{\text{grid}}^{(3)} \pi^5} \frac{\Delta^6 \nu}{\Delta q}. \quad (3.3.14)$$

Similarly, an additional term pertaining to the magnetic hyperdiffusivity is required in the induction equation 3.3.1. By employing the same methodology, we obtain:

$$f_{\text{diff}}^{(\text{hyper})} = \eta_3^{\text{mesh}} \frac{\Delta^6 \eta}{\Delta q^6} = \frac{u_{\text{rms}}}{\text{Re}_{\text{Mgrid}}^{(3)} \pi^5} \frac{\Delta^6 \eta}{\Delta q}. \quad (3.3.15)$$

3.3.2 Relation to reality

All of the output of variables in the PENCIL CODE are expressed in dimensionless code units. Consequently, a link to reality using scale factors is required in order to assess the data in physical units. The following values will be employed for the mass parameter $M_\star = 0.45 M_\odot$, stellar radius $R_\star = 0.39 R_\odot$, luminosity $L = 0.0282 L_\odot$, and central density $\rho_c^\star = 68870 \text{ [kg m}^{-3}\text{]}$, in accordance with the data set delineated in Section 3.1.3 of the MESA model.

The dimensionless luminosity is defined as

$$\mathcal{L} = \frac{L}{\sqrt{G^3 M^5 / R^5}},$$

In the current simulation have $\mathcal{L} = 5.8 \times 10^{-5}$ and the value from the star simulated is $\mathcal{L}_\star \approx 3.17 \times 10^{-14}$. From here we can get a luminosity ratio where the velocities are enhanced by this factor, and the rotation rate of the star needs to be increased by the same factor to model a realistic rotational influence on the flow. Thus, the velocities are enhanced by a factor $L_{\text{ratio}}^{1/3} \approx 1223$, where $L_{\text{ratio}} = \mathcal{L} / \mathcal{L}_\star \approx 1.83 \times 10^9$. For the rotation rate conversion factor we have:

$$\Omega^{\text{sim}} = L_{\text{ratio}}^{1/3} \left(\frac{g^{\text{sim}}}{g_\star} \frac{R_\star}{R^{\text{sim}}} \right)^{1/2} \Omega_\star.$$

The surface gravity for the star is

$$g_\star = GM_\star / R_\star^2 = \frac{0.45}{(0.39)^2} \frac{GM_\odot}{R_\odot^2} \approx 2.96 g_\odot \approx 810 \text{ [m/s}^2\text{]},$$

where $g_\odot = 274 \text{ [m/s}^2\text{]}$. Then, using the solar rotation rate $\Omega_\star = \Omega_\odot = 2.7 \times 10^{-6} \text{ [s}^{-1}\text{]}$, yields

$$\Omega_\odot^{\text{sim}} = 1.92 \left(\frac{g^{\text{sim}}}{R^{\text{sim}}} \right)^{1/2},$$

where $g^{\text{sim}} = g^{\text{sim}}(r = R) \approx 1$ and $R^{\text{sim}} = 1$ in simulation units.

Next we proceed to compute the length conversion factor, that is given by,

$$\mathbf{x} \text{ [m]} = \left(\frac{R_\star}{R^{\text{sim}}} \right) \mathbf{x} \text{ [sim.units]}.$$

For time is assumed that the rotation rate of the star in the simulation matches the one in reality, such that

$$t \text{ [s]} = \left(\frac{\Omega^{\text{sim}}}{\Omega_\star} \right) t \text{ [sim.units]}.$$

Velocity is naturally the ratio of the factors of length and time,

$$\mathbf{U} \text{ [m s}^{-1}\text{]} = \left(\frac{\Omega_\star R_\star}{\Omega^{\text{sim}} R^{\text{sim}}} \right) \mathbf{U} \text{ [sim.units]}.$$

For the magnetic field it is necessary to have a reference density in physical units (ρ_c^*), such that the conversion factor is given by,

$$\mathbf{B} = \left[\frac{\mu_0 \rho_c^* (\Omega_\star R_\star)^2}{\mu_0^{\text{sim}} \rho_0^{\text{sim}} (\Omega^{\text{sim}} R^{\text{sim}})^2} \right]^{1/2} \mathbf{B}^{\text{sim}},$$

where $\mu_0 = 4\pi \cdot 10^{-7} \text{ H m}^{-1}$ is the permeability of vacuum, $\mu_0^{\text{sim}} = 1$, $\rho_c^{\text{sim}} \approx 1$; this gives the magnetic field conversion factor to Tesla. After substitution and conversion to nicer units (Mm, yr, and kG) we find numerically:

$$\mathbf{x} [\text{Mm}] \approx 273 \mathbf{x}^{\text{sim}} [\text{sim.units}], \quad (3.3.16)$$

$$t [\text{yr}] \approx 0.0225 t^{\text{sim}} [\text{sim.units}], \quad (3.3.17)$$

$$\mathbf{U} [\text{ms}^{-1}] \approx 384 \mathbf{U}^{\text{sim}} [\text{sim.units}], \quad (3.3.18)$$

$$\mathbf{B} [\text{kG}] \approx 1130 \mathbf{B}^{\text{sim}} [\text{sim.units}]. \quad (3.3.19)$$

3.3.3 Diagnostic quantities

In our simulations, the system can be described by the dimensionless characteristics that are detailed in Section 2.2.3, specifically the Coriolis number, which is a measure of the influence rotation on the flow, is given by

$$\text{Co} = \frac{2\Omega_0}{u_{\text{rms}} k_R},$$

where u_{rms} is the volume-averaged root-mean-square velocity, and the scale of the largest convective eddies is given by $k_R = 2\pi/0.4R$, which correspond to the convective zone of the star.

The fluid and magnetic Reynolds numbers are giving by

$$\text{Re} = \frac{u_{\text{rms}}}{\nu k_R}, \quad \text{Re}_M = \frac{u_{\text{rms}}}{\eta k_R}.$$

The magnetic and SGS Prandtl number are:

$$\text{Pr}_M = \frac{\nu}{\eta}, \quad \text{Pr}_{\text{SGS}} = \frac{\nu}{\chi_{\text{SGS}}}.$$

The effect of rotation relative to viscosity is measured by the Taylor number,

$$\text{Ta} = \frac{4\Omega_0^2 R^4}{\nu^2},$$

and the SGS Péclet number are defined as:

$$\text{Pe} = \frac{u_{\text{rms}}}{\chi_{\text{SGS}} k_R}.$$

Chapter 4

Three-dimensional MHD simulations with varying magnetic diffusivity and rotation period

This chapter presents a comprehensive account of the thesis results, along with a detailed analysis of the outcomes of the simulations, which include time-latitude diagrams of the averaged toroidal magnetic field, the analysis of large-scale flows, energies, i.e., kinetic (total, differential rotation, and meridional circulation) and magnetic (total, toroidal, and poloidal), large-scale magnetic fields, and flow attributes.

4.1 Simulations

As discussed in Section 2.5, large-scale magnetic fields have been detected on the surfaces of M dwarfs. A principal instrument for understanding the underlying physics in this context is numerical simulation. This section describes the simulations used in this thesis.

Three distinct simulation setups are investigated: one with a flat diffusivity profile, i.e., the diffusivity is constant and independent of radius, one with a diffusivity profile in the core, and one with both a diffusivity profile in the core and an exterior profile. The objective of varying these profiles is to ascertain the sensitivity of the dynamo to boundary conditions. Additionally, the distinct simulation sets (A, B,

Sim	ν [m ² /s]	ν_{cz} [m ² /s]	η [m ² /s]	η_{cz} [m ² /s]	η_{ext} [m ² /s]	P_{rot} [days]	P_{cyc} [yrs]	Grid
A1*	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	-	96	17.21	288 ³
A2**	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	$5.24 \cdot 10^7$	96	32.89	288 ³
B1	$1.05 \cdot 10^7$	$1.05 \cdot 10^7$	$1.05 \cdot 10^7$	$1.05 \cdot 10^7$	-	48	28.86	128 ³
B2*	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	-	48	30.80	128 ³
B3**	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.0 \cdot 10^4$	$5.24 \cdot 10^6$	$5.24 \cdot 10^7$	48	27.06	200 ³
C1	$1.05 \cdot 10^7$	$1.05 \cdot 10^7$	$1.05 \cdot 10^7$	$1.05 \cdot 10^7$	-	24	22.24	128 ³
C2	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	-	24	-	128 ³
C3*	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	-	24	38.69	200 ³
C4**	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	$5.24 \cdot 10^7$	24	-	200 ³
C5	$1.57 \cdot 10^7$	$1.57 \cdot 10^7$	$1.57 \cdot 10^7$	$1.57 \cdot 10^7$	-	24	32.27	128 ³
D1*	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	-	16	53.72	288 ³
D2**	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	$5.24 \cdot 10^7$	16	38.04	288 ³
E1*	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	-	9.6	43.64	288 ³
E2**	$2.62 \cdot 10^4$	$2.62 \cdot 10^6$	$5.24 \cdot 10^4$	$5.24 \cdot 10^6$	$5.24 \cdot 10^7$	9.6	75.08	288 ³

Table 4.1.1: Columns from left to right indicate the name of simulation, the momentum diffusivity in the center of the star and the value in the convective zone. η , η_{cz} and η_{ext} are the magnetic diffusivity in the center of the star, the value in the convective zone, and the value of magnetic diffusivity outside the star, respectively. Then, we have the rotation period $P_{rot} = 2\pi/\Omega_0$, cycle periods, and the grid resolution that was used. One asterisk indicate runs with one jump and two asterisks indicate runs with two jumps.

C, D, and E) correspond to different rotation rates, which are analogous to the values of P_{rot} (rotation period) presented in Table 4.1.1. Simulations B1, C1 and C5 have $\eta = \nu$. In runs A1, B2, C3, D1 and E1, the value of η and ν in the center of the star was decreased by a factor 10^2 below the tachocline, which is situated at about $r = 0.6R$ (first jump). In runs A2, B3, C4, D2 and E2, a second jump in η is introduced, with a factor of 10^3 with respect to the core value. This jump occurs outside of the star, at a radius greater than 1.0. All values employed are presented in Table 4.1.1.

The standard case with $Pr_M = 1$ is selected due to its numerical convenience. Justification for reducing diffusivity in the core is provided by the need to decelerate the diffusion of flows and magnetic fields from the convection zone to the core. Similarly, the profile exterior to the star impedes rapid diffusion and effectively alters the boundary condition of the magnetic field at the surface of the star. The Pr_M values observed in the solar convection zone exhibit a range from 10^{-7} to 10^{-4} , which suggests that cool astronomical bodies with high densities present low Pr_M values (e.g. [Jermyn et al. 2022](#)). Nevertheless, it is not feasible to attain realistically low values of the magnetic Prandtl number through numerical means, as the resolution of simulations is inherently limited by the smallest scale (grid

Sim	u_{rms} [m/s]	B_{rms} [kG]	Pr_M	Pr_{Me}	Pr_{SGS}	Re_M	Re_{Me}	Re	Co	Pe	Ta
A1*	7.55	10.15	0.5	-	0.05	38.49	-	76.99	2.11	3.85	$4.09 \cdot 10^7$
A2**	7.68	10.41	0.5	0.049	0.05	39.48	3.95	78.96	2.05	3.95	$4.09 \cdot 10^7$
B1	5.38	6.88	1.0	-	0.2	13.66	-	13.66	5.93	2.73	$1.02 \cdot 10^7$
B2*	6.15	7.98	0.5	-	0.05	31.34	-	62.69	5.17	3.13	$1.64 \cdot 10^8$
B3**	6.14	7.83	0.5	0.049	0.05	30.69	3.07	61.39	5.28	3.07	$1.64 \cdot 10^8$
C1	4.61	5.81	1.0	-	0.2	11.64	-	11.64	13.93	2.33	$4.09 \cdot 10^7$
C2	6.63	8.01	0.5	-	0.05	33.79	-	67.57	9.60	3.38	$6.56 \cdot 10^8$
C3*	6.53	6.52	0.5	-	0.05	26.16	-	52.31	12.40	2.62	$6.56 \cdot 10^8$
C4**	4.99	6.29	0.5	0.049	0.05	25.52	2.55	51.03	12.71	2.55	$6.55 \cdot 10^8$
C5	4.22	5.34	1.0	-	0.6	7.09	-	7.09	15.25	4.25	$1.82 \cdot 10^7$
D1*	4.50	5.81	0.5	-	0.05	22.95	-	45.90	21.19	2.29	$1.47 \cdot 10^9$
D2**	4.99	6.15	0.5	0.049	0.05	24.46	2.45	48.93	19.88	2.45	$1.47 \cdot 10^9$
E1*	3.31	4.31	0.5	-	0.05	16.89	-	38.29	48.00	1.69	$4.09 \cdot 10^9$
E2**	3.76	4.79	0.5	0.049	0.05	19.15	1.91	38.29	42.34	1.91	$4.09 \cdot 10^9$

Table 4.1.2: Summary of the simulations and dimensionless parameters. From left to right the columns correspond to the simulation name, root-mean-square velocity, root-mean-square magnetic field strength, magnetic Prandtl number and its value in the exterior of the star, and sub-grid-scale Prandtl number, magnetic Reynolds number and its value in the exterior of the star, and fluid Reynolds number, Coriolis number which is $\text{Co} = \text{Ro}^{-1}$, Peclet and Taylor numbers.

spacing) that can be resolved, depending on the computational resources available. In practical, real-world systems, the smallest scale is significantly smaller than the smallest scale that can be represented in simulations. Furthermore, other dimensionless factors, such as the Reynolds number and the fluid Prandtl number, are also affected by this limitation. In order to achieve more realistic results, it is therefore highly beneficial to be able to operate at the highest resolution feasible.

In particular, a variety of resolutions were employed, starting with the minimum possible, which is a grid size of 128^3 . These simulations were done with a $\text{Pr}_M = 1$ and a low rotation rate ($\Omega = 1.0 - 2.0$), with the exception of B2, which had $\text{Pr}_M = 0.5$ and $\Omega = 1.0$. This approach was also applied to C2, which had faster rotation ($\Omega = 2.0$). However, this simulation was numerically unstable at a resolution 128^3 after about 60 years simulated time.

In consequence of the aforementioned reasons, the resolution was increased to 200^3 in runs B3, C3 and C4. For sets A, D and E, a grid size of 288^3 was employed, given that the last two sets feature Ω of 3.0 and 5.0, respectively. So that, the addition of diffusivity profiles necessitates an increase in resolution. Table 4.1.2 contains a summary of the simulations performed here, which are divided into five sets. Set A is for simulations with $P_{\text{rot}} = 96$ days, set B for $P_{\text{rot}} = 48$ days, set C is for $P_{\text{rot}} = 24$ days, set D for $P_{\text{rot}} = 16$ days and set E $P_{\text{rot}} = 9.6$ days. In addition

to the dimensionless parameters described before (section 2.2.3). As expected, the values of the Coriolis number increase as the rotation period increases. It should be noted that the values of Pr_{SGS} are distinct in simulations B1, C1 and C5. This is due to the absence of diffusivity profiles in the first two simulations and the variation in the values of ν and χ_{SGS} , which are 1.5×10^{-4} and 2.5×10^{-4} , respectively, in C5.

4.2 Root-mean-square magnetic fields

Large-scale magnetic fields are defined as those exhibiting coherence across extensive spatial dimensions. In the majority of the simulations presented, these fields are comparable in structure to those observed in the Sun. A seed magnetic field is enhanced in the kinematic regime, wherein the field rises exponentially before the emergence of large-scale magnetic fields. In the current simulations, the seed field is a random, small-scale field. The magnetic field growth enters a saturated and non-linear stage when the kinematic stage ends and the magnetic forces approach the turbulence forces. The study of the effects of magnetic fields in stars is interested in this final stage. Figure 4.2.1 presents the time-dependent evolution of the volume-averaged root-mean-square (rms) magnetic field, B_{rms} , for simulations where the rotation rate change inside the star (denoted by an asterisk in the Table 4.1.2). From this, it can be observed that the kinematic phase is similar for all simulations, entering a saturated stage after approximately 14 to 20 years. It can be observed that B2, C3 and E1 conclude their kinematic phase at 15 years, while in the case of A1 and D1 this occurs at approximately 19 years. From Table 4.1.2, it is visible that all simulations have very similar $\langle B_{\text{rms}} \rangle$, irrespective of the rotation rate employed. Conversely, an examination of the different diffusivity profiles reveals that those with a flat profile, specifically B1, C1, and C5, exhibit the lowest $\langle B_{\text{rms}} \rangle$ values. It should be noted, however, that the maximum values of the rms magnetic fields in the simulations presented in Fig. 4.2.1 (with one diffusivity jump) range from 6 to 8 kG, with run B2 exhibiting the highest value of 8.97 kG.

Figure 4.2.2 illustrates the same as Fig. 4.2.1, but for simulations in which the diffusivities vary both inside and outside the star (denoted by two asterisks in the table 4.1.2). In this case, runs B3, C4 and D2 reach the saturated phase

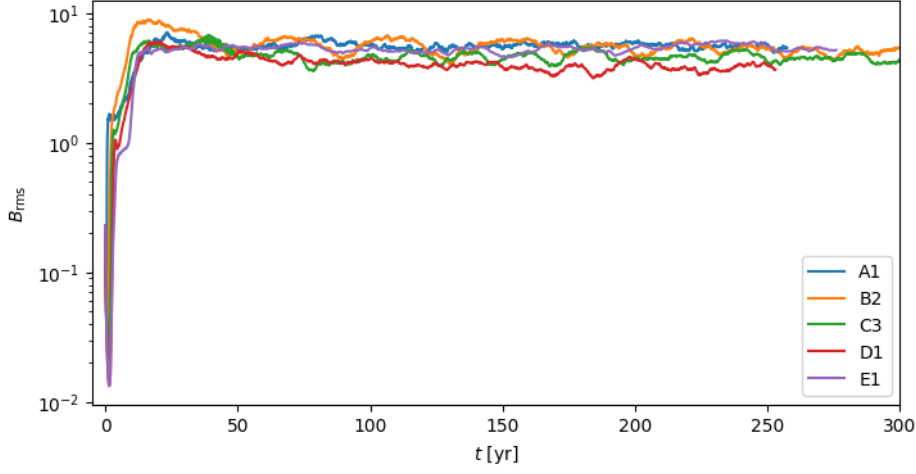


Figure 4.2.1: Time evolution of volume averaged root-mean-square magnetic field for simulations with one diffusivity jump, denoted by an asterisk in the tables.

after approximately 15 years, while simulations A2 and E2 reach this phase after approximately 20 years. In contrast to the case with a jump (Fig. 4.2.1), we can note that the evolution of the B_{rms} over time presents greater dispersion. For example, the maximum values observed fall within the range of 4 to 9 kG, where B3 is the simulation that reaches the highest value of 9.13 kG, while E2 has the lowest value of 4.30 kG.

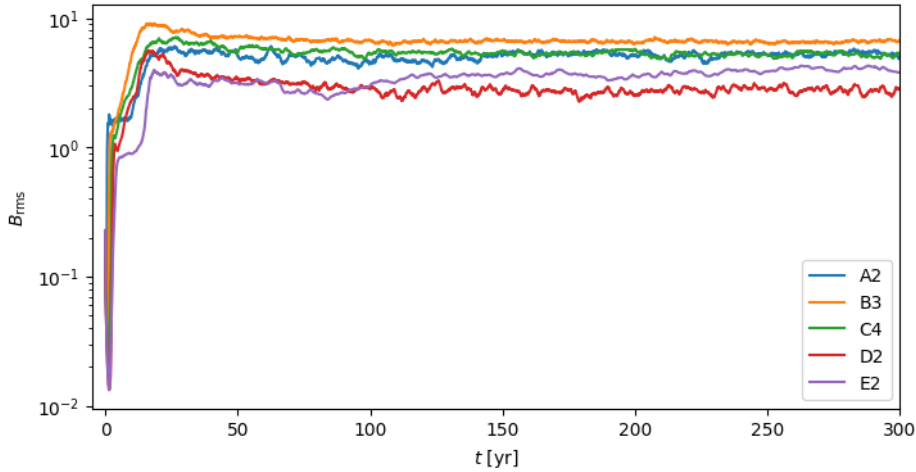


Figure 4.2.2: Time evolution of volume averaged root-mean-square magnetic field for simulations with two diffusivity jump, denoted by two asterisks in the tables.

4.3 Toroidal magnetic field

A number of the models exhibit large-scale magnetic field cycles with intervals between 20 and 55 years, with the exception of runs A2 and E2, which display cycles of 17 and 75 years, respectively. The following figures showing the azimuthally averaged toroidal magnetic field, $\overline{B}_\phi(R, \theta, t)$ at the surface of the star, illustrate these cycles. Cycles were obtained by the Fast Fourier Transform (FFT) at the equator for simulation C3, D1 and E1, and the remaining cases were calculated at latitude 15° , thereby confirming the cycles. The analysis will start by comparing each of the different diffusivity profiles within the same set. This will be followed by a comparison of the different diffusivities between simulations, which will allow us to observe how the magnetic fields behave changing only the rotation rates (Ω).

The results of the simulations of set A are shown in Figure 4.3.1; the left panel shows the result in the case with a diffusivity jump inside the star A1. The magnetic field does not exhibit clear periodicity, displaying instead a state of irregularity with no evident change in polarity. However, a field with a dipole component is discernible, although its precise nature remains ambiguous. Run A2 with diffusivity jumps inside and outside the star, shows changes in polarity until around 140 years, after which it is in a quasi-stationary mode.

Figure 4.3.2 shows the results for runs B1, B2, and B3. In run B1 with no diffusivity profiles, the magnetic field is primarily concentrated at the equator, between latitudes spanning approximately 35° north and south, where irregular reversals occur between quasi-static periods. In B2 (upper right panel), with a diffusivity jump inside the star, a quasi-static field is superimposed with a cyclic pattern with equatorward migration of activity belts towards the end of the simulation, similarly as in Käpylä et al. (2017). In run B3 (bottom panel), with diffusivity jumps inside and outside the star, the cycles disappear.

For set C the results are shown in Figure 4.3.3. For run C1 (upper left panel) the dynamo is irregular, and concentrated near the equator with sporadic activity with long quiescent periods. The upper right panel shows the dynamo of run C3 which has regular cycles near the equator with a period about $P_{\text{cyc}} = 39$ years. The bottom left panel corresponds to simulation C4 with a quasi-static dynamo in the northern hemisphere. The bottom right panel corresponds to run C5 with a quasi-static dynamo with some moments of inactivity, similar to that seen in

run C1. This phenomenon is explored further in Figure 4.3.4, which shows the evolution of u_{rms} and B_{rms} through time for runs C1 and C5, and it can be seen how the rms magnetic field and rms velocity behave in parallel. It can be noted that in both instances, the velocity demonstrates a slight increase concurrent with the decay of the magnetic field, with this phenomenon being more pronounced in the case of C5 (bottom panel). To illustrate, if we consider the year 220 of the upper panel (run C1), the value of $u_{\text{rms}}^{t=220} = 7.26 \text{ ms}^{-1}$ and $B_{\text{rms}}^{t=220} = 0.28 \text{ kG}$. A comparison with the values for C1 in Table 4.1.2 reveals that the velocity at that time is 1.5 times higher than the average, while the magnetic field is 0.048 times lower than the average. We can do the same for C5, considering $t = 1572$, which is the time when the rms magnetic field exhibits a pronounced decline, reaching a value of 0.05 kG, while the rms velocity reaches a value of 6.92 ms^{-1} . A comparison of the values in Table 4.1.2 for run C5 reveals that the same phenomenon observed in C1 occurs, whereby the velocity at that time is 1.6 times above the average, while the magnetic field value is 0.009 times below the average.

This fluctuations in the magnetic field are in alignment with the periods of inactivity observed in the butterfly diagrams illustrated in the Figure 4.3.3 for simulation C1 and C5. Also, it is notable that these two simulations exhibit the lowest magnetic Reynolds numbers (see Table 4.1.2) and the highest magnetic diffusivities (see Table 4.1.1), which is consistent with the observed quenching of the magnetic field. Note that this two simulation have flat diffusivity profiles, i.e., it is constant.

Results of set D are show in Figure 4.3.5. Run D1 exhibits an evolution of the magnetic field at the surface that is similar to that of the Sun, with activity moving equatorward and strong radial fields concentrated at latitudes $|\theta| < 40^\circ$. A similar result has been seen previously with similar simulations in (see Figure 1 of Käpylä 2022). The case of D2 has shorter and more pronounced cycles than D1, concentrated at the same latitudes and showing regular reversals.

Finally, Figure 4.3.5 shows the results for set E. The lower panel shows run E1 where a solar-like pattern of magnetic field evolution is observed in the same latitudes as in D1, with activity propagating equatorward. This can also be seen in Figure 1 of Käpylä (2022). On the other hand, run E2 has a dipole mode mixed with a quasi-stationary mode, and it is not very clear which one dominates.

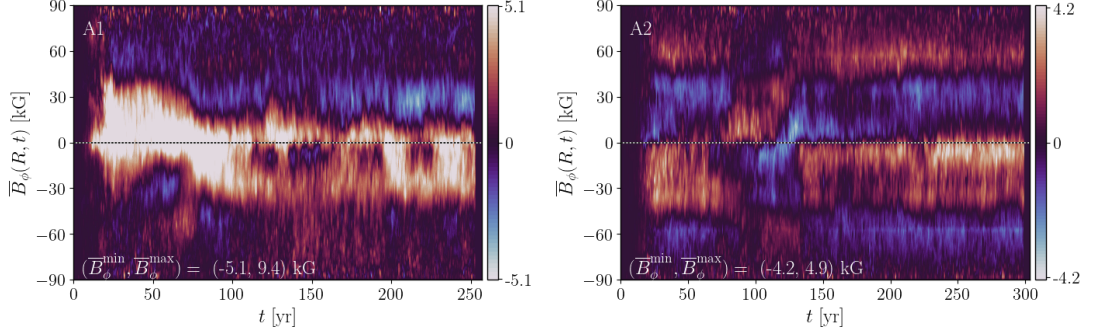


Figure 4.3.1: Time evolution of the azimuthally averaged toroidal magnetic field $\overline{B}_\phi$ at the surface of the star for simulations of the set A, where the run is indicated in the upper left, and the minimum and maximum field strength at the bottom of each panel.

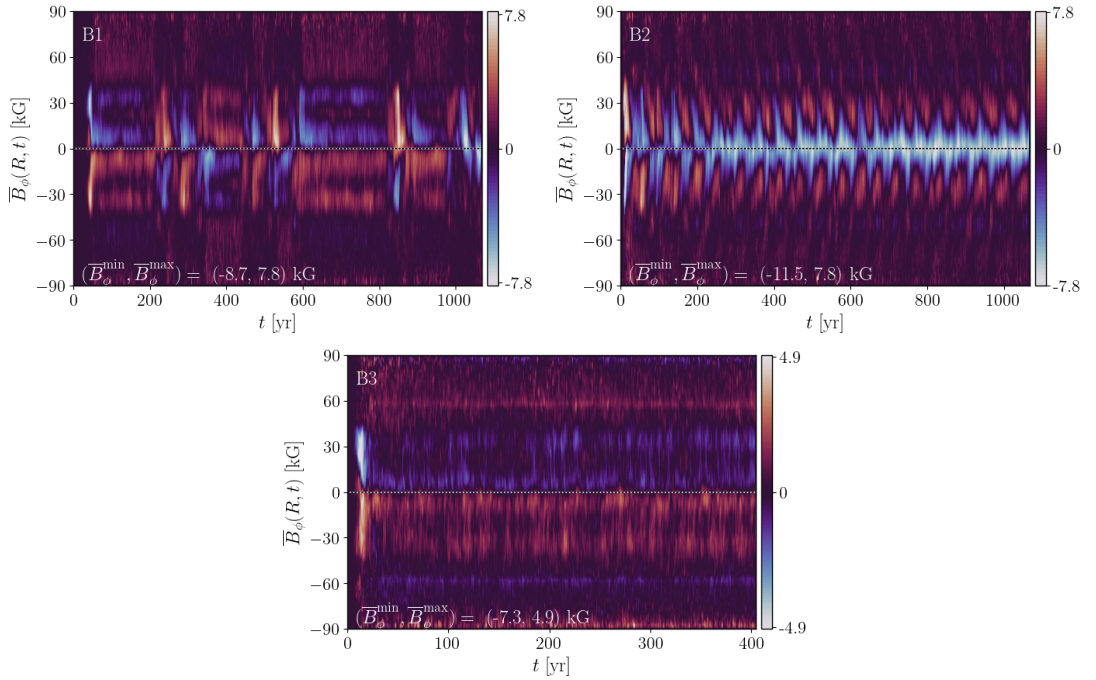


Figure 4.3.2: Time evolution of the azimuthally averaged toroidal magnetic field $\overline{B}_\phi$ at the surface of the star for simulations of set B, where the run is indicated in the upper left, and the minimum and maximum field strength at the bottom of each panel.

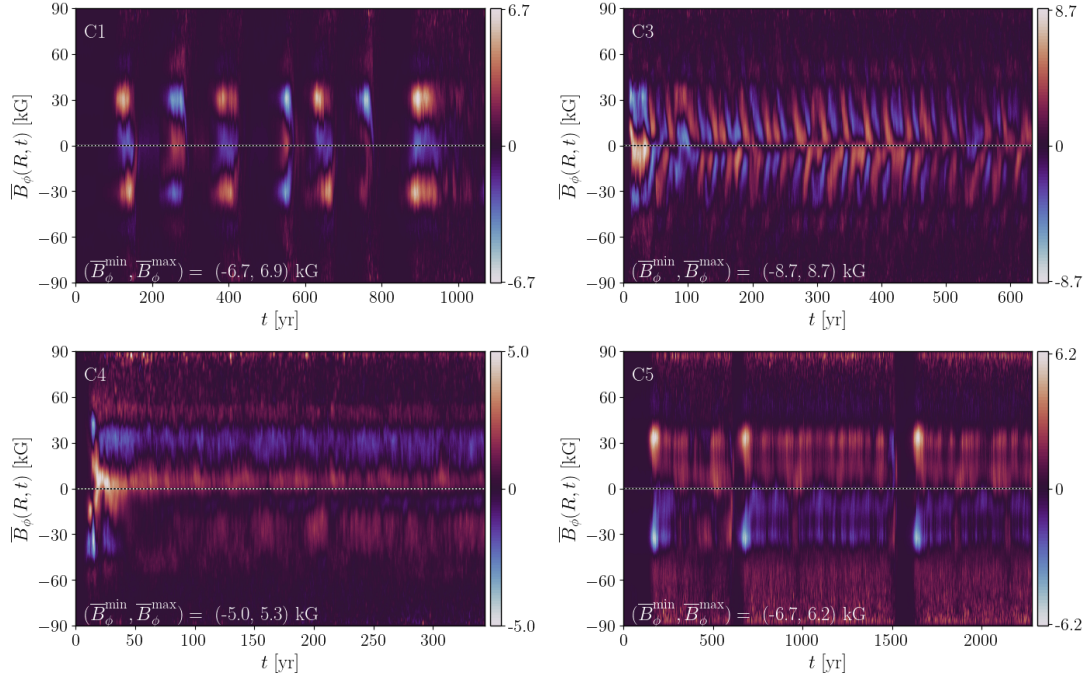


Figure 4.3.3: Time evolution of the azimuthally averaged toroidal magnetic field $\overline{B}_\phi$ at the surface of the star for simulations of the set C, where the run is indicated in the upper left, and the minimum and maximum field strength at the bottom of each panel.

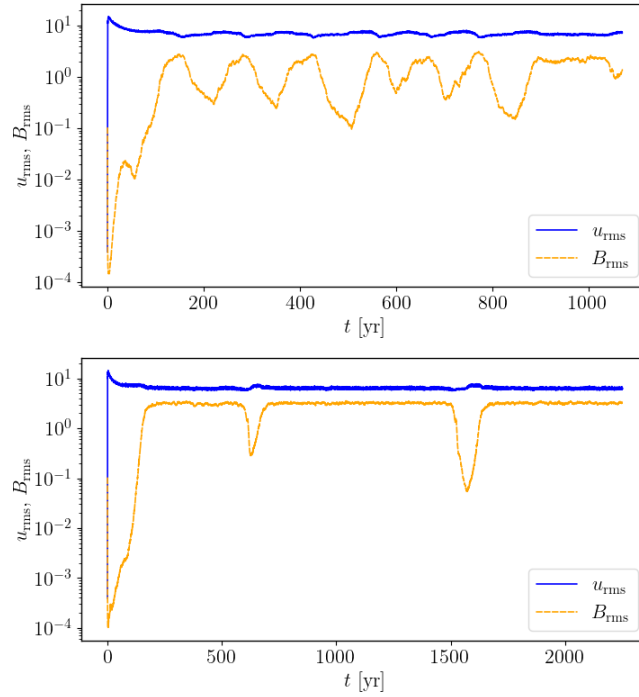


Figure 4.3.4: Evolution of root-mean-square velocity u_{rms} and magnetic field B_{rms} . The top panel is for run C1 and the bottom panel is for run C5. Time, velocity and magnetic field are measured in years, ms $^{-1}$ and kG, respectively.

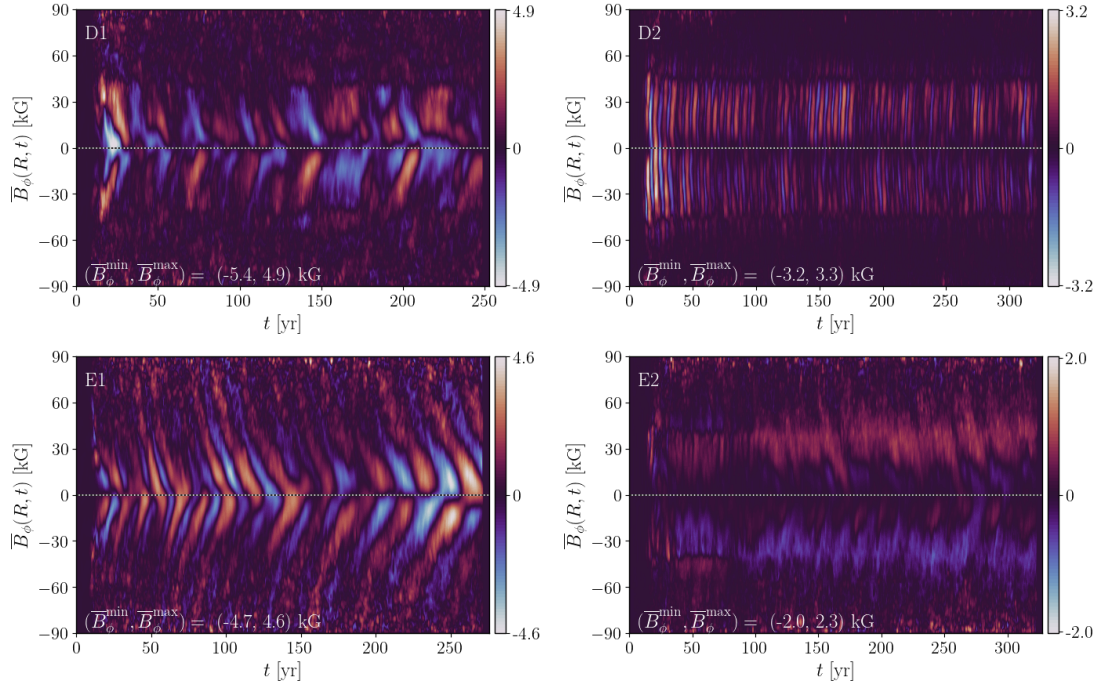


Figure 4.3.5: Time evolution of the azimuthally averaged toroidal magnetic field $\bar{B}_\phi$ at the surface of the star for simulations of the set D (upper panels) and set E (bottom panels), where the run is indicated in the upper left, and the minimum and maximum field strength at the bottom of each panel.

Comparing the sets of runs, similarities appear depending on the boundary conditions used. The runs B2, C3, D1 and E1 (with one jump) exhibit a resemblance to the solar butterfly diagram, with C3 and E1 displaying the most regular cycles. However, A1 is an exception to this, as it shows no indication of a cycle. On the other hand, runs A2, B3, C4, and E2 (with two jumps) have quasi-static dynamos, characterised by the absence of polarity changes. Nevertheless, in case D2, this phenomenon is not observed; instead, very short cycles are observed.

A plausible reason for the differences in the dynamos depending on the diffusivity profiles is that if magnetic fields cannot penetrate (or they need more time to do so) the core and the exterior, the possible dynamo modes that can be excited will also be different from the case where uniform diffusion is used. This is most clearly seen in the case where a larger diffusivity is applied in the exterior of the star (runs B3, C4 and E2) in which cases the cyclic solutions disappear. This can be understood as a lower excitation threshold for quasi-static solution in this geometry. It less clear why the dynamos in Runs C1 and C5 operates only intermittently. A possibility is that the dynamo is only weakly supercritical for

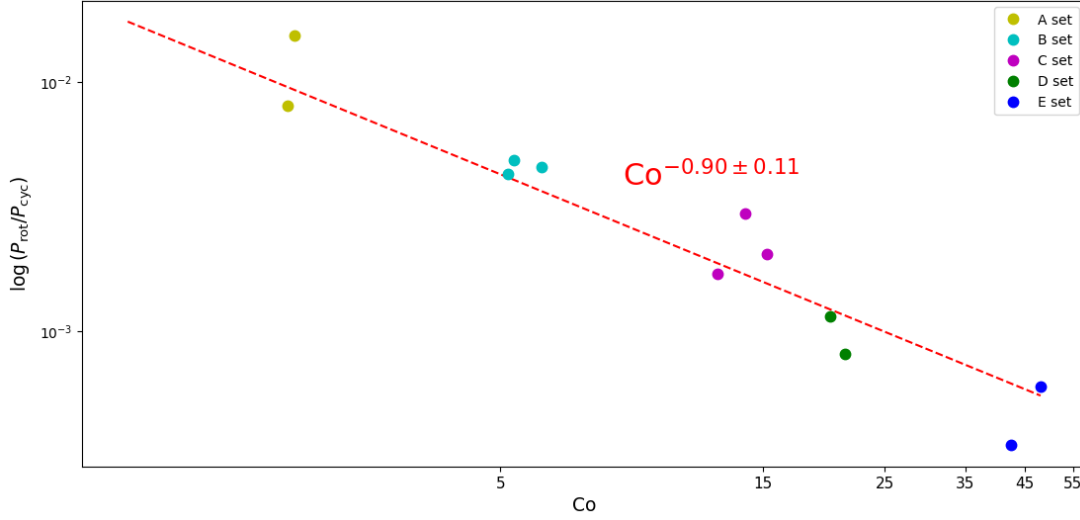


Figure 4.3.6: Ratio of rotation period to cycle period as a function of the global Coriolis number. Simulations C2 and C4 there are not in the plot because no cycles were found.

this combination of Ω and constant diffusivity. A more thorough analysis in terms of mean-field dynamo theory is needed to ascertain the precise causes for the changing behavior, which is not the aim of this thesis.

The models encompass a range of Coriolis numbers between 2.05 and 48. However, the majority of simulations indicate Coriolis numbers within the range of 2.05 and 21.19, where cyclic dynamos are found. Numerous late-type stars undergo magnetic cycles resembling the Sun, according to long-term measurements of their chromospheric emission (e.g. [Baliunas et al. 1995](#)). In addition, it has been proposed that the ratio of cycle periods to rotation periods falls into one of several branches depending on the Coriolis number (e.g. [Brandenburg et al. 1998, 2017](#)). These investigations indicate dormant and active branches with $P_{\text{rot}}/P_{\text{cyc}} \propto \text{Co}^\beta$ where $\beta > 0$. In any case, there is an ongoing debate regarding the precise nature ([Olsper et al., 2018](#)), and significance of the branches (e.g. [Boro Saikia et al. 2018](#)). Nevertheless, it is intriguing to calculate the ratio $P_{\text{rot}}/P_{\text{cyc}}$ from simulations and look for any patterns. This analysis has been employed in a limited number of studies: [Warnecke \(2018\)](#) found $\beta = -0.98 \pm 0.04$ whereas a somewhat steeper relation with $\beta = -1.6 \pm 0.14$ was reported by [Strugarek et al. \(2018\)](#). Furthermore, the results from a more heterogeneous set of simulations by [Viviani et al. \(2018\)](#) also suggest $\beta < 0$; all of them consider solar-like stars. In a recent paper, [Ortiz-Rodríguez et al. \(2023\)](#) it was found that $\beta = 1.30 \pm 0.26$

for fully convective M dwarfs. A notable exception is the study of [Guerrero et al. \(2019\)](#) who found a flat ($\beta \approx 0$) or slightly positive β for rapid rotation. Another instance in which a positive slope was identified is that of [Käpylä \(2022\)](#) for the surface cycles, where $\beta = 0.13 \pm 0.17$.

Figure 4.3.6 shows the ratio of the rotation period to cycle period as a function of the global Coriolis number. It is found that $P_{\text{rot}}/P_{\text{cyc}} \propto \text{Co}^\beta$ with $\beta = -0.90 \pm 0.11$, which is consistent with previous studies mentioned above, where also found $\beta < 0$ for global simulations of solar-like stars.

4.4 Large-scale flows

A principal element in the generation of large-scale magnetic fields is differential rotation. To this end, it is necessary to define the mean rotation rate, which can be expressed as follows:

$$\bar{\Omega}(\varpi, z) = \Omega_0 + \bar{U}_\phi(\varpi, z)/\varpi, \quad (4.4.1)$$

where $\varpi = r \sin \theta$ is the cylindrical radius.

The profiles of $\bar{\Omega}$ are shown in Figure 4.4.1 for set A, in Figure 4.4.2 for set B, in Figure 4.4.3 for set C, and in Figure 4.4.4 for sets D and E. The value of the averaged meridional flow is given as

$$\bar{U}_{\text{mer}}(\varpi, z) = (\bar{U}_\varpi, 0, \bar{U}_z). \quad (4.4.2)$$

The angular velocity varies in latitude and depth. The amplitude of the radial $\Delta_\Omega^{(r)}$ and latitudinal $\Delta_\Omega^{(\bar{\theta})}$ differential rotation can be employed as means of quantifying this phenomenon, and are given by

$$\Delta_\Omega^{(r)} = \frac{\bar{\Omega}_{\text{top,eq}} - \bar{\Omega}_{\text{bot,eq}}}{\bar{\Omega}_{\text{top,eq}}}, \quad \Delta_\Omega^{(\bar{\theta})} = \frac{\bar{\Omega}_{\text{top,eq}} - \bar{\Omega}_{\text{top,\bar{\theta}}}}{\bar{\Omega}_{\text{top,eq}}}, \quad (4.4.3)$$

where the subscripts indicate depth and latitude, respectively. The word *top* refers to a near surface layer, i.e., $r_{\text{top}} = 0.9R$, since the damping of velocities applied in the exterior is likely to alter the values at $r = R$, it is not recommended to utilize the surface velocities to measure the differential rotation. The word *bot* refers to

Sim	$\Delta_{\Omega}^{(r)}$	$\Delta_{\Omega}^{(\bar{\theta})}(60^{\circ})$	$\Delta_{\Omega}^{(\bar{\theta})}(75^{\circ})$	$\bar{U}_{\text{mer}}^{\text{max}}$ [m/s]
A1*	-0.1439	0.4833	0.4835	2.3
A2**	-0.1439	0.4847	0.4758	2.4
B1	-0.0131	-0.0201	-0.0169	1.4
B2*	-0.0362	-0.0121	-0.0096	1.4
B3**	-0.0471	0.0226	0.0219	1.7
C1	-0.0134	-0.0110	-0.0090	1.2
C2	-0.0042	-0.0034	-0.0031	2.0
C3*	-0.0308	0.0008	0.0010	0.8
C4**	-0.0087	0.0047	0.0045	1.1
C5	0.0017	-0.0106	-0.0092	1.3
D1*	-0.0177	0.4995	0.5004	0.7
D2**	-0.0164	0.4975	0.4985	1.4
E1*	-0.0015	0.5000	0.5000	0.4
E2**	-0.0064	0.5002	0.5004	1.3

Table 4.4.1: Amplitude of the temporally and azimuthally averaged angular velocity $\bar{\Omega}(r, \theta)$. From left to right the columns indicate the name of the simulation, the amplitude of the radial differential rotation (Eq. 4.4.3), the amplitude of the latitudinal differential rotation at 60° and 75° (Eq. 4.4.3), and the rms value of the meridional flow speed.

the bottom of the CZ, i.e., $r_{\text{bot}} = 0.6R$, eq corresponds to the equator, i.e., $\theta = 0^{\circ}$, and $\bar{\theta}$ is an average of $\bar{\Omega}$ between latitudes $-\theta$ and θ . Note that $\Delta_{\Omega}^{(r)}$ and $\Delta_{\Omega}^{(\bar{\theta})}$ are in spherical coordinates. These parameters are summarized in Table 4.4.1 for all sets of simulations.

A simple examination of the data in the second column of Table 4.4.1, where the negative value of $\Delta_{\Omega}^{(r)}$ indicated that the star is rotating faster near to the CZ than near to the surface. This reveals that the majority of current simulations should have an anti-solar differential rotation regime, characterised by a slower equator and faster poles. However, from the Figures it is clear that at least runs A2, B1, B3 and C5 present a solar-like differential rotation. This must be an artifact of the choice of r_{top} and r_{bot} . A more reliable diagnostic is the average $\bar{\Omega}$ at the equator, see Eq. (24) in (Käpylä, 2023). Additionally, the differential rotation in general is weak, with amplitudes in the range of a few percentage points of Ω_0 across the majority of the domain, except near the axis, where the data exhibit poor convergence.

In Figure 4.4.1 for set A we can see that both simulations (A1 and A2) present a similar angular velocity, but the distribution of the angular velocity is different,

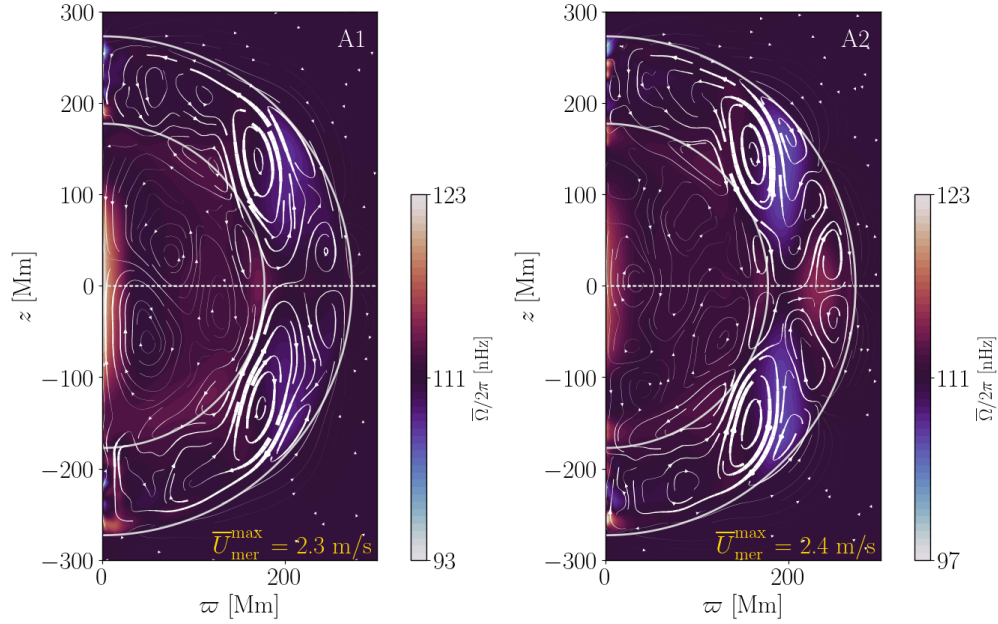


Figure 4.4.1: Temporally and azimuthally averaged angular velocity $\bar{\Omega}(\varpi, z)$ for set A. The streamlines show the meridional flow and its amplitude is indicated in the lower right corner. The surface and the bottom of the CZ are indicated by the gray line while the equator is indicated by the gray dotted line.

as A2 presents a slight reddish color near the equator on the surface of the star, which could indicate that the differential rotation is actually solar-like.

Figure 4.4.2, which presents the results of simulations B1, B2 and B3; illustrates a differential rotation analogous to that observed in the Sun for B1 and B3. A comparison of B1 and B3 reveals a high degree of similarity between the two cases. The primary distinction lies in the radiative zone of the star, with B1 exhibiting a marginally larger and redder region in proximity to the CZ compared to B2. The run B2 demonstrates a clear anti-solar differential rotation (consistent with the values of Table 4.4.1), with the core of the star exhibiting complete reddening.

In regard to Set C (Figure 4.3.3), it can be observed that there are similarities between the C1 and C5 simulations. The C5 simulation displays a robust solar-like differential rotation and a relatively lighter blue radiative zone. C1 appears to exhibit a higher rotational velocity at the surface of the equator (small white zone), and an orange radiative zone. For runs C3 and C4, the aforementioned characteristics are dispersed, with the presence of small blue areas between the bottom of the CZ and the surface. It is noteworthy that in C1 and C3, a reddening/orange effect is observed in the core of the star, similar to that observed

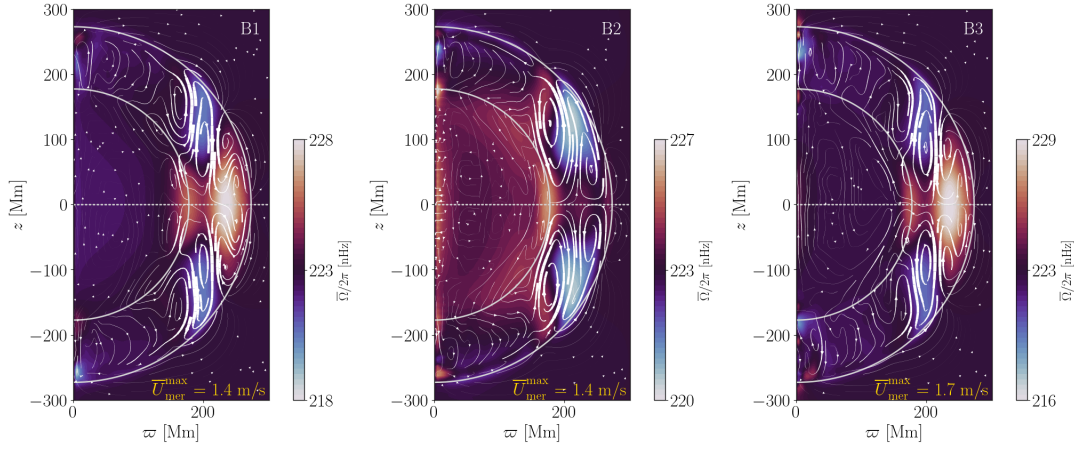


Figure 4.4.2: Temporally and azimuthally averaged angular velocity $\bar{\Omega}(\varpi, z)$ for set B. The streamlines show the meridional flow and its amplitude is indicated in the lower right corner. The surface and the bottom of the CZ are indicated by the gray line while the equator is indicated by the gray dotted line.

in B2 and C4 in a less pronounced form.

Set D and E are shown in Figure 4.4.4. In the case of D1 and D2 (upper panels), the difference in velocities in the latitudes is minimal, resulting in a similar visual appearance. However, it is evident that in D2 the features seen in D1 are more pronounced. In particular, the radiative zone of D1 exhibits a more orange color compared to D2. Furthermore, the blue stripe in the CZ of D2 is much softer in D1, as is the red spot on the surface of the CZ near the equator.

In the case of E1 and E2 (lower panels), a comparable phenomenon is observed in the E2 run, where, once more, this blue stripe is evident, but this time it is more pronounced than in D1, as well as the red spot on the surface. Conversely, in E1, there is a small difference in the velocities in the latitudes, resulting in the red color being nearly indiscernible. On closer examination, a more reddish color is discernible in the lower CZ near the equator. It is noteworthy that the maximum averaged meridional flow of E1 is the smallest of all the simulations presented (see last columns of table 4.4.1).

A comparison of the sets reveals that as the rotation rate increases, the differential rotation decreases, approaching the rotation of a solid body. It is also noteworthy that in the instances where the external conditions change (A2, B3, C4, D2 and E3), the velocity fields become more flat in comparison to the simulations which do not have the external diffusivity profile.

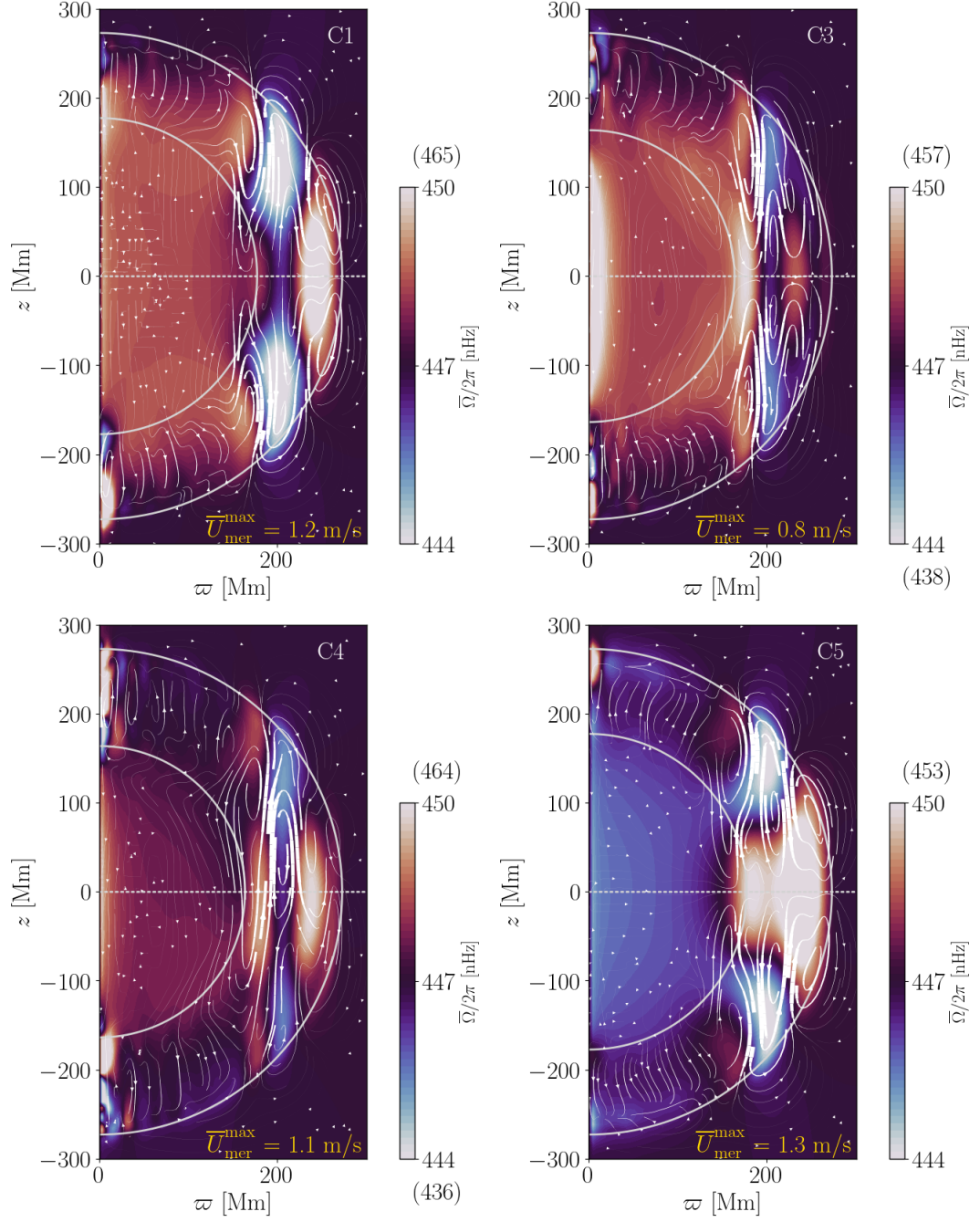


Figure 4.4.3: Temporally and azimuthally averaged angular velocity $\bar{\Omega}(\varpi, z)$ for set B. The streamlines show the meridional flow and its amplitude is indicated in the lower right corner. The surface and the bottom of the CZ are indicated by the gray line while the equator is indicated by the gray dotted line.

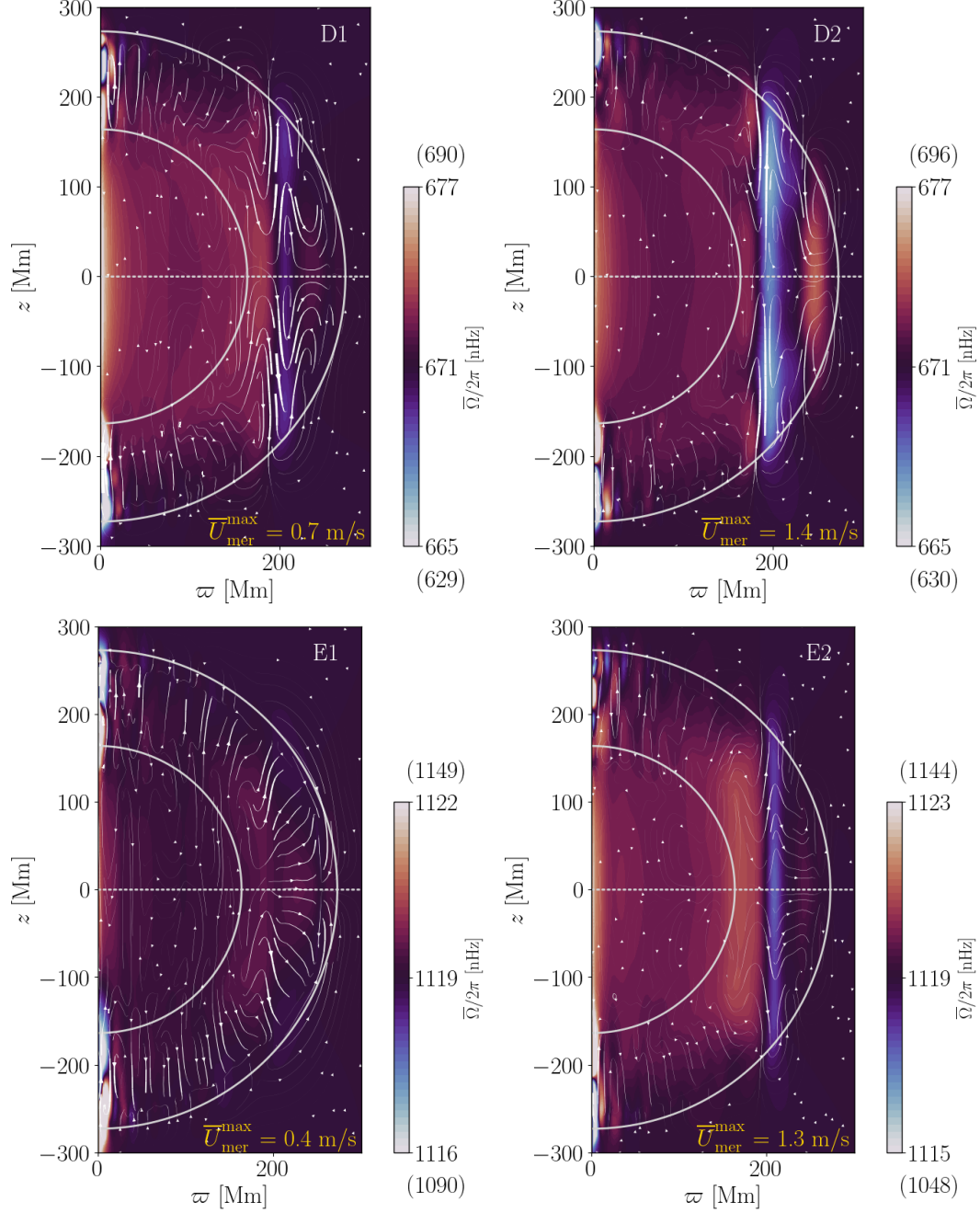


Figure 4.4.4: Temporally and azimuthally averaged angular velocity $\bar{\Omega}(\varpi, z)$ for set D (upper panel), and set E (bottom panel). The streamlines show the meridional flow and its amplitude is indicated in the lower right corner. The surface and the bottom of the CZ are indicated by the gray line while the equator is indicated by the gray dotted line.

The CZ of several simulations (D1, D2, E2, C4, C1, C3 and B2) is illustrated in orange/red, with C1 and C3 exhibiting the most pronounced orange coloration. It can be observed that the temporally and azimuthally averaged angular velocity profiles, $\bar{\Omega}(\varpi, z)$, of simulations D1, D2 and E2 are highly similar, and the main difference between these profiles is in their intensity.

The discrepancies identified in the differential rotation profiles may be attributed to the limited resolution, the minimal variation in velocities, low stratification, and Reynolds numbers. Recently, these quantities have been calculated with greater precision; for further details, refer to [Käpylä \(2023\)](#).

4.5 Flux analysis

The energy balance can be studied in terms of the respective energy fluxes. The following luminosities are associated with cooling, heating, kinetic energy, viscosity, enthalpy, and radiative fluxes ([Käpylä, 2021](#)):

$$\bar{L}_{\text{rad}} = -4\pi r^2 \langle K \rangle_{\theta\phi t} \frac{\partial \langle T \rangle_{\theta\phi t}}{\partial r}, \quad \bar{L}_{\text{enth}} = 4\pi r^2 c_p \langle (\rho u_r)' T' \rangle_{\theta\phi t}, \quad (4.5.1)$$

$$\bar{L}_{\text{kin}} = 2\pi r^2 \langle \rho \mathbf{u}^2 u_r \rangle_{\theta\phi t}, \quad \bar{L}_{\text{visc}} = -8\pi r^2 \nu \langle \rho u_i \mathbf{S}_{ir} \rangle_{\theta\phi t}, \quad (4.5.2)$$

$$\bar{L}_{\text{cool}} = - \int_0^r 4\pi r^2 \langle C \rangle_{\theta\phi t} dr, \quad \bar{L}_{\text{heat}} = \int_0^r 4\pi r^2 \langle \mathcal{H} \rangle_{\theta\phi t} dr, \quad (4.5.3)$$

where $\langle \cdot \rangle_{\theta\phi t}$ denotes horizontal and temporal averaging, index r refers to the radial component, and the prime shows deviations from the mean value. Convected luminosity as a whole is provided by $\bar{L}_{\text{conv}} = \bar{L}_{\text{kin}} + \bar{L}_{\text{enth}}$, ([Cattaneo et al., 1991](#)).

The contributions to the total luminosity from runs A1, B2, C3, D1 and E1 are shown in Figure 4.5.1. In general, the convective zone is related to positive enthalpy flux, which ranges from about 0.6 R to 0.9 R . This is because, as previously mentioned, convection is associated with this flux component, and the convective core of the star extends radially to about this level. The total convective luminosity L_{conv} matches the luminosity of the star everywhere except near the surface, where cooling is effective.

A case-by-case analysis reveals that in A1, the dominant components are due to the outward enthalpy flow (L_{enth}) and the inward kinetic energy flow (L_{kin}). Moreover, it can be observed that the kinetic energy flux tends towards zero as

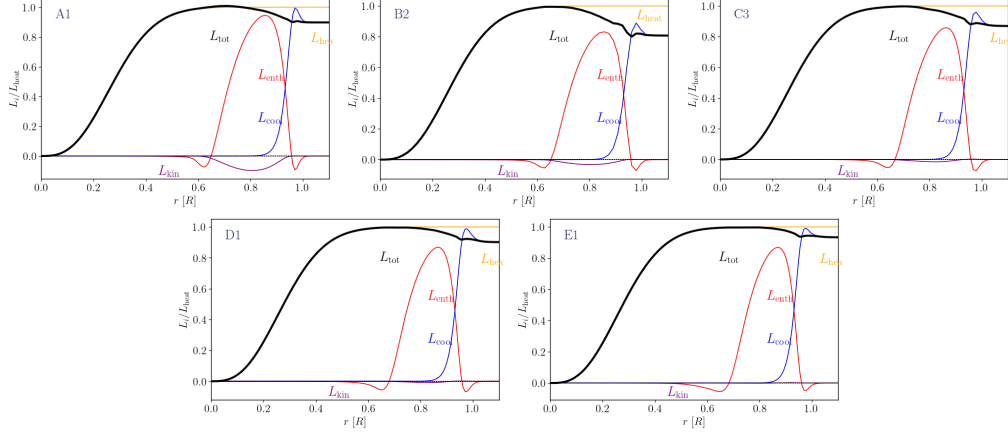


Figure 4.5.1: Contributions to the luminosity from enthalpy (red), kinetic energy (purple), cooling (blue), and heating (orange) fluxes in run A1, B2, C3, D1 and E1. The name of the simulation can be found in the upper left corner.

the rotation rate of the simulation increases. A detailed examination of E1 reveals that L_{kin} is no longer negative and approaches zero.

The luminosity contributions for the remaining simulations exhibit comparable profiles to those shown in Fig. 4.5.1, with a non-zero enthalpy flux at the location of the convective envelope, and L_{kin} displays the same phenomenon observed in their corresponding sets. Furthermore, it is evident that the cooling process is most effective in the closer proximity to the surface.

4.6 Energy analysis

The total magnetic and kinetic energies, respectively, are given by:

$$E_{\text{mag}} = \frac{1}{2} \int \langle \mathbf{B}^2 \rangle dV, \quad E_{\text{kin}} = \frac{1}{2\mu_0} \int \langle \rho \mathbf{U}^2 \rangle dV \quad (4.6.1)$$

where the brackets denote volume average for radii $r < R$. The toroidal and poloidal magnetic energies are given by:

$$E_{\text{mag}}^{\text{tor}} = \frac{1}{2\mu_0} \int \langle \overline{B}_\phi^2 \rangle dV, \quad E_{\text{mag}}^{\text{pol}} = \frac{1}{2\mu_0} \int \langle (\overline{B}_\varpi^2 + \overline{B}_z^2) \rangle dV \quad (4.6.2)$$

The energies for the differential rotation (DR) and meridional circulation (MC), are defined as:

$$E_{\text{kin}}^{\text{DR}} = \frac{1}{2} \int \langle \rho \bar{U}_\phi^2 \rangle dV, \quad E_{\text{kin}}^{\text{MC}} = \frac{1}{2} \int \langle \rho (\bar{U}_\varpi^2 + \bar{U}_z^2) \rangle dV \quad (4.6.3)$$

These energies from all sets of simulation are listed in Table 4.6.1.

The simulations with the lowest rotation rate (Set A) exhibit the highest kinetic energies values. Indeed, as the rotation rate increases in each of the sets, a decrease in kinetic energy is observed. This set also has the highest energy contribution from the meridional circulation, at 1.9% and 1.8% for A1 and A2, respectively. It is noteworthy that the energy associated with differential rotation does not exceed 16% in any of the cases, with the highest values observed in simulations C1 and C5, at 15.8% and 11%, respectively.

Sim	E_{mag}	$E_{\text{mag}}^{\text{tor}}/E_{\text{mag}}$	$E_{\text{mag}}^{\text{pol}}/E_{\text{mag}}$	E_{kin}	$E_{\text{kin}}^{\text{DR}}/E_{\text{kin}}$	$E_{\text{kin}}^{\text{MC}}/E_{\text{kin}}$	$\tilde{E}_{\text{mag}}$
A1*	5663	0.080	0.063	17739	0.045	0.019	0.319
A2**	4798	0.068	0.076	18707	0.054	0.018	0.256
B1	3475	0.119	0.131	8332	0.094	0.010	0.417
B2*	4510	0.088	0.065	11029	0.042	0.009	0.408
B3**	8505	0.095	0.102	10734	0.067	0.011	0.792
C1	449	0.074	0.055	5893	0.158	0.006	0.076
C2	3189	0.061	0.048	12677	0.154	0.009	0.251
C3*	3301	0.062	0.069	7418	0.043	0.006	0.444
C4**	5057	0.092	0.114	6945	0.030	0.007	0.728
C5	1961	0.153	0.297	5018	0.111	0.007	0.391
D1*	2864	0.034	0.047	5906	0.051	0.005	0.484
D2**	1203	0.040	0.047	6713	0.084	0.005	0.179
E1*	4881	0.033	0.057	3235	0.026	0.005	1.508
E2**	2276	0.061	0.158	4083	0.079	0.004	0.557

Table 4.6.1: Kinetic and magnetic energies, averaged over time and volume. The total kinetic and magnetic energies have units of 10^5 Jm^{-3} , while the rest of energies are written as fractions of these quantities. The last column is defined as $\tilde{E}_{\text{mag}} = E_{\text{mag}}/E_{\text{kin}}$.

Run C5 exhibits the highest toroidal energy, comprising 15% of the total energy, while 29% of the magnetic energy can be attributed to poloidal energy, this makes sense if we look at the Fig. 4.3.3 as there is intermittency in the toroidal component. These represent the highest values, with simulation B1 exhibiting 11% and 13% for toroidal and poloidal energies, respectively. Nevertheless, the

simulation with the second highest poloidal energy value is E2 at 15%, but its toroidal energy is lower, at 6.1%. The simulation with the highest magnetic energy value is B3, followed by A1 and C4.

On the other hand, the simulation exhibiting the lowest magnetic energy is C1, which is in accordance with the plot of the Fig. 4.3.3, given that its magnetic field displays multiple periods of intermittency. Moreover, as previously discussed, the simulations with the highest rotation rate (in this case, with an $\Omega = 5$) exhibit the lowest kinetic energies. This is in agreement with the $\bar{\Omega}(\varpi, z)$ plots (Fig. 4.4.4) for set E.

Finally, the majority of simulations exhibit magnetic energies below equipartition (where equipartition is $\tilde{E}_{\text{mag}} = 1$). The values for each simulation are in the last column of the Table 4.6.1, where C1 has the lowest value with 0.076, followed by run D2 with 0.179. In contrast, E2 reaches super-equipartition, where the magnetic energy is approximately 1.5 times that of the kinetic energy.

Chapter 5

Summary and conclusions

In this thesis, we employed the star-in-a-box model of [Käpylä \(2022\)](#) to perform 3D magnetohydrodynamical simulations. This approach was adopted to facilitate a comprehensive understanding of magnetism in partially convective M dwarf stars, and to investigate the sensitivity to boundary conditions with the employed model. The non-ideal fully compressible MHD equations were solved using PENCIL CODE ([Pencil Code Collaboration et al., 2021](#)).

To ascertain the sensitivity of the magnetic field to boundary conditions, three diffusivity profiles have been explored. The first profile under consideration is flat, the second occupies a diffusivity profile in the core; and the last profile presents a diffusivity profile in the interior and exterior of the star. Additionally, five rotation periods have been considered, ranging from 9.6 to 96 days. The root-mean-square magnetic fields were found to range from 4.3 to 10.4 kG, while the toroidal magnetic fields exhibited values between 2.0 and 12 kG.

It has been demonstrated that there is a degree of similarity in dynamo solutions depending on the boundary conditions and diffusivity profiles that were used; with some dynamos displaying regular cycles and others exhibiting quasi-static behavior. The results of the simulations configured with a magnetic diffusivity jump demonstrate a high degree of similarity to the solar butterfly diagram, with C3 and E1 exhibiting the most regular and pronounced cycles. In contrast, the A1 run does not show any cyclic behavior. For the simulations characterized by two magnetic diffusivity jumps (inside and outside), the dynamos tend to be quasi-static, marked by the absence of polarity reversals. An exception to this is

observed in the D2 series, which exhibits very short magnetic cycles. It is possible that these behaviors may be attributable to other factors in the simulations, such as the resolution or the duration of the simulations. Moreover, an intermittent magnetic field phenomenon was identified in two simulations (C1 and C5). The underlying cause of this phenomenon remains to be elucidated. This phenomenon has been previously documented in [Guerrero et al. \(2016\)](#), where intermittent outages of magnetic activity were observed in dynamos operating exclusively in the convection zone.

A correlation has been identified between the rotation period and the cycle period, as a function of the global Coriolis number. The relationship is expressed as $P_{\text{rot}}/P_{\text{cyc}} \propto \text{Co}^\beta$, where $\beta = -0.90 \pm 0.11$, indicating a concurrence with earlier studies (e.g., [Warnecke 2018](#); [Strugarek et al. 2018](#); [Viviani et al. 2018](#); [Ortiz-Rodríguez et al. 2023](#)), where $\beta < 0$ was also determined for global simulations of solar-like and fully convective stars.

Large-scale flows were also studied. At least in four of the simulations presented here, a solar-like differential rotation in the CZ is observed, according to the figures showing the time and azimuthally averaged angular velocity, $\overline{\Omega}(\varpi, z)$. Conversely, the remaining nine simulations exhibited anti-solar differential rotation, which was analyzed radially and latitudinally. Moreover, the data show poor convergence near the axis, indicating weak differential rotation overall, with amplitudes of the averaged meridional flow in the range of 0.4 to 2.4 m/s. A comparison of the sets reveals that the rotation approaches that of a solid body as the rotation rate increases. Furthermore, it is noteworthy that the velocity fields exhibit a flattening tendency in comparison to simulations performed without an external diffusivity profile, particularly in scenarios where external conditions change, in the case of two jumps. The temporally and azimuthally averaged angular velocity profiles of simulations D1, D2, and E2 are found to be highly analogous; the principal differentiating factor between them is the amplitude of DR.

No systematic change in the total magnetic energy density is observed in the explored parameter regime. However, in most of the simulations, the poloidal magnetic energy dominates over the toroidal magnetic energy, with only simulations A1, B2 and C1 displaying the opposite effect. The kinetic energy density increases as the rotation rate decreases in the sets. Furthermore, in all simulations, differential rotation dominates over the meridional circulation. The

majority of the simulations indicate magnetic energies that are below equipartition, while E2 demonstrates super-equipartition fields, with the magnetic energy being approximately 1.5 times that of the kinetic energy.

Convective luminosity L_{conv} approaches the stellar luminosity L in the convective zone. The effect of boundary condition and rotation is quite weak in the parameters regime studied here.

The methodology employed in the study of stellar dynamos via numerical simulations in a Cartesian domain, encompassing the entire star, has been shown to facilitate a more detailed analysis of the interactions between the inside and outside regions of the star when compared to traditional models in spherical shells. Furthermore, the capacity to modify the internal and external conditions, in addition to the spatial resolution of the simulations, provides a substantial advantage for the exploration of complex phenomena. It is important to mention that this methodology has not been used before to explore how external boundary conditions affect the dynamos that are generated, so it is a breakthrough to have this kind of initial results to be explored in more detail in the future. The results obtained are of relevance not only for the understanding of magnetic fields in M dwarfs, but also for extrapolation to other classes of partially and fully convective stars, opening new lines of research in the field of stellar astrophysics.

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