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Logística de una Lancha Sustentable que Recorre la Antártica. Modelos y Métodos de Solución

por
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Resumen

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La presente tesis doctoral aborda la planificación logística en la Antártica, una región sumamente exigente por sus condiciones climáticas extremas, grandes distancias y sensibles restricciones medioambientales. El trabajo se enmarca en el Maritime Inventory Routing Problem (MIRP), integrando ruteo de embarcaciones y gestión de inventarios en puertos o bases científicas a lo largo de un horizonte de planificación de 60 a 120 días. A diferencia de la mayoría de los estudios, aquí se considera una flota de embarcaciones heterogéneas y, de manera novedosa, se introduce la propulsión híbrida con selección de velocidad adaptativa para reducir el consumo de combustible y el impacto ambiental.

Para ello, se proponen dos formulaciones de Programación Entera Mixta (MILP/MIBLP). La primera refuerza la capacidad de resolver MIRP en horizontes largos con múltiples barcos, incorporando inecuaciones válidas y técnicas de symmetry breaking que mejoran notablemente la eficiencia computacional. La segunda agrega la dimensión energética de embarcaciones híbridas y velocidad variable, lo que implica modelar el consumo de combustible y la gestión de la batería mediante términos bilineales. Se validan estas propuestas con instancias realistas basadas en datos de bases científicas chilenas en la Antártica, mostrando reducciones de combustible y emisiones, aunque con retos de escalabilidad en los casos de mayor magnitud.

En conjunto, la tesis demuestra que la integración simultánea de ruteo, inventarios y gestión energética de buques híbridos mejora la sostenibilidad y eficiencia de la logística antártica. Además, sienta bases metodológicas para la aplicación en entornos remotos con restricciones ambientales, aportando soluciones que equilibran el abastecimiento confiable de las bases con la protección de uno de los ecosistemas más frágiles del planeta.

Palabras clave: Maritime Inventory Routing Problem, Antarctica, Hybrid Electric Propulsion, Speed Selection, Mixed-Integer Programming.

Abstract

Logistics of a Sustainable Boat Touring Antarctica. Models and Solution Methods

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Concepción, April 2025

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This doctoral thesis addresses logistical planning in Antarctica, a highly demanding region due to extreme climatic conditions, vast distances, and stringent environmental restrictions. The study is framed within the Maritime Inventory Routing Problem (MIRP), integrating vessel routing and inventory management at ports or scientific bases over a planning horizon of 60 to 120 days. Unlike most existing research, this thesis considers a heterogeneous fleet of vessels and, innovatively, introduces hybrid propulsion with adaptive speed selection to reduce fuel consumption and environmental impact.

To achieve this, two Mixed-Integer Programming (MILP/MIBLP) formulations are proposed. The first one enhances the capacity to solve MIRP over long horizons with multiple ships, incorporating valid inequalities and symmetry-breaking techniques that significantly improve computational efficiency. The second adds an energy dimension—hybrid vessels with variable speeds—requiring bilinear modeling of fuel consumption and battery management. These approaches are validated using realistic instances based on data from Chilean scientific bases in Antarctica, demonstrating reductions in fuel usage and emissions, though facing scalability challenges in larger instances.

Overall, the thesis shows that simultaneously integrating routing, inventory, and energy management of hybrid vessels improves sustainability and efficiency in Antarctic logistics. It also provides a methodological foundation for application in remote environments with environmental constraints, delivering solutions that balance reliable base resupply with the protection of one of the most fragile ecosystems on the planet.

Keywords: Maritime Inventory Routing Problem, Antarctica, Hybrid Electric Propulsion, Speed Selection, Mixed-Integer Programming.

Capítulo 1

Introducción

Desde la firma del Tratado Antártico en 1959, la Antártica se ha convertido en un escenario de creciente relevancia científica a nivel global, por la contribución a la comprensión del cambio climático y por la diversidad de investigaciones que allí se llevan a cabo en ámbitos como glaciología, biología marina, geología, meteorología, entre otros [ant, 1959]. En este contexto la logística antártica, necesaria para realizar expediciones científicas enfrenta variados desafíos como: condiciones climáticas extremas, grandes distancias entre bases científicas, restricciones ambientales rigurosas, entre otros [Instituto Antártico Chileno, 2010]. La planificación adecuada de: rutas de transporte, provisión de suministros, retiro de desechos y manejo de inventarios es un factor crítico de éxito para las misiones científicas y de mantenimiento de infraestructura en esta región.

Una condición determinante en la Antártica es su fragilidad ambiental, por lo que reducir las emisiones en el transporte de carga y pasajeros es esencial para preservar los ecosistemas únicos, que allí subsisten. La Organización Marítima Internacional (OMI), en su rol de organismo regulador del transporte marítimo internacional, supervisa la prevención de la contaminación oceánica generada por los barcos y ha reforzado normas para mitigar las emisiones de: CO_2, óxidos de azufre y otros contaminantes [mar, 1973]. En regiones muy sensibles como la Antártica, cualquier alteración puede producir efectos irreversibles en la fauna, flora y dinámica de los hielos; por esto, la importancia de adoptar tecnologías y prácticas de navegación limpias, que contribuyan a mantener la pureza de los océanos y la integridad de este continente.

En Chile, el Instituto Antártico Chileno (INACH) es el organismo gubernamental encargado de seleccionar proyectos de investigación y proporcionar apoyo logístico a los equipos respectivos [Instituto Antártico Chileno, 2025]. El apoyo logístico consiste en la asignación de los equipos de investigación a alguna de las seis bases científicas que dispone INACH, el transporte de pasajeros y carga entre las distintas bases, y el aseguramiento de la disponibilidad continua de suministros y equipos para el período de duración de la misión.

Esta tesis doctoral considera: diseñar, analizar y resolver modelos de logística marítima de una flota de lanchas híbridas, que recorra las bases chilenas en la Antártica, optimizando las rutas de navegación y la gestión de inventarios asociados al abastecimiento de bases científicas, minimizando las emisiones de la flota en el horizonte de planificación. Se proponen métodos exactos que permitan resolver de manera eficiente y sostenible problemas de ruteo e inventarios de larga duración y con múltiples embarcaciones. Con énfasis en los requerimientos y restricciones que se presentan en condiciones antárticas.

La motivación se inicia con la necesidad de integrar dos áreas comúnmente estudiadas de forma separada: la gestión de rutas de embarcaciones (routing) y la planificación de niveles de inventario en los puertos o bases. Este problema conjunto es conocido como Maritime Inventory Routing Problem (MIRP). Aunque han existido avances importantes en la literatura, como la incorporación de: horizontes de planificación extensos, flotas heterogéneas de múltiples embarcaciones y producción/consumo continuos, no se han encontrado estudios que aborden el MIRP conjuntamente con la gestión energética de barcos eléctricos híbridos, que genera un vacío que este estudio propone cubrir. Los principales aspectos tratados en esta tesis son los siguientes:

- Horizontes de planificación extensos: La mayoría de los estudios de MIRP o IRP (Inventory Routing Problem) abordan ventanas de hasta 60 unidades de tiempo (por ejemplo, 60 días), pero las campañas antárticas y los requerimientos de largo plazo exigen horizontes de 120 o más días.

- Restricciones medioambientales y enfoques sostenibles: Considerar la sustentabilidad de la lancha y reducir emisiones de CO_2 y contaminantes locales es relevante en áreas tan frágiles como la Antártica.

- Escalabilidad y solución eficiente de instancias de gran tamaño: Los métodos exactos (basados en Programación Entera Mixta) pueden enfrentar grandes dificultades de tiempo de cómputo cuando el problema aumenta de tamaño. Es importante incluir inecuaciones válidas para mejorar la factibilidad de resolver instancias realistas.

A continuación, se describe el contenido de esta tesis y la forma en que se organizan los capítulos para abordar estos desafíos.

1.1. Estructura y contenido de la tesis

La presente tesis se organiza en cuatro capítulos principales:

- **Capítulo 1: Introducción**

  El capítulo actual considera: el contexto global del problema, relevancia tanto científica como práctica, los principales objetivos de la investigación y la estructura del documento. También, se detallan los aportes esperados y se delinean los desafíos a enfrentar en la logística en la Antártica.

- **Capítulo 2: Enhanced MILP Approach for Long-Term Multi-Vessel Maritime Inventory Routing with Application to Antarctic Logistics**

  Este capítulo corresponde al primer artículo *(Paper 1)* que ofrece la tesis [Cifuentes-Lobos et al., En este, se propone un modelo de Programación Entera Mixta (MILP) mejorado para el Maritime Inventory Routing Problem (MIRP), y con horizonte de planeación extenso y múltiples embarcaciones. El modelo introduce restricciones de inventario y ruteo con horizonte continuo. Además incorpora inecuaciones válidas que mejoran la cota inferior de la relajación lineal y aceleran la convergencia de la solución. Se presenta un análisis computacional que compara el rendimiento de la formulación con distintas variantes y métodos de la literatura. Para validar la aplicabilidad en entornos reales, se incluyen instancias basadas en bases científicas chilenas en la Antártica, con resultados que muestran reducciones significativas en los tiempos de cómputo y GAP de optimalidad aceptables incluso en horizontes de 120 días.

- **Capítulo 3: Optimizing Maritime Logistics for Antarctic Research: A Hybrid Electric Fleet Approach with Adaptive Speed Selection**

  El segundo artículo generado extiende la línea de investigación anterior, abordando un elemento relevante en la navegación polar: la sustentabilidad energética. En particular, se propone un modelo de Programación Entera Mixta Bilineal (MIBLP) que considera: i) Barcos con propulsión híbrida (motor de combustión interna y motor eléctrico), donde la batería se recarga o descarga según la estrategia de navegación, ii) Selección de velocidad, que afecta de forma directa el consumo de combustible y por lo tanto, la duración de los trayectos, y iii) Parametrización y linearización parcial de los términos bilineales para resolver el problema con un enfoque cercano a la Programación Entera Mixta, manteniendo viables los cálculos en un solver comercial.

  Este nuevo HS-SS-MIRP (Hybrid Ships with Speed Selection Maritime Inventory Routing Problem) trata con instancias de la Antártica, evidenciando que emplear velocidades más bajas reduce significativamente el uso de diésel (y por ende, las emisiones), pero incrementa la complejidad de satisfacer las restricciones de inventario en horizontes de planificación muy extensos. Para horizontes de hasta 60 días, se obtienen soluciones óptimas en la mayoría de las instancias; en horizontes de 120 días, algunas instancias de gran tamaño presentan gaps considerables o alcanzan el límite de tiempo, resaltando la necesidad de futuras mejoras.

- **Capítulo 4: Discusión**

  Este capítulo analiza en profundidad los resultados obtenidos en los dos modelos propuestos (MV-MIRP y HS-SS-MIRP), comparando su desempeño en distintos escenarios logísticos realistas basados en las bases científicas chilenas en la Antártica. Se discuten las ventajas y limitaciones de cada formulación, enfatizando la efectividad de las técnicas utilizadas, como las inecuaciones válidas y la parametrización del modelo bilineal, para mejorar la eficiencia computacional y escalabilidad. Además, se abordan los desafíos pendientes para resolver instancias de gran magnitud, especialmente en horizontes extensos de 120 días.

- **Capítulo 5: Conclusiones y Líneas Futuras de Investigación**

  Finalmente, se presentan los aportes de la tesis al campo de la logística marítima en entornos polares con reflexión sobre las limitaciones y oportunidades de mejora. Se plantean posibles líneas de investigación futura, tales como: incorporación de modelos de incertidumbre (consumos variables, demanda estocástica, efectos del clima), optimización multiobjetivo para balancear costos y emisiones, y el desarrollo de métodos híbridos de descomposición y heurísticas avanzadas que permitan escalar el tamaño de los problemas abordables.

1.2. Hipótesis de estudio

La integración de la planificación de inventarios y rutas marítimas (MIRP) con la gestión energética y la selección de velocidad de embarcaciones híbridas permite reducir de manera significativa el consumo de combustible y las emisiones, manteniendo un abastecimiento confiable en bases antárticas con restricciones operativas extremas.

1.3. Objetivos

1.3.1. Objetivo general

Proponer, formular y probar modelos de logística marítima para la Antártica que integre la gestión de inventarios, el ruteo de múltiples embarcaciones y la propulsión híbrida, para reducir el consumo de combustible y las emisiones contaminantes, asegurando simultáneamente la continuidad operativa de las bases científicas en condiciones extremas.

1.3.2. Objetivos específicos

1. Desarrollar una formulación de programación entera mixta que incorpore: gestión de inventarios y logística de embarcaciones híbridas (ruteo, velocidad adaptativa, estado de carga de la batería, etc.), reflejando las restricciones propias del entorno antártico.

2. Determinar cotas inferiores que permitan reducir los tiempos de cómputo en instancias de gran tamaño.

3. Proponer e implementar estrategias de resolución exacta que permitan abarcar horizontes de planificación largos (60-120 días) y múltiples barcos con distintos perfiles de carga y capacidades energéticas.

4. Generar y resolver instancias basadas en datos logísticos y geográficos de las bases científicas chilenas en la Antártica, evaluando la eficiencia operativa (tiempos de reabastecimiento, número de visitas) y la sustentabilidad (consumo de combustible, emisiones).

Capítulo 2

Enhanced MILP Approach for Long-Term Multi-Vessel Maritime Inventory Routing with Application to Antarctic Logistics

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Article

Enhanced MILP Approach for Long-Term Multi-Vessel Maritime Inventory Routing with Application to Antarctic Logistics

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Abstract: The maritime inventory routing problem (MIRP) integrates vessel routing and inventory management over a planning horizon to optimize logistical operations in marine environments. While existing models predominantly address short-term planning with single vessels, this research advances the field by presenting a tightened mixed-integer linear programming (MILP) model designed for long-term planning with multiple vessels. The proposed model leverages an improved mathematical formulation and state-of-the-art optimization solvers to enhance computational performance. To demonstrate its applicability, the model was evaluated using benchmark instances from the literature and new instances derived from the logistics of Chilean scientific bases in Antarctica, a challenging and underexplored maritime environment. The results show computational time reductions of up to 98% for small to medium-sized instances, achieved through the incorporation of valid inequalities into the model and the use of advanced hardware and solvers. For larger instances, optimal or near-optimal solutions were achieved within one hour for a planning horizon of 60 time units, with optimality gaps below 24.7% for a 120-time-unit horizon. These findings highlight the potential of the model to support decision-making in complex maritime logistics scenarios, extending its application to long-term, multi-vessel operations in remote and environmentally sensitive regions. The proposed framework provides a valuable tool for enhancing the sustainability and efficiency of maritime logistics systems.

Keywords: multi-vessel maritime inventory routing problem; Antarctic science; mixed-integer linear programming; logistics; shipping industry

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1. Introduction

The coordination of the route of cargo vessels, considering the levels of inventory of products in the ports along the route, presents a complex management task. This situation leads to the formulation of the Maritime Inventory Routing Problem (MIRP), which represents a challenging optimization problem that can be solved through mathematical modeling. The MIRP is critical for different types of industries, such as oil and gas [1], chemicals [2], bulk commodities [3], and waste recovery [4]. The MIRP generally aims to optimize transportation and inventory costs while reducing environmental impact by minimizing fuel consumption, emissions, and inventory maintenance costs. The MIRP involves various factors, such as demand and supply locations, production and consumption rates, vessel capacities, or sailing times. When addressing the combined issue of distribution and inventory management, it is possible to reduce costs by 10% and the number of vehicles employed [5].

Researchers have extensively studied Inventory Routing Problems (IRPs) in the literature, proposing various formulations and solution methods for nearly four decades. These formulations vary in planning horizon, policies, objectives, and decisions [6]. Alternative IRP approaches include partially satisfying a customer's demand by dropping cargo to a nearby second customer [7] or considering time-variable demands [8]. The MIRP is a particular case of IRP in which the transportation of goods is restricted to maritime vessels. A classical formulation of the MIRP considers discrete time and multi-vessel, with valid inequalities to tighten the search space [9]. Determining an optimal solution for large-scale instances of the problem, particularly those with extended planning horizons, multiple vessels, and numerous ports, remains a significant computational challenge.

Despite extensive research on IRPs and their maritime counterparts (MIRP), a significant research gap exists in addressing large-scale, multi-vessel scenarios over extended planning horizons. Most existing studies focus on short-term planning horizons (e.g., 60 time units or shorter) or consider only a single vessel, which limits their applicability to real-world situations where coordination of multiple vessels over longer periods is essential. Additionally, achieving optimal solutions for larger instances remains a computational challenge due to the increased complexity associated with additional vessels and extended time frames.

To address this gap, we propose an improved and tightened MILP model for the Multi-Vessel Maritime Inventory Routing Problem (MV-MIRP). Our model extends existing formulations [10–14] by incorporating multiple heterogeneous vessels and longer planning horizons, capturing the complexities and dynamics of real-world maritime logistics operations. By introducing valid inequalities and leveraging advanced computational tools, our model enhances computational efficiency and scalability, enabling it to solve large-size instances that were previously intractable. To evaluate the practicality of the proposed model, we generated problem instances using real-world data from Chilean Antarctic bases. These data include the geographical coordinates of seven bases, derived from Google Earth, and logistical requirements based on the daily meal demands of the scientific personnel stationed at these locations.

Antarctic logistics presents a unique and challenging context for maritime inventory routing due to the extreme environment, long distances between bases, and the need for efficient and sustainable resource allocation. The Chilean scientific bases serve as an ideal case study to validate the applicability of our model to real-world scenarios requiring multi-vessel coordination and long-term planning.

The key contributions of this paper are as follows:

- A novel MILP model for the MIRP: We introduce a new mixed-integer linear programming (MILP) model that addresses the Multi-Vessel Maritime Inventory Routing Problem (MV-MIRP), incorporating a fleet of heterogeneous vessels, multiple depots, and a continuous time horizon.
- Tightened formulation: Our model incorporates valid inequalities that improve computational efficiency, enabling it to solve large-scale instances that previously resulted in unsolved gaps and reducing the optimality gap in other challenging cases.
- Comparative computational experiments: We conducted experiments to evaluate the performance of our model using benchmark data sets from the existing literature, specifically discrete-time formulations.

- Original real-world instances: We generate new problem instances based on real-world data from Chilean scientific bases in Antarctica, adding a layer of practical application to our theoretical model.
- Evaluation of new instances: Our model solves several generated instances to optimality, while others show reduced gaps, demonstrating its capability to address real-world complexities and offering valuable insights into its scalability and efficiency.

The remainder of this paper is structured as follows. Section 2 reviews the relevant literature, Section 3 offers an in-depth description of the problem and explains the formulations used, Section 4 presents the findings, and Section 5 presents a discussion about scalability, uncertainty management, and sustainability. Section 6 draws conclusions and outlines potential directions for future research.

2. Literature Review

The MIRP is a complex challenge faced by shipping companies and logistics providers involved in maritime transport. This problem involves intricate decision-making processes due to various factors, such as multiple products, time windows, vessel capacities, port constraints, inventory holding costs, and uncertain demand. The MIRP is a computationally complex and multidimensional problem that requires the expertise of various fields, such as operations research, transportation, logistics, and supply chain management. Efficient logistics of shipping inventory directly impacts the cost-effectiveness of global trade and logistics operations. By optimizing inventory allocation, vessel routes, and schedules, companies can minimize transportation costs, reduce inventory holding costs, and improve overall supply chain performance. For a more in-depth view, see [15,16].

The MIRP can be formulated considering a continuous or discrete time horizon. In models that use continuous time, the entire planning period is treated in a continuous way. Decision variables can be made for any time between zero and the end of the planning period, and constraints, such as inventory limits, are applied at all times. However, the parameters must remain constant over time. In contrast, the discrete-time formulation subdivides the planning horizon into intervals. Continuous-time formulations are more appropriate for applications with constant production and consumption rates. In contrast, discrete-time formulations are better suited for applications where production and consumption rates vary over time [17]. In practice, the choice of formulation depends on the specific characteristics of the problem considered. For a given problem instance, the best algorithmic performance occurs with a tight continuous-time formulation [18].

Regardless of the formulation, the MIRP often must address end-of-horizon difficulties, which can be addressed by defining a parameter to limit the difference between initial and final inventories in ports [10] or by producing a repeatable cyclic solution [14]. However, these approaches have only been tested in instances with up to six ports and five vessels, which did not reach optimality even after two hours of computational time. Optimality was achieved only in instances with up to four ports and three vessels. Stochasticity can be modeled using a robust optimization approach and a Benders decomposition algorithm [19], which has achieved optimality in instances with up to eight ports and six vessels, with the largest instances with sixteen ports and seven vessels. Furthermore, different techniques have been compared to model uncertainty in the MIRP, including deterministic models with safety stocks, stochastic programming models, robust optimization models, and models based on conditional value-at-risk measures [20]. These techniques have been tested in instances with up to six ports and five vessels, with deterministic and stochastic approaches achieving the least computational time. In turn, robust optimization and conditional value-at-risk approaches obtained better results, but with a higher computational cost. The critical parameters in the formulations are each port's consumption and production rates, which can be treated as decision variables to reduce costs over the entire time horizon. This approach achieved an average cost reduction of 11.4% in all instances with up to six ports and five vessels [11]. Other critical parameters of the model are the speed and load of the vessels, which influence fuel consumption. Taking these parameters into account, the costs of sailing can be reduced to 38% [12]. Furthermore, ref. [21] aimed to minimize fuel consumption by modeling speed as a continuous decision variable; however, they ended up with a nonlinear formulation. They solved instances with up to ten ports and six vessels using a metaheuristic algorithm.

In discrete-time formulations, ref. [22] developed a practical approach with a flexible modeling framework for the more general IRP, and successfully applied it to a MIRP. Furthermore, ref. [18] developed discrete- and continuous-time formulations for the MIRP, and ref. [23] tested several matheuristics approaches to solving the MIRP, which outperformed MILP formulations solved with commercial solvers. Some works use alternative time approaches, such as [24], which developed a hybrid continuous-discrete-time formulation. Ref. [25] proposes a formulation for the short-term planning of LNG deliveries, solving instances with up to 62 vessels using a metaheuristic approach. In summary, although novel and innovative modeling techniques have been devised, achieving optimality remains a challenge for larger instances, particularly when compared to other problems such as the vehicle routing problem and the traveling salesman problem.

Valid inequalities are linear constraints that are added to an MILP formulation to reduce the size of the feasible region and improve the quality of the linear relaxation [26]. They are often used to solve complex optimization problems that arise in maritime transportation, such as the MIRP. Ref. [9] proposes a discrete-time formulation for the MIRP and derives several classes of valid inequalities based on flow, inventory, and capacity constraints, improving the performance of the branch-and-cut algorithm for large-scale instances. Ref. [27] introduces a new formulation for the MIRP based on customer delivery patterns, solving it with a branch-price-and-cut algorithm using valid inequalities based on loading operations and timing cuts. Their algorithm produce lower bounds that are stronger than those of the default solving algorithm on a commercial solver. Ref. [28] proposes a discrete-time formulation for a crude oil distribution problem using cascading knapsack inequalities, which are linear constraints obtained by reformulating inventory balance constraints. This type of inequality has a structure that many commercial solvers can exploit, improving the quality of the solution.

Despite numerous studies proposing models and solution methods for the Inventory Routing Problem (IRP), scalability remains a critical challenge. Researchers have employed decomposition techniques such as two-phase decomposition methods [29], Benders decomposition [30,31], and Lagrangian decomposition [32] to address this issue. Other approaches include leveraging parallel computing resources [33] and developing novel algorithms like the Three-Front Parallel Branch-and-Cut Algorithm (3FP-B&C) [34]. Additionally, various metaheuristic algorithms—such as Simulated Annealing and Iterated Local Search [35], Genetic Algorithms (GAs) [36], and Modified Adaptive GAs [37]—have been employed. Matheuristic methods that combine mathematical programming and heuristics have also been proposed [38].

Although these methods have shown effectiveness in finding high-quality solutions within acceptable computational times, they often sacrifice the optimality guarantees inherent in exact methods. Our work contributes to this field by presenting an exact optimization model capable of solving medium-sized instances efficiently. Recognizing the scalability limitations of exact approaches, we aim to build upon this foundation by incorporating strategies to enhance computational performance for larger instances in future research.

A comparison of the main characteristics of the existing literature on the MIRP is presented in Table 1. The table includes the following aspects: the nature of the time horizon (discrete (D) or continuous (C)), the length of the time horizon, the type of algorithm used (exact (E) or metaheuristic (M)), and the number of considered vessels. The first column lists the authors of each work, whereas the second and third columns indicate whether the time horizon is discrete or continuous. The fourth column indicates time horizons of 60 time units or shorter, while the fifth column indicates time horizons longer than 60 time units. The sixth and seventh columns classify the algorithm used as exact or metaheuristic. The eighth and ninth columns report the number of vessels involved in the problem. Table 1 shows that only two studies have considered time horizons longer than 60 time units: Agra et al. (2022) [10], who used a single vessel and a metaheuristic method to solve instances of up to 240 time units, and Papageorgiou et al. (2018) [23], who used a metaheuristic method and fictitious markets in each port to solve instances of up to 360 time units. This work fills a gap in the literature by proposing an exact approach to solving multi-vessel problems with time horizons exceeding 60 time units.

Table 1. Literature review comparison.

	Time Horizon		**TH Length**		**Algorithm**		**Vessels**	
Reference	**D**	**C**	**$\leq$ 60**	**>60**	**E**	**M**	**1**	**>1**
[1]	✓		✓		✓			✓
[9]	✓	✓	✓		✓			✓
[18]	✓	✓	✓		✓			✓
[10]		✓		✓	✓		✓	
[11]		✓	✓		✓			✓
[7]	✓		✓		✓			✓
[21]	✓		✓			✓		✓
[12]		✓	✓		✓			✓
[23]	✓			✓		✓		✓
[27]	✓		✓			✓		✓
[13]		✓	✓			✓		✓
[8]	✓		✓		✓			✓
[22]	✓		✓			✓		✓
[3]	✓		✓		✓			✓
[14]		✓	✓		✓			✓
This work		✓		✓	✓			✓

3. Mathematical Model

In this section, we consider a new approach to address the MV-MIRP. We adopt a multi-vessel approach, capturing the complexities and dynamics in real-world shipping industries, such as fleet optimization and scheduling. With this approach, our objective is to provide a more comprehensive framework for solving the MIRP, improving the decision support for maritime logistic practitioners.

The MV-MIRP considers a set $N = \{1, \ldots, n\}$ of ports, where each port can be visited a maximum of $\overline{m}_i$ times. Consequently, we define a visit by the pair (i, m), with $i \in N$ and $m \in \{1, \ldots, \overline{m}_i\}$. The port i either produces ($J_i = 1$) or consumes ($J_i = -1$) inventory at a constant rate R_i. Let $G = (V, A)$ be a directed graph where the set V contains an origin node $\{O\}$ and all nodes formed by the ports and their respective visits. The set of arcs A comprises all possible combinations of nodes. However, an arc $((i, m), (j, n)) \in A$ is considered valid only if no consecutive visits occur at the same port i, satisfying the condition $i \neq j$. The port visits are executed by a fleet of heterogeneous vessels, denoted $K = \{1, \ldots, k\}$, each with capacity C_k. The trip of these vessels starts at the node of origin

$\{O\}$ and can end at any node on the graph. Finally, we define the parameters and variables used in our model to facilitate further analysis.

3.1. Parameters

- J_i: 1 if port i is a supplier, and -1 if port i is a consumer;
- $\overline{m}_i$: Maximum possible visits to port i;
- T_i^Q: Rate of *time/(unit of cargo)* loaded or unloaded in port i;
- C_{ij}^k: Variable cost for traveling from port i to port j by vessel k;
- C_0^k: Fixed cost for operating vessel k;
- C_y^k: Fixed cost per visit to port y by vessel k;
- $\overline{S}_i$: Maximum inventory level in port i;
- $\underline{S}_i$: Minimum inventory level in port i;
- S_i^0: Initial stock level in port i;
- R_i: Production or consumption rate in port i;
- Q_k^0: Initial cargo in vessel k on the origin O;
- C_k: Maximum capacity of vessel k;
- T_{ij}^S: Travel time from port i to port j;
- T_i^0: Travel time from the origin O to port i;
- T: Planning horizon.

Although time and cost units are arbitrary, it is crucial to maintain consistency throughout the entire model. For instance, if the planning horizon T is measured in days, then the travel times T_i^0 and T_{ij}^S must also be measured in days, and the rate T_i^Q must be measured in days per unit of cargo. A similar approach is required for costs C_{ij}^k, C_0^k, and C_y^k.

3.2. Binary Variables

- x_{imjnk}: 1 if vessel k travels directly from the node (i,m) to the node (j,n), 0 otherwise;
- x_{imk}^0: 1 if vessel k travels directly from the origin to the node (i,m), 0 otherwise;
- y_{imk}: 1 if vessel k makes the m^{th} visit to port i, 0 otherwise;
- z_{imk}: 1 if vessel k ends its journey at the node (i,m), 0 otherwise.

3.3. Continuous Variables

- q_{imk}: Quantity collected or delivered to the node (i,m) by vessel k;
- f_{imjnk}: Quantity transported from the node (i,m) to the node (j,n) by vessel k;
- f_{imk}^0: Quantity transported from the origin to the node (i,m) by vessel k;
- f_{imk}^D: Quantity transported from the node (i,m) to the destination by vessel k;
- t_{im}: Start time of operation at the node (i,m);
- s_{im}: Stock level at the beginning of the operation at the node (i,m).

We present an example in which two vessels navigate the graph; thus, $K = \{k_1, k_2\}$. Figure 1 illustrates the spatial distribution of ports 1, 2, and 3, forming the network over which the vessels route. Understanding the layout of these ports is essential to model routing decisions and travel times in the MIRP. Figure 2 displays an expanded graph that includes the origin node, the destination node, and the nodes representing each pair of port and visit, demonstrating the scheduling and routing complexities inherent in coordinating multiple vessels over multiple visits in the MV-MIRP. We consider that ports 1 and 2 can each be visited a maximum of two times—thus, $\overline{m}_1 = \overline{m}_2 = 2$—while port 3 can be visited up to three times, which means $\overline{m}_3 = 3$.

The solution to this example unfolds as follows: vessel 1 departs from the origin to the node $(1,1)$; thus, $x_{111}^0 = 1$. Then, it travels directly from the node $(1,1)$ to the node $(3,1)$, indicated by $x_{11311} = 1$. The remainder of the voyage of vessel 1 is as follows:

$(3,1) \rightarrow (2,1) \rightarrow (3,2) \rightarrow (1,2)$. Since $(1,2)$ is the last node visited by vessel 1 before proceeding to the destination, we have $z_{121} = 1$.

Vessel 2 departs from the origin to the node $(3,3)$, resulting in $x^0_{332} = 1$. Then, it travels directly to the node $(2,2)$; thus, $x_{33222} = 1$. Since the node $(2,2)$ is the last node visited by vessel 2 before heading to the destination, we have $z_{222} = 1$. To improve the understandability of Figure 2, the edges used by vessel 2 are colored blue.

Regarding the cargo transported, in this example, vessel 1 carries 10 units of cargo from the origin ($f^0_{111} = 10$), while vessel 2 carries 0 units ($f^0_{332} = 0$). In the example, vessel 1 unloads 5 units of cargo at the node $(1,1)$—therefore, $q_{111} = 5$—and transports the remaining 5 units to the node $(3,1)$, which is represented by $f_{11311} = 5$. To avoid cluttering Figure 2, we proceed directly to the variable f^D for vessel 1. In this case, vessel 1 transports 1 unit of cargo to the destination; thus, $f^D_{121} = 1$. For vessel 2, the variables function analogously: $f^0_{332} = 0$, $q_{332} = 6$, $f_{33222} = 6$, $q_{222} = 6$, and $f^D_{222} = 0$.

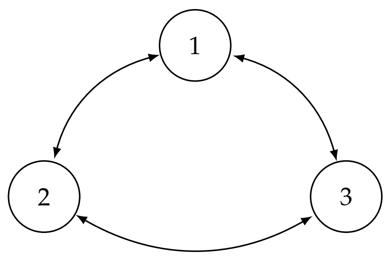

Figure 1. A geographical representation of the three ports in the MV-MIRP example.

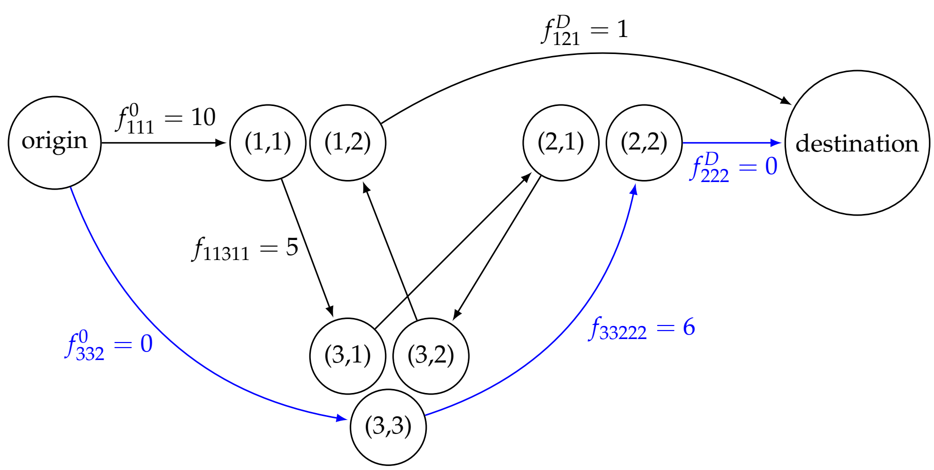

Figure 2. An expanded graph representation of the MV-MIRP example with two vessels, including the origin and destination nodes.

Next, we present the mathematical programming model.

3.4. Objective Function

$$\min z = \sum_{k \in K} \sum_{((i,m),(j,n)) \in A} C^k_{ij} x_{imjnk} + \sum_{k \in K} \sum_{(i,m) \in V} C^k_0 x^0_{imk} + \sum_{k \in K} \sum_{(i,m) \in V} C^k_y y_{imk} \quad (1)$$

The objective function (1) minimizes total costs, considering travel costs and fixed costs per used vessel, and the cost per visit.

To enhance readability, the constraints are grouped into thematic subsections that align with key operational aspects of the MV-MIRP. Each group addresses a specific set of requirements:

- Routing constraints (Section 3.5) ensure vessel connectivity and visit feasibility. This group primarily involves the variables x, y, and z.
- Pickup and delivery constraints (Section 3.6) maintain cargo balance and respect vessel capacities. This group primarily involves the variables f and q, along with x, y, and z.
- Time constraints (Section 3.7) regulate service times at ports and travel durations. This group primarily involves the variables t and x.
- Inventory constraints (Section 3.8) manage stock levels and production/consumption at ports. This group primarily involves the variables s, q, and t.

3.5. Routing Constraints

$$\sum_{(i,m)\in V} x^0_{imk} \leq 1 \qquad \forall\, k \in K \tag{2}$$

$$y_{imk} - \sum_{(j,n)\in V|j\neq i} x_{jnimk} - x^0_{imk} = 0 \qquad \forall\, (i,m) \in V, k \in K \tag{3}$$

$$y_{imk} - \sum_{(j,n)\in V|j\neq i} x_{imjnk} - z_{imk} = 0 \qquad \forall\, (i,m) \in V, k \in K \tag{4}$$

$$\sum_{k\in K} y_{imk} \leq 1 \qquad \forall\, (i,m) \in V \tag{5}$$

$$\sum_{k\in K} y_{i(m-1)k} - \sum_{k\in K} y_{imk} \geq 0 \qquad \forall\, (i,m) \in V|m \geq 2 \tag{6}$$

$$x^0_{imk}, y_{imk}, z_{imk} \in \{0,1\} \qquad \forall\, (i,m) \in V, k \in K \tag{7}$$

$$x_{imjnk} \in \{0,1\} \qquad \forall\, ((i,m),(j,n)) \in A, k \in K \tag{8}$$

Constraints (2) states that all vessels are located first at the origin. Constraints (3) and (4) are vessel connectivity constraints. Constraints (5) ensures that each node (i,m) can be visited by at most one vessel. Constraints (6) ensures the feasibility of consecutive visits to port i (that is, node $(1,2)$ cannot be visited before node $(1,1)$). Constraints (7) and (8) are variable definitions.

3.6. Pickup and Delivery Constraints

$$\sum_{k\in K} f^0_{imk} + \sum_{k\in K}\sum_{(j,n)\in V|j\neq i} f_{jnimk} + J_i \sum_{k\in K} q_{imk} = \sum_{k\in K}\sum_{(j,n)\in V|j\neq i} f_{imjnk} + \sum_{k\in K} f^D_{imk} \qquad \forall\, (i,m) \in V \tag{9}$$

$$f^0_{imk} = Q^0_k x^0_{imk} \qquad \forall\, (i,m) \in V, k \in K \tag{10}$$

$$f_{imjnk} \leq C_k x_{imjnk} \qquad \forall\, ((i,m),(j,n)) \in A, k \in K \tag{11}$$

$$f^D_{imk} \leq C_k z_{imk} \qquad \forall\, (i,m) \in V, k \in K \tag{12}$$

$$q_{imk} \leq C_k y_{imk} \qquad \forall\, (i,m) \in V, k \in K \tag{13}$$

$$f^0_{imk}, f^D_{imk}, q_{imk} \geq 0 \qquad \forall\, (i,m) \in V, k \in K \tag{14}$$

$$f_{imjnk} \geq 0 \qquad \forall\, ((i,m),(j,n)) \in A, k \in K \tag{15}$$

Constraints (9) ensures the balance of the cargo at each node (i, m). Constraints (10) states that the cargo transported from the origin node to the node (i, m) is equal to the initial cargo on vessel k. Constraints (11) and (12) ensure that the cargo transported cannot exceed the capacity of vessel k. Constraints (13) imposes upper limits on the pickup or delivery of cargo from each node (i, m). Constraints (14) and (15) ensure that the variables are nonnegative.

3.7. Time Constraints

$$t_{im} + T_i^Q q_{imk} + (T + T_{ij}^S)x_{imjnk} \le T + t_{jn} \qquad \forall\, ((i,m),(j,n)) \in A, k \in K \tag{16}$$

$$T_i^0 \sum_{k \in K} x_{imk}^0 \le t_{im} \qquad \forall\, (i,m) \in V \tag{17}$$

$$0 \le t_{im} \le T \qquad \forall\, (i,m) \in V \tag{18}$$

Constraints (16) gives a lower limit on the start time of service at node (j, n) by vessel k, after sailing directly from the node (i, m). Constraints (17) gives a lower bound to the start time of service on the node (i, m) by vessel k. Constraints (18) provides a minimum and maximum value for the t variables.

3.8. Inventory Constraints

$$S_i^0 + J_i R_i t_{i1} = s_{i1} \qquad \forall\, i \in N \tag{19}$$

$$s_{i(m-1)} - \sum_{k \in K} J_i q_{i(m-1)k} + J_i R_i (t_{im} - t_{i(m-1)}) = s_{im} \qquad \forall\, (i,m) \in V | m \ge 2 \tag{20}$$

$$s_{im} + \sum_{k \in K} q_{imk} - \sum_{k \in K} R_i T_i^Q q_{imk} \le \overline{S}_i \qquad \forall\, (i,m) \in V | J_i = -1 \tag{21}$$

$$s_{im} - \sum_{k \in K} q_{imk} + \sum_{k \in K} R_i T_i^Q q_{imk} \ge \underline{S}_i \qquad \forall\, (i,m) \in V | J_i = 1 \tag{22}$$

$$s_{i\overline{m}} + \sum_{k \in K} q_{i\overline{m}k} - R_i (T - t_{i\overline{m}}) \ge \underline{S}_i \qquad \forall\, i \in N | J_i = -1 \tag{23}$$

$$s_{i\overline{m}} - \sum_{k \in K} q_{i\overline{m}k} + R_i (T - t_{i\overline{m}}) \le \overline{S}_i \qquad \forall\, i \in N | J_i = 1 \tag{24}$$

$$\underline{S}_i \le s_{im} \le \overline{S}_i \qquad \forall\, (i,m) \in V \tag{25}$$

Constraints (19) specifies that the inventory level at the first visit to port i is determined by the initial inventory at port i (S_i^0) and the inventory produced (for $J_i = 1$) or consumed (for $J_i = -1$) between the beginning of the time horizon and the time of the first visit. Constraints (20) ensures that the inventory level at port i during visit m depends on the inventory level from the previous visit $m - 1$. This relationship accounts for the amount of cargo collected (for $J_i = 1$) or delivered (for $J_i = -1$) during the visit $m - 1$, and the inventory produced or consumed at port i between visits $m - 1$ and m. Constraints (21) and (22) ensure that inventory levels remain within their prescribed lower and upper limits at each port at the end of each visit. These constraints account for the cargo collected or delivered and the inventory produced or consumed during the handling of the cargo q_{imk}. Constraints (23) and (24) ensure that the inventory levels during the final visits remain within the allowed limits. This takes into account the cargo handled during the last visit and the inventory produced or consumed between the last visit and the end of the time horizon. Constraints (25) provides an upper and lower bound for the inventory level at each port at the beginning of each visit.

The constraints presented ensure a comprehensive formulation of the MV-MIRP by rigorously addressing its multifaceted requirements. Although the detailed structure of the constraints reflects the complexity of the problem, their thematic grouping provides a systematic framework for understanding their purpose. This detailed formulation is essential to achieve optimal solutions and accurately capture the intricacies of real-world maritime logistics.

3.9. Valid Inequalities

To improve the computational efficiency of our MV-MIRP model, we incorporate four sets of valid inequalities. These inequalities are designed to tighten the formulation by reducing the feasible region of the linear relaxation without excluding any feasible integer solutions. In doing so, they improve the quality of the lower bounds and accelerate the convergence of the solver. The valid inequalities included are the following: (i) minimum number of visits to a port (Section 3.9.1), (ii) logic inequalities (Section 3.9.2), and (iii) symmetry-breaking inequalities (Section 3.9.3).

3.9.1. Minimum Number of Visits to Port

The first set of valid inequalities ensures that the minimum required number of visits to each port is accounted for, based on the net demand over the planning horizon. This approach prevents the solver from considering solutions that do not meet the essential supply or demand requirements of each port, thereby eliminating infeasible or suboptimal solutions early in the process.

For their formulation, we calculate the net demand Q_i^N for each port i over the time horizon T. This net demand represents the total quantity that must be transported to or from port i to satisfy its production or consumption needs while respecting inventory limits.

For supplier ports ($J_i = 1$), we use Equation (26):

$$Q_i^N = \max\{TR_i + S_i^0 - \overline{S}_i, 0\} \qquad \forall\, i \in N, | J_i = 1 \tag{26}$$

For consumer ports ($J_i = -1$), we use Equation (27):

$$Q_i^N = \max\{TR_i - S_i^0, 0\} \qquad \forall\, i \in N, | J_i = -1 \tag{27}$$

Here, TR_i represents the total production or consumption over the planning horizon, S_i^0 is the initial inventory level, and $\overline{S}_i$ is the maximum inventory level at port i.

Next, with Equation (28), we estimate the minimum number of visits μ_i required for each port i by considering the largest vessel capacity available:

$$\mu_i = \lceil Q_i^N / \max C_k \rceil \qquad \forall\, i \in N, k \in K \tag{28}$$

To enforce these minimum visits, we introduce the following constraints ((29) and (30)):

$$\sum_{k \in K} y_{i\mu_i k} = 1 \qquad \forall i \in N \tag{29}$$

$$\sum_{n<m} z_{ink} + y_{imk} \le 1 \qquad \forall (i,m) \in V | m > \mu_i, k \in K \tag{30}$$

Constraint (29) ensures that a visit μ_i is made by a vessel $k \in K$. Constraint (30) ensures that if the vessel $k \in K$ visits port i for the m^{th} time, it cannot have made its last visit before in the same port.

These inequalities are based on the capacity and demand balance principles commonly used in inventory routing problems. By enforcing the minimum number of visits required to meet net demand, we eliminate infeasible or suboptimal solutions from the feasible region. This approach is supported by previous research that demonstrates the effectiveness of such constraints in strengthening optimization models [9].

3.9.2. Logic Inequalities

The logic inequalities capture the dependency between a vessel's departure from the origin and its ability to make port visits. They ensure that a vessel cannot perform any operations unless it has first departed from the origin, which is in line with real-world operational logic.

$$\sum_{(i,m)\in V} y_{imk} \leq M \sum_{(i,m)\in V} x^0_{imk} \qquad \forall k \in K \tag{31}$$

Here, M represents a sufficiently large constant, such as the total number of nodes in V. For instance, if there are five ports and the maximum allowable number of visits per port is four, then V would contain 21 nodes, including the origin node, and M would be assigned the value 21.

- If vessel k does not depart from the origin ($\sum_{(i,m)\in V} x^0_{imk} = 0$), then $\sum_{(i,m)\in V} y_{imk} \leq 0$, implying $y_{imk} = 0$ for all (i, m). This ensures that no visits are scheduled for a vessel that remains at the origin.
- If vessel k leaves ($\sum_{(i,m)\in V} x^0_{imk} = 1$), the constraints allow for any number of visits up to M.

By adding logical constraint (31), we eliminate infeasible scenarios where a vessel makes port visits without departing from the origin. This refinement reduces the solution space and assists the solver in converging more quickly.

Logic constraints are a standard technique in integer programming to model dependencies and conditional relationships. They have been shown to effectively reduce computational complexity by pruning infeasible or illogical solutions [39].

3.9.3. Symmetry Breaking Inequalities

Symmetry in optimization models can lead to redundant exploration of equivalent solutions, increasing computational effort without providing additional insights. Symmetry-breaking inequalities reduce this redundancy by imposing an order on homogeneous vessels.

Constraint (32) stipulates that vessel k can only leave the origin if vessel $(k-1)$ has already departed. It effectively orders the departures of identical vessels, breaking symmetry.

$$\sum_{(i,m)\in V} x^0_{imk} \leq \sum_{(i,m)\in V} x^0_{im,k-1} \qquad \forall k \in S | k > 1 \tag{32}$$

where $S \subseteq K$ is the subset of homogeneous vessels.

By reducing the number of symmetric solutions, we decrease the computational burden on the solver. These constraints help prevent the exploration of multiple permutations of vessel departures that are equivalent in terms of the objective function.

Symmetry-breaking constraints are widely recognized for improving the efficiency of solving integer programming models. They have been successfully applied in various contexts to reduce solution times without sacrificing optimality [40].

3.10. Case Study: Antarctic Logistics

To evaluate the practicality of the proposed model, we created a set of problem instances based on real-world data from Chilean Antarctic scientific bases. This case study reflects the logistical challenges of supplying research stations in a remote and environmentally sensitive region.

1. Data Sources: The geographical coordinates of seven Chilean Antarctic bases were obtained using Google Earth, providing accurate distance matrices for modeling travel times and costs. Chile maintains 10 scientific bases and 2 shelters in Antarctica, and we selected 7 specific bases for this study to ensure a representative and diverse set of scenarios.
   The selection was not random; the chosen bases are strategically distributed across key locations, including the South Shetland Islands (e.g., Base Escudero) and continental Antarctica (e.g., Base Yelcho). This distribution allows the model to account for both short-distance travels within the islands and long-distance travels between the islands and continental Antarctica, reflecting realistic logistical challenges. The locations of the selected bases are shown in Figure 3. These bases are Base Yelcho, Base Presidente Eduardo Frei Montalva, Base Escudero, Risopatrón Base, Captain Arturo Prat Base, Doctor Guillermo Mann Base, and Base General Bernardo O'Higgins.
   Logistical requirements, including cargo demand, were estimated based on the daily meal needs of the scientific personnel at each base. Each time period in the model represents one day, aligning with the planning horizon of a typical scientific mission in Antarctica (approximately four months for a 120-day horizon).
2. Instance Generation: The data were used to define the parameters of the problem, such as the number of ports (Antarctic bases), the capacities of the vessels, the production and consumption rates, and the levels of inventory. The generated instances reflect realistic logistical scenarios that capture the unique operational conditions of Antarctic supply chains.
3. Model Application: The Antarctic instances were solved using the proposed MILP model to optimize multi-vessel routing and inventory management over long planning horizons. The results, including computational performance and optimality gaps, are presented in Section 4, which compares the performance of weak and strong formulations.
4. Assumptions: For simplicity, the analysis assumes constant production and consumption rates at the bases, as well as fixed vessel capacities. These assumptions allow the model to focus on routing and inventory decisions while maintaining computational tractability.

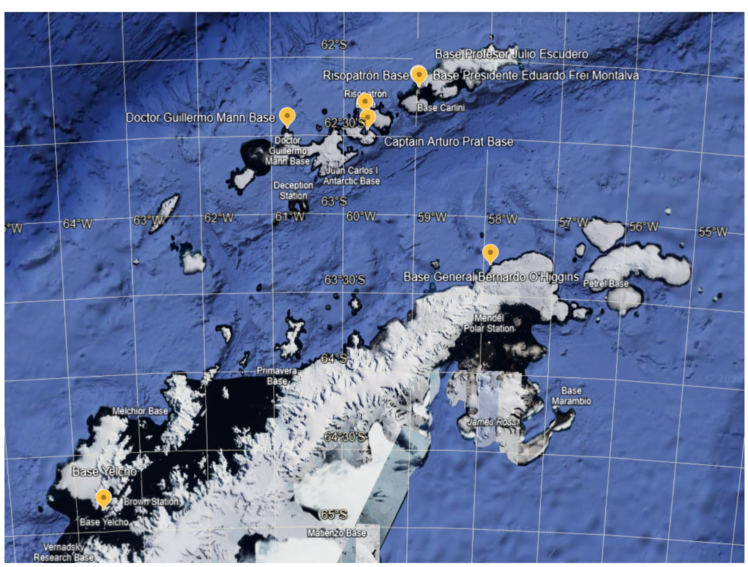

Figure 3. Locations of the seven Chilean Antartic scientific bases considered in this work.

4. Results

We present the experimental results of our mathematical model proposed for the MV-MIRP. Our experimental setup consists of two phases. In the first phase, we compare the elapsed times of our model with those of the literature [10], as shown in Tables

2–4. Although we had access to key data from the instances provided by Agra et al. (2022)—including the distance matrix, ship speeds, and supply and demand at each port—shared with us upon request, we did not have access to the exact parameters of the objective function. As a result, we were unable to compute an initial lower bound value or a gap in this phase. In the second phase, we apply our model to our original instances, whose results are shown in Tables 5–8. The results show that our method can solve most large instances optimally and in a computationally efficient time.

We implemented our approach with Gurobi 10 optimization software on a Windows 10 machine with a 12-core and 24-thread Intel Xeon E5-2696v2 at 2.5 GHz CPU, and 16 GB of RAM at 1866 MHz. We selected the default tolerance level of 10^{-6} and set a maximum solver time of 3600 s. We recorded the elapsed time taken by Gurobi 10 to find an optimal solution or prove infeasibility on each MV-MIRP instance, providing insights into our methodology's computational complexity and efficiency. By following this setup, we expected to deliver a high-quality analysis of the performance of our formulation to solve problem instances of different sizes.

4.1. Model Performance Comparison

Table 2 provides an overview of the time elapsed for each instance of both studies, over a time horizon of $T = 60$. The first column of the table indicates the name of each instance. The following two columns show the number of loading ports and the number of unloading ports. We compare our work with three different methods utilized in [10]: (i) branch-and-cut with a weak formulation (BCw), (ii) branch-and-cut with a tightened formulation (BCs), and (iii) a custom algorithm developed by the authors (ALG1). The results are given in the following three columns. The results of this work (TW) are presented in the last column.

Table 2. Elapsed time comparison between branch-and-cut weak formulation (BCw), branch-and-cut tightened formulation (BCs), custom algorithm (ALG1), and this work (TW) for a time horizon $T = 60$.

			Elapsed Time [s]			
Instance	**Loading Ports**	**Unloading Ports**	**BCw**	**BCs**	**ALG1**	**TW**
A1	1	3	31.6	21.7	20.4	1.4
A2	1	3	35.2	21.7	17.3	1.6
B1	1	2	11.5	7.7	8.0	1.4
B2	1	2	13.3	9.2	9.0	2.0
C1	2	2	20.5	19.9	29.1	1.4
C2	2	2	20.4	14.0	16.5	3.6
D1	2	3	89.8	32.5	24.3	2.6
D2	2	3	195.0	37.2	37.0	5.2
E1	2	3	56.7	30.2	59.4	0.8
E2	2	3	166.0	43.0	72.2	0.8
F1	2	2	31.4	19.0	23.8	0.3
F2	2	2	27.8	16.0	17.6	1.0
G1	3	3	20.5	11.5	13.0	3.9
G2	3	3	21.0	13.7	13.2	2.0

The custom algorithm (ALG1) is a two-level branching approach to solve the MIRP to optimality. The main idea is to divide the problem into two subproblems based on the total number of visits by the vessel. The first subproblem has a constraint that limits the number of visits to be less than or equal to half of a given upper bound, while the second subproblem has a constraint that requires the number of visits to be greater than or equal to half of the upper bound plus one. The first subproblem is solved using branch-and-cut considering a strong formulation with valid inequalities (BCs). The best feasible solution found is used as a cut-off value for the second subproblem. The algorithm returns the best solution among the two subproblems. Since each subproblem has a time limit of 3600 s, the entire algorithm has a maximum time limit of 7200 s.

We present the results for a longer planning horizon ($T = 120$) in Table 3. This table shows the name of the instance, the number of loading ports (LP), the number of unloading ports (UP), and the solution time for each method. If the optimal solution is not found within 3600 [s], we report the % gap between the best value of the upper bound (UB) and the best value of the lower bound (LB), where % gap = ((UB - LB) / UB) 100. We label instances that were proven to be unfeasible within the time limit as (UNF). We use a hyphen to denote inconclusive instances, which means that the model does not reach optimality, find a gap, or prove infeasibility. We do not use any tightening constraints for these model tests. For the time horizon $T = 60$, we obtain optimal solutions for all instances, with significantly shorter solving times than those reported in the literature. For the $T = 120$ time horizon, our approach outperforms previous studies by solving nine instances to optimality, while only five were previously solved. Moreover, we observe a gap in two instances, whereas they observed a gap in three. Four instances were unresolved in the previous study and zero in ours. The shorter running times achieved with Gurobi and a high-speed processor highlight the importance of using efficient computational tools.

Table 3. Performance comparison between branch-and-cut weak formulation (BCw), branch-and-cut tightened formulation (BCs), custom algorithm (ALG1), and this work (TW) for a time horizon $T = 120$.

			Elapsed Time [s]				% Gap			
Instance	**LP**	**UP**	**BCw**	**BCs**	**ALG1**	**TW**	**BCw**	**BCs**	**ALG1**	**TW**
A1	1	3	3600.0	3600.0	3922.5	460.8	-	-	-	0.0
A2	1	3	3600.0	3600.0	5113.1	883.5	-	-	19.0	0.0
B1	1	2	3600.0	2870.9	3619.4	1722.4	-	UNF	-	UNF
B2	1	2	195.0	19.5	21.0	9.8	0.0	0.0	0.0	0.0
C1	2	2	204.0	104.2	115.6	10.6	0.0	0.0	0.0	0.0
C2	2	2	2907.0	242.2	320.9	47.5	0.0	0.0	0.0	0.0
D1	2	3	3600.0	3600.0	3736.1	3600.0	-	18.9	-	8.1
D2	2	3	3600.0	3600.0	3698.9	3600.0	-	-	-	21.8
E1	2	3	3600.0	2795.0	5566.7	80.2	-	0.0	0.0	0.0
E2	2	3	3600.0	3600.0	7200.0	2.8	-	18.9	21.5	0.0
F1	2	2	3600.0	180.2	210.7	1.2	-	0.0	0.0	0.0
F2	2	2	3600.0	3.9	3.4	14.2	-	UNF	UNF	UNF
G1	3	3	3600.0	617.8	198.2	18.8	-	UNF	UNF	UNF
G2	3	3	3600.0	3600.0	6451.4	6.9	-	-	0.8	0.0

We compare the performance of three formulations of our work in the same instances: (i) a weak formulation without valid inequalities (WF), (ii) a strong formulation with valid inequalities (29) and (30), which consider the minimum number of visits to a port (SA); and (iii) a strong formulation with logic and symmetry-breaking valid inequalities (31) and (32) (SO). Table 4 shows the results. The first column shows the name of the instance. The next three columns show the elapsed time in seconds for each formulation. The SO formulation is the fastest on four instances, and the SA formulation is the fastest on instances E1 to G2. The WF formulation is faster only in instances A1 and C1. Columns 5 to 7 show the nodes explored by each formulation. Usually, fewer explored nodes mean less computational time, but this is not true for instances B1 and C1. The SA formulation shows the fewer nodes explored in two instances, while the SO formulation does so in four. Columns 8 to 10 show the initial lower bound of the linear relaxation for each formulation. The SA formulation always improves the initial lower bound of the WF formulation, while the SO formulation does so only in instances B1 and B2.

Table 4. Performance comparison between formulations WF, SA, and SO for time horizon $T = 60$.

	Elapsed Time [s]			Explored Nodes			Initial Lower Bound		
Instance	**WF**	**SA**	**SO**	**WF**	**SA**	**SO**	**WF**	**SA**	**SO**
A1	1.4	3.7	4.1	1179	4426	3871	601,510	683,252	601,510
A2	1.6	1.2	1.2	2145	1243	1015	412,552	553,423	412,552
B1	1.4	1.4	1.4	2030	793	1176	791,184	800,783	796,769
B2	2.0	1.6	1.1	2342	2461	973	859,426	961,331	865,491
C1	1.4	1.5	1.8	215	292	185	413,037	426,614	413,037
C2	3.6	1.7	2.7	2614	373	1611	509,115	524,363	509,115
D1	2.6	1.4	2.0	96	1	1	449,030	644,628	449,030
D2	5.2	5.0	3.7	2040	2506	959	580,720	713,767	580,720
E1	0.8	0.5	0.9	1	1	1	231,233	433,827	231,233
E2	0.8	0.6	1.2	1	1	1	8451	352,682	8451
F1	0.3	0.2	0.2	1	1	1	79,196	156,100	79,196
F2	1.0	0.2	1.2	1	1	1	200,225	390,100	200,225
G1	3.9	1.3	4.8	1	1	1	140,548	720,611	140,548
G2	2.0	1.1	2.6	1	1	1	120,296	583,900	120,296

4.2. Model Performance with New Instances

To evaluate the practical applicability of our MILP formulation, we create a set of new MV-MIRP instances using real-world data from Chilean scientific bases in Antarctica. These bases require regular deliveries of supplies to support their research activities, making them ideal candidates for our case study.

We use Google Earth to obtain the geographical coordinates of seven Antarctic Chilean bases. The coordinates allow us to calculate accurate distance matrices between the bases, which serve as ports in our model. The cargo at each base is estimated on the basis of the number of meals required to sustain the scientific personnel. We assume an average number of personnel at each base and calculate daily meal requirements, which is translated into cargo demand in our model. This approach reflects the logistical needs of supplying food and essentials to the bases. We consider each time period as one day. The longer planning horizon of 120 time periods corresponds to the typical duration of a scientific mission in Antarctica, which spans approximately four months.

We report the experimental results for the new set of problem instances on a time horizon of $T = 60$ in Table 5. The name of the instance, the number of loading and unloading ports, and the number of vessels are given in columns 1 to 4, respectively. Columns 5 to 7 show the running times for each formulation until optimality or the time limit is reached. Columns 8 to 10 show the percentage gap between the best value and the lower bounds for each formulation, or a hyphen if no gap was achieved.

The computational performance of the solver varies depending on the characteristics of the instances. We notice that larger instances (from 1 to 6) generally require more time, nodes, and iterations than smaller ones. For example, while instance 1 is solved optimally in less than a second, instance 6 only reaches a gap after an hour of computation. However, instance 6b shows that the size of the instance is not the only factor affecting the solver efficiency, as it obtains an optimal solution in less than 5 s for $T = 60$. Therefore, other aspects, such as production/consumption rates, vessel capacities, and inventory levels at the bases, also play a significant role in the solver performance.

Table 5. Elapsed time and percentage gap for MV-MIRP on original instances for $T = 60$.

				Elapsed Time [s]			% Gap		
Instance	**LP**	**UP**	**V**	**WF**	**SA**	**SO**	**WF**	**SA**	**SO**
1	1	3	1	0.4	0.2	0.6	0.0	0.0	0.0
1b	1	3	1	0.2	0.2	0.5	0.0	0.0	0.0
2	1	2	2	0.8	0.0	0.6	0.0	0.0	0.0
2b	1	2	2	1.8	0.4	1.6	0.0	0.0	0.0
3	2	2	2	7.1	6.7	0.6	0.0	0.0	0.0
3b	2	2	2	6.9	5.9	0.9	0.0	0.0	0.0
4	2	3	2	585.0	2452.6	1.9	0.0	0.0	0.0
4b	2	3	2	894.1	377.7	1.7	0.0	0.0	0.0
5	2	3	3	2049.2	3600.0	374.4	0.0	15.6	0.0
5b	2	3	3	2615.0	801.7	318.9	0.0	0.0	0.0
6	2	4	2	3600.0	28.1	3600.0	24.7	0.0	24.9
6b	2	4	2	124.6	224.4	9.4	0.0	0.0	0.0
7	2	5	3	1146.2	726.5	389.6	0.0	0.0	0.0
7b	2	5	3	3600.0	956.3	133.2	5.9	0.0	0.0

We present additional results on node exploration and the initial lower bound for each instance in Table 6. The table lists the name of the instance, the number of loading ports (LP), the number of unloading ports (UP), and the number of vessels (V) in the first four columns. The next three columns show the nodes explored by Gurobi with different formulations, and the last three columns show the initial lower bound obtained with each formulation. The SA formulation reduces node exploration in two instances, but improves the initial lower bound in all instances. The SO formulation reduces node exploration in 12 instances, but improves the initial lower bound only in 4 instances.

Table 6. Explored nodes and initial lower bound for MV-MIRP on original instances for $T = 60$.

				Explored Nodes			Initial Lower Bound		
Instance	**LP**	**UP**	**V**	**WF**	**SA**	**SO**	**WF**	**SA**	**SO**
1	1	3	1	1	1	1	261,558	315,411	261,558
1b	1	3	1	1	1	1	285,973	337,822	285,973
2	1	2	2	436	1	1	374,223	649,000	374,223
2b	1	2	2	2331	1	1242	687,550	711,333	687,550
3	2	2	2	18,185	37,813	1	26,570	88,900	30,231
3b	2	2	2	14,968	26,280	1	26,787	88,900	31,109
4	2	3	2	828,851	363,8473	1	31,542	49,100	34,828
4b	2	3	2	1,042,948	610,822	1	14,438	34,423	17,384
5	2	3	3	1,630,249	3,390,406	363,247	86,074	202,829	86,074
5b	2	3	3	2,105,200	585,315	333,075	116,571	214,369	116,571
6	2	4	2	1,317,710	23,196	1,396,386	165,538	600,939	165,538
6b	2	4	2	175,889	312,753	12,237	229,373	300,622	229,373
7	2	5	3	731,809	448,182	302,965	156,077	424,932	156,077
7b	2	5	3	1,453,616	749,996	74,234	99,454	196,153	99,454

The results in Table 7 show that increasing the time horizon from $T = 60$ to $T = 120$ has a significant impact on the computational performance of the model. Most instances require much longer running times to reach optimality or terminate without finding an optimal solution in 3600 s. Only instance 3 has a negligible difference in runtime, suggesting that the complexity and difficulty of the problem vary depending on the instance characteristics. Comparison with Table 5 confirms that all instances are more difficult to solve with a longer time horizon and that only five instances achieve optimality in this setting. These findings highlight the trade-off between the accuracy and tractability of the model and the need for further research on efficient solution methods for the MIRP.

Table 7. Elapsed time and percentage gap for MV-MIRP on original instances for $T = 120$.

				Elapsed Time [s]			% Gap		
Instance	**LP**	**UP**	**V**	**WF**	**SA**	**SO**	**WF**	**SA**	**SO**
1	1	3	1	3600.0	3600.0	3600.0	INC	INC	INC
1b	1	3	1	3600.0	3600.0	3600.0	INC	INC	INC
2	1	2	2	1775.4	1266.8	1185.7	0.0	0.0	0.0
2b	1	2	2	42.0	114.3	27.5	0.0	0.0	0.0
3	2	2	2	11.5	8.7	7.2	0.0	0.0	0.0
3b	2	2	2	15.2	13.4	3.1	0.0	0.0	0.0
4	2	3	2	3600.0	3600.0	3154.0	17.8	14.6	0.0
4b	2	3	2	3600.0	3600.0	3600.0	13.8	12.7	3.3
5	2	3	3	3600.0	3600.0	3600.0	7.8	13.5	7.8
5b	2	3	3	3600.0	309.9	3600.0	21.5	0.0	29.1
6	2	4	2	3600.0	419.1	3600.0	6.1	0.0	7.2
6b	2	4	2	3600.0	3600.0	3600.0	4.5	1.6	4.6
7	2	5	3	3600.0	3600.0	3600.0	INC	INC	INC
7b	2	5	3	3600.0	3600.0	3600.0	20.0	24.7	22.1

Table 8 shows the number of nodes explored and the initial lower bound values for the original instances with a time horizon of $T = 120$. The table has the same columns as Table 6. The results suggest that longer time horizons increase the difficulty of solving the instances, as the number of nodes explored increases significantly compared to Table 6. The SA formulation outperforms the other formulations in seven instances, by exploring fewer nodes and enhancing all initial lower bound values. The SO formulation also explores fewer nodes in seven instances, but only enhances the initial lower bound value in three instances.

Table 8. Explored nodes and initial lower bound for MV-MIRP on original instances for $T = 120$.

				Explored Nodes			Initial Lower Bound		
Instance	**LP**	**UP**	**V**	**WF**	**SA**	**SO**	**WF**	**SA**	**SO**
1	1	3	1	4,435,944	8,900,858	1,890,043	3,036,711	3,043,122	3,036,711
1b	1	3	1	7,715,299	5,524,034	8,524,051	2,955,044	2,956,662	2,955,044
2	1	2	2	6,755,676	5,945,934	1,323,876	2,289,638	2,525,500	2,289,638
2b	1	2	2	77,532	381,348	43,824	2,899,500	2,963,133	2,899,500
3	2	2	2	7399	5566	1892	164,670	182,612	196,086
3b	2	2	2	17,407	13,489	693	164,887	182,804	196,964
4	2	3	2	2,646,762	1,832,312	2,603,886	279,596	390,409	287,353
4b	2	3	2	2,523,325	2,131,800	2,927,427	289,798	358,631	289,798
5	2	3	3	2,205,410	2,646,163	1,756,032	651,129	830,380	651,129
5b	2	3	3	1,996,743	282,437	1,417,788	772,523	1,230,566	772,523
6	2	4	2	1,915,121	299,373	1,664,805	1,382,327	1,946,887	1,382,327
6b	2	4	2	2,129,875	1,680,105	1,950,939	1,438,538	1,942,467	1,438,538
7	2	5	3	931,465	1,181,111	982,733	1,782,577	2,326,201	1,782,577
7b	2	5	3	716,540	688,821	760,832	1,196,418	1,468,096	1,196,418

5. Discussion and Future Research

In this section, we delve into three critical aspects of the proposed MILP model and its applicability to the MV-MIRP. Section 5.1 addresses the scalability of the model, exploring its computational performance as the complexity of the problem increases, and discussing potential strategies to improve the scalability. Section 5.2 considers the incorporation of uncertainty into the problem, emphasizing the challenges posed by dynamic and stochastic elements in real-world maritime logistics and suggesting future directions for extending the model. Finally, Section 5.3 discusses environmental considerations, highlighting the potential to integrate sustainability objectives, such as reducing emissions and optimizing fuel efficiency, into the model. Together, these subsections provide a comprehensive discussion on improving the practical relevance, robustness and alignment of the model with modern logistical and environmental demands.

5.1. Scalability

The scalability of our MILP model is a critical factor for its practical applicability in real-world maritime logistics. As the number of vessels, ports, and time units increases, the computational complexity of the model grows significantly. Specifically, the number of binary variables related to routing decisions and scheduling increases combinatorially with the addition of more vessels and ports. This growth leads to longer computational times and a greater demand for memory resources.

Our computational experiments, presented in Tables 7 and 8, demonstrate that instances with larger sizes or longer planning horizons (e.g., $T = 120$) require substantially more computational time, and in some cases, the solver could not find optimal solutions within the allotted time. For example, instances 7 and 7b showed increased difficulty as the number of ports and vessels increased, indicating that scalability is a limiting factor of our current approach.

To address scalability concerns and provide a broader perspective on our model's performance relative to other solution techniques, we compared our results with other works that use metaheuristics to solve related problems. Ref. [35] addressed the IRP using Simulated Annealing, solving instances with up to 50 customers, five potential vehicles, and six time periods. For the longest time horizon of six time periods, they achieved a best optimality gap of 0.66% for instance H6 with one potential vehicle. Ref. [37] tackled a Multi-Product Multi-Vehicle Inventory Routing Problem using a Modified Adaptive Genetic Algorithm, solving instances with up to 30 customers, five vehicles, and seven time periods. They reported an average "closeness" value of 29.05, a performance metric they used to evaluate their results, but did not reach optimality in any instance. Ref. [36] solved instances of a location-inventory-routing problem with up to 50 customers and five time periods using a Genetic Algorithm; however, the number of vehicles was not explicitly stated.

Although our work encounters gaps of up to 24.9% on the largest tested instance 7b, we consider instances with up to 120 time periods, which is over 17 times longer than the longest time horizon considered by [37]. This demonstrates that our exact optimization approach can handle significantly longer planning horizons than those addressed by metaheuristic methods, albeit with a trade-off in computational performance and solution optimality for larger instances.

Although metaheuristic approaches have shown promise in addressing large-scale problems with acceptable computational times, they often sacrifice optimality guarantees. Our exact MILP model provides optimal solutions for small to medium-sized instances and longer planning horizons, making it a valuable tool for strategic decision-making where accuracy is paramount. Future research should focus on integrating advanced decomposition techniques, parallel computing, and hybrid methods to address scalability challenges and extend the model's applicability to even larger instances. In addition to tackling computational complexity, addressing the inherent uncertainty present in real-world maritime logistics is crucial to enhancing the robustness of the model.

5.2. Uncertainty Management

An important avenue for future research is the incorporation of dynamic and stochastic elements into the MV-MIRP. In real-world maritime logistics, parameters such as demand in ports, production and consumption rates, vessel capacities, and operational conditions are rarely constant and often subject to uncertainty. Factors such as time-varying demand, fluctuating supply rates, and unforeseen events such as vessel breakdowns or adverse weather conditions can significantly impact routing and inventory decisions.

Extending our deterministic model to accommodate these uncertainties would enhance its practical applicability and robustness. One potential approach is to develop a stochastic programming version of the MV-MIRP, where uncertain parameters are modeled as random variables with known probability distributions. This would allow for the optimization of expected costs or service levels while considering the variability in demand and supply. Alternatively, robust optimization techniques could be employed to find solutions that perform well under the worst-case scenarios of uncertainty, providing a safeguard against extreme events.

Incorporating dynamic elements, such as the time-varying demand and supply rates, would require adapting the model to handle parameters that change over time within the planning horizon. This could involve segmenting the time horizon into smaller intervals where the parameters are assumed to be constant or developing continuous-time models that can capture dynamic behaviors more effectively.

Implementing these extensions poses several challenges, including increased computational complexity and the need for advanced solution methodologies. Techniques such as scenario decomposition, sampling methods, or hybrid approaches combining exact and heuristic algorithms might be necessary to solve the stochastic MV-MIRP efficiently.

5.3. Environmental Considerations

An important aspect of modern maritime logistics is the consideration of environmental sustainability, particularly in reducing emissions and optimizing fuel efficiency. While our current model focuses on minimizing total costs—which inherently accounts for fuel consumption through variable travel costs—it does not explicitly incorporate environmental objectives or constraints. Integrating environmental considerations into the MV-MIRP could involve introducing emission factors associated with fuel consumption, vessel speed, and cargo load, as suggested by [12].

For example, the model could be extended to minimize greenhouse gas emissions by incorporating emission coefficients into the objective function or by adding constraints that limit the total emissions over the planning horizon. Furthermore, optimizing vessel speeds for fuel efficiency, as explored by [21], could be integrated into the model by treating speed as a decision variable and modeling its nonlinear relationship with fuel consumption and emissions.

Incorporating environmental sustainability into the MV-MIRP would enable practitioners to optimize both operational efficiency and ecological impact, aligning with growing global regulatory demands for greener shipping practices. Future work could integrate emission factors, fuel efficiency, and vessel speeds into the model, contributing to a reduction in carbon footprints while maintaining cost-effectiveness. By combining sustainability objectives with scalability and uncertainty management, our model can evolve into a comprehensive tool that balances economic, operational, and environmental considerations, thus addressing the multifaceted challenges of modern maritime logistics.

6. Conclusions

Our research extends the best-performance continuous-time model for the MIRP, introducing several novel features. Unlike the base model, which allows only the use of a single vessel, our model enables the routing of a fleet of heterogeneous vessels. Furthermore, our model incorporates an optimization component for the size of the fleet, selecting the combination of vessels that minimizes the objective value. To enhance the efficiency of the model, we applied tightening constraints based on valid inequalities from the existing literature and also proposed two new valid inequalities. Using these cuts, we improved the initial lower bounds and the efficiency of the model. To evaluate the effectiveness of our extended model, we created a new set of instances based on real-world data on Chilean scientific bases in Antarctica, obtained from Google Earth. These instances reflect the logistical challenges faced by Antarctic bases, such as long distances, and the need for efficient resource allocation to maintain an appropriate inventory level of meals at all times. Through these enhancements and the use of real-world data, our model represents a significant advance in addressing the MIRP. These advancements provide maritime logistics practitioners with a more powerful tool to optimize fleet routing and inventory management, reducing both operational costs and environmental impact.

To evaluate the performance of our model, we tested it on reference instances from the literature and obtained computational times lower than those reported in previous studies. Since the exact parameters of the reformulated instances for one vessel are not available, we cannot make a direct comparison of the results. However, we can demonstrate that computational times can be significantly reduced by using more advanced hardware and a commercial solver, such as Gurobi.

We ran the model with our original instances, obtaining results that met our expectations based on the results of the instances in the literature, which are presented in Tables 5–8. Using the weak formulation, we could solve all but one instance to optimality within a time horizon of 60 time units, while the remaining instance had a small gap. However, when we extended the time horizon to 120 time units, the weak formulation became less effective and could only solve four instances to optimality. The strong formulations (SA and SO) performed better and solved six and five instances to optimality, respectively. In total, we solved seven instances to optimality using SA or SO. For the remaining instances, we either obtained a gap (five instances) or could not determine the feasibility or optimality (three instances). We also observed that the strong formulations reduced the computational time or the gap for most instances after 3600 s, except for instance 2b, where the SA formulation increased the elapsed time.

The authors acknowledge that the computational complexity of the instance, which includes the number of ports and vessels, cannot be determined solely by its size. We observe that instance 6b had a faster running time considering a time horizon of 60 time units, compared with smaller instances, such as 3 and 3b. Therefore, the smallest instances (1 and 1b) exhibited the highest complexity with a time horizon of 120 time units, making it challenging to achieve a gap or demonstrate infeasibility.

We evaluated the performance of MV-MIRP, a method for solving the multi-vessel MIRP with long time horizons and constant port rates, using a continuous-time formulation. Our results showed that MV-MIRP was effective, fast, and reliable. This method has theoretical and practical implications for researchers and practitioners in the shipping industry, as it can handle complex problems with fewer computational and timely resources.

It also opens new avenues for future research on stochasticity, dynamic effects, climate prediction, etc.

In addition to the contributions presented in this paper, future research could further explore the integration of stochastic elements and environmental sustainability, as discussed. These enhancements would extend the model's applicability to real-world dynamic scenarios, making it even more robust for maritime logistics challenges.

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Abbreviations

The following abbreviations are used in this manuscript:

IRP	Inventory Routing Problem
MILP	Mixed-Integer Linear Programming
MIRP	Maritime Inventory Routing Problem
MV-MIRP	Multi-Vessel Maritime Inventory Routing Problem

References

1. Agra, A.; Christiansen, M.; Delgado, A. Mixed integer formulations for a short sea fuel oil distribution problem. *Transp. Sci.* **2013**, *47*, 108–124. https://doi.org/10.1287/TRSC.1120.0416.
2. Prause, F.; Prause, G.; Philipp, R. Inventory Routing for Ammonia Supply in German Ports. *Energies* **2022**, *15*, 6485. https://doi.org/10.3390/EN15176485.
3. Yusuf, F.K.; Ridwan, A.Y.; Pambudi, H.K. Maritime Inventory Routing Problem: Application on Discharge the Load of the Ship in Cement Companies to Minimize the Total Transportation Cost. *IOP Conf. Ser. Mater. Sci. Eng.* **2020**, *982*, 012056. https://doi.org/10.1088/1757-899X/982/1/012056.
4. Montagné, R.; Gamache, M.; Gendreau, M. A shortest path-based algorithm for the inventory routing problem of waste vegetable oil collection. *J. Oper. Res. Soc.* **2019**, *70*, 986–997. https://doi.org/10.1080/01605682.2018.1476801.
5. Archetti, C.; Speranza, M.G. The inventory routing problem: The value of integration. *Int. Trans. Oper. Res.* **2016**, *23*, 393–407. https://doi.org/10.1111/itor.12226.
6. Bertazzi, L.; Speranza, M.G. Inventory routing problems: An introduction. *EURO J. Transp. Logist.* **2012**, *1*, 307–326. https://doi.org/10.1007/S13676-012-0016-7.
7. Baller, A.C.; Dabia, S.; Desaulniers, G.; Dullaert, W.E. The Inventory Routing Problem with Demand Moves. *Oper. Res. Forum* **2021**, *2*, 6. https://doi.org/10.1007/s43069-020-00042-z.
8. Skålnes, J.; Andersson, H.; Desaulniers, G.; Stålhane, M. An improved formulation for the inventory routing problem with time-varying demands. *Eur. J. Oper. Res.* **2022**, *302*, 1189–1201. https://doi.org/10.1016/J.EJOR.2022.02.011.

9. Agra, A.; Andersson, H.; Christiansen, M.; Wolsey, L. A maritime inventory routing problem: Discrete time formulations and valid inequalities. *Networks* **2013**, *62*, 297–314. https://doi.org/10.1002/NET.21518.
10. Agra, A.; Christiansen, M.; Wolsey, L. Improved models for a single vehicle continuous-time inventory routing problem with pickups and deliveries. *Eur. J. Oper. Res.* **2022**, *297*, 164–179. https://doi.org/10.1016/J.EJOR.2021.04.027.
11. Algendi, A.; Urrutia, S.; Hvattum, L.M. Optimizing production levels in maritime inventory routing with load-dependent speed optimization. *Flex. Serv. Manuf. J.* **2023**, *35*, 111–141. https://doi.org/10.1007/S10696-022-09460-Z.
12. Eide, L.; Ardal, G.C.H.; Evsikova, N.; Hvattum, L.M.; Urrutia, S. Load-dependent speed optimization in maritime inventory routing. *Comput. Oper. Res.* **2020**, *123*, 105051. https://doi.org/10.1016/j.cor.2020.105051.
13. Shaabani, H.; Hoff, A.; Hvattum, L.M.; Laporte, G. A matheuristic for the multi-product maritime inventory routing problem. *Comput. Oper. Res.* **2023**, *154*, 106214. https://doi.org/10.1016/J.COR.2023.106214.
14. Zojaji, A.; Soltaniani, K.; Hvattum, L.M.; Urrutia, S. Cyclic solutions to a maritime inventory routing problem. *Marit. Transp. Res.* **2022**, *3*, 100074. https://doi.org/10.1016/J.MARTRA.2022.100074.
15. Fagerholt, K.; Hvattum, L.M.; Papageorgiou, D.J.; Urrutia, S. Maritime inventory routing: Recent trends and future directions. *Int. Trans. Oper. Res.* **2023**, *30*, 3013–3056. https://doi.org/https://doi.org/10.1111/itor.13313.
16. Christiansen, M.; Fagerholt, K. Ship Routing and Scheduling in Industrial and Tramp Shipping. In *Vehicle Routing*; MOS-SIAM Series on Optimization; SIAM: University City, PA, USA, 2014; Chapter 13, pp. 381–408. https://doi.org/10.1137/1.9781611973594.ch13.
17. Andersson, H.; Hoff, A.; Christiansen, M.; Hasle, G.; Løkketangen, A. Industrial aspects and literature survey: Combined inventory management and routing. *Comput. Oper. Res.* **2010**, *37*, 1515–1536. https://doi.org/10.1016/J.COR.2009.11.009.
18. Agra, A.; Christiansen, M.; Delgado, A. Discrete time and continuous time formulations for a short sea inventory routing problem. *Optim. Eng.* **2017**, *18*, 269–297. https://doi.org/10.1007/S11081-016-9319-0.
19. Liu, B.; Zhang, Q.; Yuan, Z. Two-stage distributionally robust optimization for maritime inventory routing. *Comput. Chem. Eng.* **2021**, *149*, 107307. https://doi.org/10.1016/J.COMPCHEMENG.2021.107307.
20. Rodrigues, F.; Agra, A.; Christiansen, M.; Hvattum, L.M.; Requejo, C. Comparing techniques for modelling uncertainty in a maritime inventory routing problem. *Eur. J. Oper. Res.* **2019**, *277*, 831–845. https://doi.org/10.1016/J.EJOR.2019.03.015.
21. De, A.; Mamanduru, V.K.R.; Gunasekaran, A.; Subramanian, N.; Tiwari, M.K. Composite particle algorithm for sustainable integrated dynamic ship routing and scheduling optimization. *Comput. Ind. Eng.* **2016**, *96*, 201–215. https://doi.org/10.1016/j.cie.2016.04.002.
22. Song, J.H.; Furman, K.C. A maritime inventory routing problem: Practical approach. *Comput. Oper. Res.* **2013**, *40*, 657–665. https://doi.org/10.1016/J.COR.2010.10.031.
23. Papageorgiou, D.J.; Cheon, M.S.; Harwood, S.; Trespalacios, F.; Nemhauser, G.L. Recent progress using matheuristics for strategic maritime inventory routing. *Intell. Syst. Ref. Libr.* **2018**, *131*, 59–94. https://doi.org/10.1007/978-3-319-61801-2_3.
24. Misra, S.; Kapadi, M.; Gudi, R.D. Hybrid Time-Based Framework for Maritime Inventory Routing Problem. *Ind. Eng. Chem. Res.* **2020**, *59*, 20394–20409. https://doi.org/10.1021/acs.iecr.0c03186.
25. Msakni, M.K.; Haouari, M. Short-term planning of liquefied natural gas deliveries. *Transp. Res. Part C Emerg. Technol.* **2018**, *90*, 393–410. https://doi.org/10.1016/j.trc.2018.03.013.
26. Wolsey, L., Strong Valid Inequalities. In *Integer Programming*; John Wiley & Sons, Ltd.: Hoboken, NJ, USA, 2020; Chapter 9, pp. 167–194. https://doi.org/10.1002/9781119606475.ch9.
27. Rakke, J.G.; Andersson, H.; Christiansen, M.; Desaulniers, G. A New Formulation Based on Customer Delivery Patterns for a Maritime Inventory Routing Problem. *Transp. Sci.* **2015**, *49*, 384–401.
28. Rocha, R.; Grossmann, I.E.; de Aragão, M.V.S.P. Cascading Knapsack Inequalities: reformulation of a crude oil distribution problem. *Ann. Oper. Res.* **2013**, *203*, 217–234.
29. Wang, S.; Chu, F. A Decomposition-Based Heuristic Method for Inventory Routing Problem. *IEEE Trans. Intell. Transp. Syst.* **2022**, *23*, 18352–18360. https://doi.org/10.1109/TITS.2022.3170569.
30. Alkaabneh, F.; Diabat, A.H.; Gao, H.O. Benders decomposition for the inventory vehicle routing problem with perishable products and environmental costs. *Comput. Oper. Res.* **2020**, *113*, 104751. https://doi.org/10.1016/J.COR.2019.07.009.
31. Lefever, W.; Touzout, F.A.; Hadj-Hamou, K.; Aghezzaf, E. Benders' decomposition for robust travel time-constrained inventory routing problem. *Int. J. Prod. Res.* **2021**, *59*, 342–366. https://doi.org/10.1080/00207543.2019.1695167.
32. Cordeau, J.; Laganà, D.; Musmanno, R.; Vocaturo, F. A decomposition-based heuristic for the multiple-product inventory-routing problem. *Comput. Oper. Res.* **2015**, *55*, 153–166. https://doi.org/10.1016/j.cor.2014.06.007.
33. Adulyasak, Y.; Cordeau, J.; Jans, R. Formulations and Branch-and-Cut Algorithms for Multivehicle Production and Inventory Routing Problems. *INFORMS J. Comput.* **2012**, *26*, 103–120. https://doi.org/10.1287/ijoc.2013.0550.
34. Schenekemberg, C.M.; Guimarães, T.A.; Chaves, A.A.; Coelho, L.C. A Three-Front Parallel Branch-and-Cut Algorithm for Production and Inventory Routing Problems. *Transp. Sci.* **2023**, *58*, 687–707. https://doi.org/10.1287/trsc.2022.0261.

35. Alvarez, A.; Munari, P.; Morabito, R. Iterated local search and simulated annealing algorithms for the inventory routing problem. *Int. Trans. Oper. Res.* **2018**, *25*, 1785–1809. https://doi.org/10.1111/itor.12547.
36. Hiassat, A.; Diabat, A.H.; Rahwan, I. A genetic algorithm approach for location-inventory-routing problem with perishable products. *J. Manuf. Syst.* **2017**, *42*, 93–103. https://doi.org/10.1016/J.JMSY.2016.10.004.
37. Mahjoob, M.; Fazeli, S.S.; Milanlouei, S.; Tavassoli, L.S.; Mirmozaffari, M. A modified adaptive genetic algorithm for multi-product multi-period inventory routing problem. *Sustain. Oper. Comput.* **2021**, *3*, 1–9. https://doi.org/10.1016/j.susoc.2021.08.002.
38. Solyalı, O.; Süral, H. An Effective Matheuristic for the Multivehicle Inventory Routing Problem. *Transp. Sci.* **2022**, *56*, 1044–1057. https://doi.org/10.1287/trsc.2021.1123.
39. Williams, H.P. *Model Building in Mathematical Programming*; Wiley: Hoboken, NJ, USA, 2013.
40. Margot, F. Symmetry in Integer Linear Programming. In *50 Years of Integer Programming 1958-2008: From the Early Years to the State-of-the-Art*; Jünger, M.; Liebling, T.M.; Naddef, D.; Nemhauser, G.L., Pulleyblank, W.R., Reinelt, G., Rinaldi, G., Wolsey, L.A., Eds.; Springer: Berlin/Heidelberg, Germany, 2010; pp. 647–686. https://doi.org/10.1007/978-3-540-68279-0_17.

Capítulo 3

Optimizing Maritime Logistics for Antarctic Research: A Hybrid Electric Fleet Approach with Adaptive Speed Selection

ARTICLE TEMPLATE

Optimizing maritime logistics for antarctic research: a hybrid electric fleet approach with adaptive speed selection

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ABSTRACT
In the challenging environment of the Chilean Antarctic Territory, the maritime logistics to support scientific research entail navigating through complex operational constraints, including harsh weather conditions and environmental sustainability considerations. This paper introduces a groundbreaking mixed integer bilinear programming model tailored for a fleet of hybrid electric ships, emphasizing adaptive speed selection to enhance fuel efficiency and logistical efficacy. We leverage real-world data to explore diverse operational scenarios across different time horizons, unveiling strategies that significantly reduce fuel consumption while ensuring timely and reliable supply chain operations. Our findings advance the operational efficiency of maritime inventory routing and contribute to the sustainable management of logistics in extreme conditions. The study's insights are poised to inform future developments in hybrid maritime transport systems, offering a scalable solution to optimize resource allocation and minimize environmental impact in the logistics sector.

KEYWORDS
maritime inventory routing problem; hybrid electric ships; speed selection, mixed integer bilinear programming, parameterization

1. Introduction

Antarctica, a unique frontier for scientific research, presents formidable logistical challenges due to its harsh climate, including extreme cold, strong winds, and isolation. These conditions require a detailed and exhaustive logistics planning to support research activities. The agency tasked with logistical operations must manage the intricate coordination of transporting researchers, supplies, medical assistance, and other services to maintain Antarctic bases efficiently throughout the operational season. A key aspect of this logistics is the use of ships with dual energy sources, which improves sustainability and operational flexibility. These ships navigate the complexities of delivering resources to bases where storage capacity is limited, ensuring the feasibility of loading and unloading procedures. This logistical support is vital for the safe and timely delivery of essential supplies to facilitate research. Logistic planning

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begins before the season by evaluating research project proposals by simulating various scenarios to ascertain project viability. In the context of these research projects, it is crucial to meticulously plan and continuously adjust logistics, including periodic scheduling of dual-energy-source ships. This careful coordination ensures seamless operations throughout project execution.

This logistic situation faces several difficulties, such as weekly time windows, severe climatic conditions, and the environmental impact of human activity. The research bases require periodic replenishment of consumables goods and removal of disposables. The replenishment visits must scheduled to avoid exceeding the inventory capacity or running out of supplies at the bases. Moreover, the ships performing the visits must respect their loading capacity and ensure sufficient fuel and battery levels for completing their journeys. In addition, ship speed is a critical factor in this problem, as it strongly affects the power consumption. Specifically, the ships have a parallel hybrid propulsion system, including an internal combustion engine (ICE) and an electric motor, as shown in the schematic view of this system in Figure 1. Thus, the ICE can generate electricity to charge the ship's batteries but cannot simultaneously charge the battery using electric propulsion. Figure 1 depicts the ICE connected to the generator and the propeller, allowing it to perform both functions. The diagram also indicates the flow of mechanical (MP) and electrical (EP) power, highlighting the operational dynamics of this hybrid system. Considering all these elements, we characterize a hybrid ship with a speed selection maritime inventory routing problem (HS-SS-MIRP).

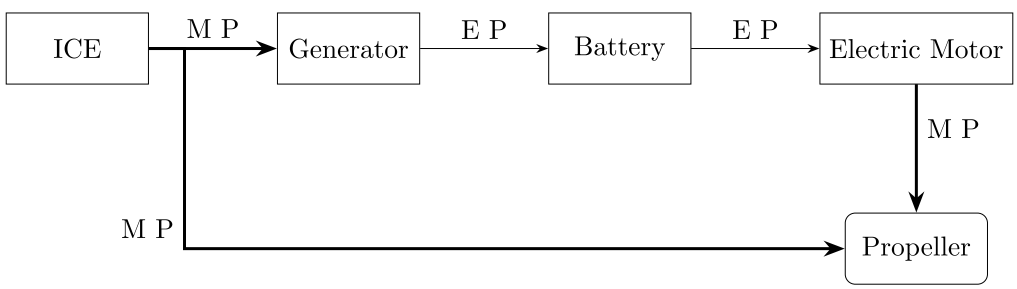

Figure 1. Schematic of a parallel hybrid propulsion system for a hybrid electric ship

The logistical challenge HS-SS-MIRP is directly related to the Maritime Inventory Routing Problem (MIRP), a subject extensively explored in the academic literature. However, the Antarctic scenario adds a unique dimension to this well-studied problem, combining the need for precise timing, constrained by the short summer period, with the critical requirement of supporting scientific research in one of the world's most remote and challenging environments, considering transportation activities with multiple energy sources. A scientific review of the existing literature reveals various frameworks that researchers have used to address related problems, such as the MIRP, the vehicle routing problem (VRP), and the periodic routing problem (PRP), and their extensions, using different solution methods such as exact algorithms, metaheuristics, and matheuristics, with or without speed selection, and for different types of ships. Despite extensive research, this study is original in that it applies the MIRP to a fleet of hybrid electric ships with speed selection operating in the Antarctic region.

We propose a mathematical model and solutions for the HS-SS-MIRP, a problem that involves routing a fleet of hybrid electric ships with different capacities and variable speeds. We use real-world data to generate realistic instances of the problem. Our model extends the continuous-time location event model from Agra *et al.* [1] and incorporates some energy constraints from Duan *et al.* [2]. We formulate a mixed integer bilinear programming model (MIBLP) which contains bilinear constraints for battery level tracking and bilinear terms in the objective function. We also propose parameterizing the model by fixing the speed levels in each arc. We aim to minimize fuel consumption by optimizing the schedule, speed, and energy source combination in each area of the route while meeting inventory requirements in the ports and battery limits onboard. We simultaneously solve parameterized routing and energy problems to obtain globally optimal solutions, compared to solving those problems separately. We tested our model in real-world instances developed with data from Chilean scientific bases in Antarctica.

This manuscript makes the following contributions: i) we formulate a problem based on a challenge that government agencies face when they plan scientific expeditions to Antarctica and ensure a constant supply of goods and waste removal for the research stations by using hybrid electric ships that adjust their speed according to the operating conditions. ii) we propose a mixed integer bilinear programming model for the HS-SS-MIRP and further parameterize the bilinear terms in the objective function and the bilinear constraints of the model; iii) we generate real-world instances using data from the Chilean Antarctic Territory, and iv) we test the model in these instances varying the speed parameter in three levels and the number of maximum visits.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature and identifies the knowledge gap. Section 3 describes the problem in detail and presents the mathematical formulations. Section 4 reports the results of the test of our formulations in various instances. The implications and limitations of our findings are discussed in Section 5, along with some suggestions for future research directions on this topic. Section 6 summarizes the main contributions and conclusions of this work.

2. Literature Review

The maritime transportation industry involves complex optimization problems related to routing and scheduling. These problems have been addressed by many researchers over the years, proposing different formulations and solution methods. Some studies focus solely on ship routing or port production, consumption, and inventory management, while other studies integrate both aspects. Regardless of the approach used, all studies aim to improve the efficiency of the global maritime shipping industry. The routing aspect of this problem is addressed by applying variations of the Vehicle Routing Problem (VRP) [3–5] and the Periodic Routing Problem (PRP) [6–8]. Other routing approaches include Voyage Optimization Using Genetic Algorithm and Dynamic Programming [9] and Particle Swarm Optimization in a bi-objective formulation [10]. Integrated approaches include incorporating production scheduling at ports to the VRP [11], and combining heuristics and mixed integer programming [12], among others.

The MIRP, which is a particular case of the IRP, is the most common integrated approach found in the literature and is characterized by the fact that deliveries take place over time, without a fixed schedule, and the quantities to be picked up or delivered at a certain location are modeled as decision variables. Furthermore, the route of the ships is bound to change according to the selected planning horizon [13]. Depending on whether we choose the time horizon to be continuous or discrete, different formulations of the MIRP can be used. Continuous formulation is more effective for problems involving constant parameters, whereas discrete formulation can handle problems that have variable parameters over time [14]. Other MIRP variants include single-product [15] or multi-product [16,17] considerations, and stochastic [18,19] or deterministic [1] formulations. Typically, the MIRP is modeled as a problem with a finite horizon, which can lead to end-of-horizon effects, complicating the possibility of using solutions in a periodic fashion [14]. Some approaches to handling these effects include the definition of a parameter that forces the final inventory at the ports to be a linear combination of their initial inventory and maximum capacity [1], and explicitly designing the formulation to produce cyclic solutions [20]. However, most articles do not explicitly acknowledge these effects.

Resolution methods for the different MIRP formulations include exact algorithms, where most articles use the default branch-and-bound algorithm found in most solvers. Some recent examples include Algendi *et al.* [21] solving instances with up to 5 ports and 3 ships, considering up to 60 days of planning horizon, and Diz *et al.* [22] solving instances with up to 7 ports and 11 ships. Other researchers develop exact models such as branch-and-cut, solving instances with up to 12 months of planning horizon, 46 ships, and 4 ports [23]; and branch-and-price solving instances with up to 5 ships and 16 ports [24]. Approximate methods include metaheuristics such as VNS [25], hybrid metaheuristics [26], multi-start heuristic with further improvement procedures [27], and a genetic algorithm (GA) for a nonlinear formulation [28]. Other approximate methods include matheuristics such as extended fix-and-relax [29], relax-and-fix and fix-and-optimize [30], and an ALNS-based matheuristic [31]. Some studies also add valid inequalities related to symmetry break, in the case of the presence of identical ships [23], the visit timing [1,23], and the minimum number of operations and departures [30].

An important challenge in the MIRP is to determine the optimal speed of the ships, which affects both fuel consumption and travel time. Some studies have addressed this issue by incorporating speed optimization into MIRP formulations, such as Eide *et al.* [32]. When considering a fixed schedule for ports visits and fuel consumption as a convex function of ship speed, optimal speeds can be found in quadratic time [33]. This leads to the possibility of simultaneously optimizing speeds and routes that can generate a reduction in fuel cost compared to considering only fixed speeds [34,35]. Speed optimization has an impact on sustainability because fuel consumption has a non-linear relation to the ship's speed, typically considered as a cubic function. With a reduction in speed, fuel consumption can also be significantly reduced. This practice is called "slow steaming". Furthermore, speed is an important parameter that can influence the entire logistic chain, which can lead to crowded ports or increase inventory costs, both in transit and in ports [36]. Adding these and other characteristics to objective functions or constraints can lead to different optimal values for speeds [37].

Most MIRP models simplify the propulsion system and the energy source of the ships, usually assuming diesel as the only option. However, some ships have hybrid propulsion systems that can operate in different modes, such as hybrid diesel-electric, pure electric, or solar [2]. These modes affect fuel efficiency and the reduction of ships' emissions according to their speed. Therefore, existing optimization models must incorporate the features of hybrid ships and their influence on MIRP performance. Some previous work addressing hybrid ships on maritime routing is Pan *et al.* [38], who develop a MILP model for a fleet of ships with four modes of travel and hydrogen fuel cells, diesel and solar power as energy sources; Ritari *et al.* [39], who propose a MILP formulation to optimize the route and speeds of a fleet of ships with diesel and electricity; and Domachowski *et al.* [40], who design a MILP formulation that minimizes

Table 1. Literature review comparison.

	Routing formulation			Speed selection		Ship type		Algorithm	
Authors	VRP	PRP	IRP	No	Yes	C	H	E	A
Adhi *et al.* [26]			✓	✓		✓			✓
Agra *et al.* [1]			✓	✓		✓		✓	
Algendi *et al.* [21]			✓		✓	✓		✓	
Charalambopoulos *et al.* [4]	✓			✓		✓			✓
De *et al.* [15]			✓		✓	✓			✓
Duan *et al.* [2]	✓				✓		✓		✓
Fadda *et al.* [3]	✓			✓		✓			✓
Friske *et al.* [30]			✓	✓		✓			✓
Nikolaisen *et al.* [18]			✓	✓		✓			✓
Pan *et al.* [38]	✓			✓			✓		✓
Ritari *et al.* [39]	✓				✓		✓	✓	
Sanghikian *et al.* [25]			✓	✓		✓			✓
Shaabani *et al.* [17]			✓	✓		✓			✓
Vieira *et al.* [7]		✓		✓		✓		✓	✓
Xu Xin and Chen [24]	✓			✓		✓		✓	
Zojaji *et al.* [20]			✓	✓		✓		✓	
This work			✓		✓		✓	✓	

emissions by adding a constraint on fuel consumption and considering hybrid electric ships. To our knowledge, no work combined a MIRP formulation with various types of energy sources for ships, which makes our work novel.

The objective function and the group of constraints that track the battery state of charge (SoC) through the journey of the ships present bilinear terms. Although some commercial solvers such as Gurobi are capable of solving mixed integer nonlinear problems (MINLP) directly, some researchers have used different techniques to linearize or relax this type of constraint, such as McCormick envelopes [41,42] and the Big-M approach [43].

We compare our work with recent works in the related literature, focusing on four main aspects: i) the routing formulation, ii) the speed selection in the arcs, iii) the type of propulsion system of the ship, and iv) the resolution algorithm. Table 1 summarizes these aspects for each of the articles reviewed. The first column includes the reviewed work. The second, third, and fourth columns report the routing formulation of each work: vehicle routing problem (VRP), periodic routing problem (PRP), or inventory routing problem (IRP), respectively. The fifth and sixth columns report whether the model allows to choose different speeds on different arcs or not. The seventh column reports if the ship has a conventional (C) propelling system, which means that it only uses diesel, or if there is no information about it. The eighth column shows whether the ship uses a hybrid propelling system (H), which means that it uses at least two different energy sources. The ninth and tenth columns report whether the resolution algorithm used is exact (E) or approximate (A). As shown in the table, our work is the only one that uses the IRP as the routing formulation, incorporates speed selection on the arcs, and considers hybrid ships.

3. Mathematical Model

In this section, we present the mixed integer bilinear programming (MIBLP) model that we developed to solve HS-SS-MIRP, and its posterior parameterization. The model aims to optimize the routing and scheduling decisions of a fleet of ships that transport products from a set of suppliers to a set of customers, while minimizing total diesel consumption and respecting operational constraints. The model is based on the formulation proposed by Agra *et al.* [1] and Duan *et al.* [2], and considers the selection of the speed as an integer variable. We extended the model to account for the presence of batteries on board the ships, which can provide additional energy sources and reduce the dependence on diesel. We also introduce a set of constraints that ensure that the state-of-charge (SOC) of the battery remains above a certain threshold at the beginning of each operation.

HS-SS-MIRP considers a set P of ports denoted by the letter i, with each port able to be visited a maximum of $\overline{m}_i$ times and with each visit marked by the letter m. To illustrate this concept, an extended directed graph $G = (N, A)$ is introduced, where the set N contains all nodes (i, m) formed by the ports and their respective visits, and an additional origin node $\{O\}$. The set A contains all possible arcs $((i, m), (j, n))$ formed by combinations of nodes $(i, m) \in N$ and $(j, n) \in N$ that satisfy the condition $i \neq j$, which means that it is not possible to visit a port twice consecutively. A set of heterogeneous ships K is presented, each with a maximum cargo capacity C_k and a battery capacity $\overline{L}_k$. The ships begin their journey at the node of origin $\{O\}$ and can end at any node $(i, m) \in N$ within the graph. The set V contains all possible speed levels v that ships can select on an arc $((i, m), (j, n)) \in A$. The set W contains all the powers w_v required for ships to travel at the speed level $v \in V$. The variables and parameters used in the model are then presented to facilitate further analysis.

Parameters

- J_i: 1 if the port i is a supplier, and -1 if the port i is a consumer,
- $\overline{m}_i$: Maximum possible visits to the port i,
- T_i^Q: Rate of *time/(unit of cargo)* loaded or unloaded in the port i,
- C_{ij}^k: Variable cost for traveling from the port i to the port j by the vessel k,
- C_0^k: Fixed cost for operating the vessel k,
- C_y^k: Fixed cost per port visit of the vessel k,
- $\overline{S}_i$: Maximum inventory level in the port i,
- $\underline{S}_i$: Minimum inventory level in the port i,
- S_i^0: Initial stock level in the port i,
- R_i: Production or consumption rate in the port i,
- Q_k^0: Initial cargo in the vessel k on the origin O,
- C_k: Maximum capacity of the vessel k,
- T_{ij}^S: Travel time from the port i to the port j,
- T_i^0: Travel time from the origin O to the port i,
- T: Planning horizon.

Binary variables

- x_{imjnk}: 1 if the vessel k travels directly from the node (i, m) to the node (j, n), 0 otherwise,
- x_{imk}^0: 1 if the vessel k travels directly from the origin to the node (i, m), 0 otherwise,
- y_{imk}: 1 if the vessel k makes the m^{th} visit to port i, 0 otherwise,
- z_{imk}: 1 if the vessel k ends its journey at the node (i, m), 0 otherwise.

Continuous variables

- q_{imk}: Quantity collected or delivered to the node (i, m) by vessel k,
- f_{imjnk}: Quantity transported from the node (i, m) to the node (j, n) by vessel k,
- f^0_{imk}: Quantity transported from the origin to the node (i, m) by vessel k,
- f^D_{imk}: Quantity transported from the node (i, m) to the destination by vessel k,
- t_{im}: Start time of operation at the node (i, m),
- s_{im}: Stock level at the beginning of the operation at the node (i, m).

In Figure 2 we show an example of a graph used in this study. For the sake of simplicity, we consider a problem with 2 ports and 2 ships. Port 1 can be visited twice, represented by the nodes $(1, 1)$ and $(1, 2)$, while port 2 can be visited at most once, represented by the single node $(2, 1)$. The ships can choose between two speed options and have a common origin and destination, denoted by the nodes O and D, respectively.

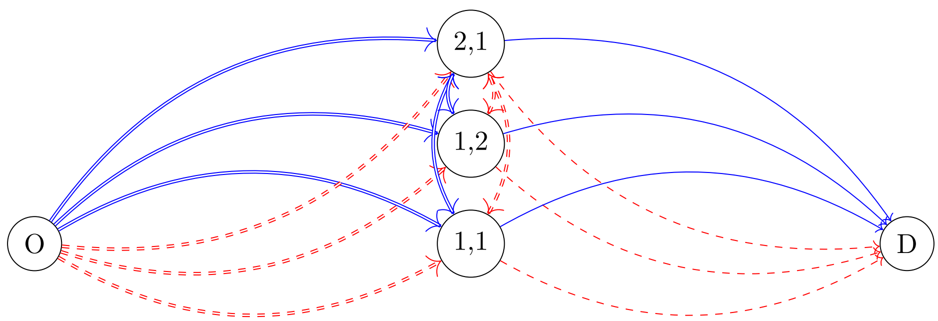

Figure 2. Example graph for the HS-SS-MIRP

The solid arrows represent the valid arcs of the ship k_1 and the dashed arrows represent the valid arcs of the ship k_2. The double arrows represent the two possible speed options of the ships. Each node (i, j) is connected to the origin with 4 arcs; 2 arcs per ship, and connected to the destination with 2 arcs, 1 per ship. This is because ships can select a speed to depart from the origin, but the destination is a fictitious node, meaning that there is no real journey. The nodes $(1, 1)$ and $(1, 2)$ are connected by four arcs to the node $(2, 1)$, again 2 per ship. However, nodes $(1, 1)$ and $(1, 2)$ are not connected because consecutive visits are not allowed at the same port. Next, we show the constraints and objective function of the formulation.

3.1. Routing constraints

$$\sum_{v\in V}\sum_{(i,m)\in N} x^0_{imkv} \leq 1 \qquad \forall\, k\in K \tag{1}$$

$$y_{imk} - \sum_{v\in V}\sum_{(j,n)\in N|j\neq i} x_{jnimkv} - \sum_{v\in V} x^0_{imkv} = 0 \qquad \forall\,(i,m)\in N, k\in K \tag{2}$$

$$y_{imk} - \sum_{v\in V}\sum_{(j,n)\in N|j\neq i} x_{imjnkv} - z_{imk} = 0 \qquad \forall\,(i,m)\in N, k\in K \tag{3}$$

$$\sum_{k\in K} y_{imk} \leq 1 \qquad \forall\,(i,m)\in N \tag{4}$$

$$\sum_{k\in K} y_{i(m-1)k} - \sum_{k\in K} y_{imk} \geq 0 \qquad \forall\,(i,m)\in N|m\geq 2 \tag{5}$$

$$x^0_{imkv}, y_{imk}, z_{imk} \in \{0,1\} \qquad \forall\,(i,m)\in N, k\in K, v\in V \tag{6}$$

$$x_{imjnkv} \in \{0,1\} \quad \forall\,((i,m),(j,n))\in A, k\in K, v\in V \tag{7}$$

The constraints 1 state that all ships will potentially start at the node of origin. The constraints 2 and 3 state that if node (i,m) is visited by a ship k, it came from the origin or from a node (j,n), and that it will continue its journey to another node (j,n) or end its journey at node (i,m). The constraints 4 state that node (i,m) can be visited up to once. The constraints 5 ensure the ascending order of visits to the port i. The constraints 6 and 7 state the binary nature of the variables related to the routing.

3.2. Pickup and delivery constraints

$$\begin{aligned}\sum_{k\in K} f^0_{imk} + \sum_{k\in K}\sum_{(j,n)\in N|j\neq i} f_{jnimk} + J_i \sum_{k\in K} q_{imk} \\ = \sum_{k\in K}\sum_{(j,n)\in N|j\neq i} f_{imjnk} + \sum_{k\in K} f^D_{imk} \qquad \forall\,(i,m)\in N\end{aligned} \tag{8}$$

$$f^0_{imk} = Q^0_k \sum_{v \in V} x^0_{imkv} \qquad \forall\,(i,m) \in N, k \in K \tag{9}$$

$$f_{imjnk} \leq C_k \sum_{v \in V} x_{imjnkv} \qquad \forall\,((i,m),(j,n)) \in A, k \in K \tag{10}$$

$$f^D_{imk} \leq C_k z_{imk} \qquad \forall\,(i,m) \in N, k \in K \tag{11}$$

$$q_{imk} \leq C_k y_{imk} \qquad \forall\,(i,m) \in N, k \in K \tag{12}$$

$$f_{imjnk} \geq 0 \qquad \forall\,((i,m),(j,n)) \in A, k \in K \tag{13}$$

The constraints 8 ensure the balance of the cargo at the node (i,m). The constraints 9 define the initial cargo of each ship k as the amount transported from its origin node. The constraints 10 and 11 impose that cargo flow in any arc $((i,m),(j,n))$ and at the last node visited by a ship k cannot exceed its capacity. The constraints 12 limit the maximum amount of cargo that can be picked up or delivered to any node (i,m) by the capacity of the ship k. The constraints 13 ensure that all variables are nonnegative.

3.3. Time constraints

$$t_{im} + T^Q_i q_{imk} + (T + T^S_{ijv}) x_{imjnkv} \leq T + t_{jn} \quad \forall\,(i,m,j,n) \in A, k \in K, v \in V \tag{14}$$

$$\sum_{v \in V} \sum_{k \in K} T^0_{iv} x^0_{imkv} \leq t_{im} \qquad \forall\,(i,m) \in N \tag{15}$$

$$0 \leq t_{im} \leq T \qquad \forall\,(i,m) \in N \tag{16}$$

The constraints 14 provide the lowest possible start time of service at node (j,n) after being visited directly from node (i,m). The constraints 15 provide the lowest possible start time for the service at node (i,m) after being visited from the origin node. The constraints 16 provide lower and upper limits for the variables.

3.4. Inventory constraints

$$S^0_i + J_i R_i t_{i1} = s_{i1} \qquad \forall\, i \in P \tag{17}$$

$$s_{i(m-1)} - \sum_{k \in K} J_i q_{i(m-1)k} + J_i R_i (t_{im} - t_{i(m-1)}) = s_{im} \qquad \forall\,(i,m) \in N | m \geq 2 \tag{18}$$

$$s_{im} + \sum_{k \in K} q_{imk} - \sum_{k \in K} R_i T^Q_{ik} q_{imk} \leq \overline{S}_i \qquad \forall\,(i,m) \in N | J_i = -1 \tag{19}$$

$$s_{im} - \sum_{k \in K} q_{imk} + \sum_{k \in K} R_i T^Q_{ik} q_{imk} \geq \underline{S}_i \qquad \forall\,(i,m) \in N | J_i = 1 \tag{20}$$

$$s_{i\overline{m}} + \sum_{k \in K} q_{i\overline{m}k} - R_i (T - t_{i\overline{m}}) \geq \underline{S}_i \qquad \forall\, i \in P | J_i = -1 \tag{21}$$

$$s_{i\overline{m}} - \sum_{k \in K} q_{i\overline{m}k} + R_i (T - t_{i\overline{m}}) \leq \overline{S}_i \qquad \forall\, i \in P | J_i = 1 \tag{22}$$

$$\underline{S}_i \leq s_{im} \leq \overline{S}_i \qquad \forall\,(i,m) \in N \tag{23}$$

The constraints 17 initialize the inventory level at node $(i,1)$. The constraints 18 states the inventory level at node (i, m) with respect to the inventory level at node $(i, m-1)$. The constraints 19 and 20 ensure that the inventory level at port i is between its given limits at the end visit time at each node (i, m). The constraints 21 and 22 ensure that the inventory level at port i on its last visit is between its given limits. The constraints 23 ensure that the inventory level in port i is between its given limits at the start time of each visit.

3.5. Energy constraints

$$u^{D,L}_{imjnk} + u^{B,L}_{imjnk} = \sum_{v \in V} w_v x_{imjnkv} \qquad \forall\, (i,m,j,n) \in A, k \in K \tag{24}$$

$$u^{D,L,0}_{imk} + u^{B,L,0}_{imk} = \sum_{v \in V} w_v x^0_{imkv} \qquad \forall\, (i,m) \in N, k \in K \tag{25}$$

$$u^{D,L}_{imjnk} + u^{D,B}_{imjnk} \leq P^{D,B}_k + P^{D,L}_k \qquad \forall\, (i,m,j,n) \in A, k \in K \tag{26}$$

$$u^{D,L,0}_{imk} + u^{D,B,0}_{imk} \leq P^{D,B}_k + P^{D,L}_k \qquad \forall\, (i,m) \in N, k \in K \tag{27}$$

$$u^{B,L}_{imjnk} \leq P^{B,L}_k \qquad \forall\, (i,m,j,n) \in A, k \in K \tag{28}$$

$$u^{B,L,0}_{imk} \leq P^{B,L}_k \qquad \forall\, (i,m) \in N, k \in K \tag{29}$$

$$e^{B,C}_{imjnk} + e^{B,D}_{imjnk} \leq \sum_{v \in V} x_{imjnkv} \qquad \forall (i,m,j,n) \in A, k \in K \tag{30}$$

$$e^{B,C,0}_{imk} + e^{B,D,0}_{imk} \leq \sum_{v \in V} x^0_{imkv} \qquad \forall (i,m) \in N, k \in K \tag{31}$$

$$u^{D,B}_{imjnk} \leq P^{C,B}_k e^{B,C}_{imjnk} \qquad \forall (i,m,j,n) \in A, k \in K \tag{32}$$

$$u^{D,B,0}_{imk} \leq P^{C,B}_k e^{B,C,0}_{imk} \qquad \forall (i,m) \in N, k \in K \tag{33}$$

$$u^{D,L}_{imjnk}, u^{B,L}_{imjnk}, u^{D,B}_{imjnk} \geq 0 \qquad \forall ((i,m),(j,n)) \in A, k \in K \tag{34}$$

$$u^{D,L,0}_{imk}, u^{B,L,0}_{imk}, u^{D,B,0}_{imk} \geq 0 \qquad \forall (i,m) \in N, k \in K \tag{35}$$

$$e^{B,C}_{imjnk}, e^{B,D}_{imjnk} \in \{0,1\} \qquad \forall ((i,m),(j,n)) \in A, k \in K \tag{36}$$

$$e^{B,C,0}_{imk}, e^{B,D,0}_{imk} \in \{0,1\} \qquad \forall (i,m) \in N, k \in K \tag{37}$$

The constraints 24 and 25 equal the power demanded from the ship k traveling with speed v through the arc $((i,m),(j,n))$, or from the origin node to node (i,m), to the power delivered to the ship motor by solar panels, diesel generator, and battery. The power delivered from each energy source, solar, diesel, and battery, is limited to the maximum capacity of each source when the ship k travels through the arc $((i,m),(j,n))$ or when it moves from the origin node to the node (i,m). This is enforced by the constraints 26, 27, 28 and 29. The constraints 30 and 31 guarantee that the battery cannot be charged and discharged in the same arc $((i,m),(j,n))$ or when it first arrives at node (i,m). Constraints 32 and 33 are logic constraints that limit the charging and discharge power of the battery. Constraints 34, 35, 36, and 37 provide the nature of the variables.

3.5.1. Battery level at the nodes

Let l_{jnk} denote the level of the battery of the ship $k \in K$ at the beginning of the operation at the node $(j, n) \in N$. The ship k can arrive at the node (j, n) from the node (i, m) or from the origin node. The power flows from and to the battery are represented by $u^{B,L}_{imjnk}$, $u^{PV,B}_{imjnk}$ and $u^{D,B}_{imjnk}$ if the ship k comes from node (i, m), and by $u^{B,L,0}_{jnk}$, $u^{PV,B}_{jnk}$ and $u^{D,B}_{jnk}$ if the ship k comes from the origin node. To determine the level of the battery at node (j, n), we need to calculate the energy the battery receives or delivers. Since power is the rate of change in energy over time, we obtain energy by multiplying the power delivered or received by the battery by the time spent in the arc $((i, m), (j, n))$ or on the journey from the origin node to the node (j, n). Because time depends on the selected speed, we obtain a set of bilinear constraints.

$$\begin{aligned} l_{jnk} \leq l_{imk} - \frac{1}{\mu^d_k}(u^{B,L}_{imjnk} \sum_{v \in V} T^S_{ijv} x_{imjnkv} + u^{B,L,0}_{jnk} \sum_{v \in V} T^0_{jv} x^0_{jnkv}) \\ + \mu^c_k(u^{D,B}_{imjnk} \sum_{v \in V} T^S_{ijv} x_{imjnkv} + u^{D,B,0}_{jnk} \sum_{v \in V} T^0_{jv} x^0_{jnkv}) \\ + \overline{L}_k(1 - \sum_{v \in V} x_{imjnkv}) \qquad \forall((i, m), (j, n)) \in A, k \in K \end{aligned} \tag{38}$$

$$l_{imk} \geq \underline{L}_k \qquad \forall(i, m) \in N, k \in K \tag{39}$$

The bilinear constraints 38 associate the level of the battery of the ship k at the node (j, n) with the level of the battery at the node (i, m). The constraints 39 impose a lower bound on the variables.

3.6. Objective function

$$\min z = \sum_{k \in K} \sum_{((i,m),(j,n)) \in A} \left((u^{D,B}_{imjnk} + u^{D,L}_{imjnk}) \sum_{v \in V} T^S_{ijv} x_{imjnkv} + (u^{D,B,0}_{imk} + u^{D,L,0}_{imk}) \sum_{v \in V} T^0_{iv} x^0_{imkv} \right) \tag{40}$$

The objective function 40 aims to minimize the total energy developed using diesel of the entire fleet. The objective function also contains bilinear terms that further complex the model.

3.7. Model parameterization

Since there are bilinear terms on the constraints 38 and in the objective function 40, the solvers cannot directly apply the branch-and-bound algorithm to solve the mixed integer bilinear program (MIBP) optimally. To overcome this problem, we parameterize the speed subindex in variables x_{imjnkv} and x^0_{imkv}, and on parameters T^S_{ijv}, T^0_{iv}, and w_v. Therefore, the routing constraints 1, 2, and 3 are replaced by the constraints 41, 42, and 43, respectively.

$$\sum_{(i,m)\in N} x^0_{imk} \leq 1 \qquad \forall\, k \in K \tag{41}$$

$$y_{imk} - \sum_{(j,n)\in N|j\neq i} x_{jnimk} - x^0_{imk} = 0 \qquad \forall\,(i,m)\in N, k\in K \tag{42}$$

$$y_{imk} - \sum_{(j,n)\in N|j\neq i} x_{imjnk} - z_{imk} = 0 \qquad \forall\,(i,m)\in N, k\in K \tag{43}$$

The pickup and delivery constraints 9 and 10 are replaced by the constraints 44 and 45, respectively.

$$f^0_{imk} = Q^0_k x^0_{imk} \qquad \forall\,(i,m)\in N, k\in K \tag{44}$$

$$f_{imjnk} \leq C_k x_{imjnk} \qquad \forall\,((i,m),(j,n))\in A, k\in K \tag{45}$$

The time constraints 14 and 15 are replaced by the constraints 46 and 47, respectively.

$$t_{im} + T^Q_i q_{imk} + (T + T^S_{ij})x_{imjnk} \leq T + t_{jn} \qquad \forall\,(i,m,j,n)\in A, k\in K \tag{46}$$

$$\sum_{k\in K} T^0_i x^0_{imk} \leq t_{im} \qquad \forall\,(i,m)\in N \tag{47}$$

The energy constraints 24, 25, 30, and 31 are replaced by the constraints 48, 49, 50, and 51, respectively.

$$u^{D,L}_{imjnk} + u^{B,L}_{imjnk} = w x_{imjnk} \qquad \forall\,(i,m,j,n)\in A, k\in K \tag{48}$$

$$u^{D,L,0}_{imk} + u^{B,L,0}_{imk} = w x^0_{imk} \qquad \forall\,(i,m)\in N, k\in K \tag{49}$$

$$e^{B,C}_{imjnk} + e^{B,D}_{imjnk} \leq x_{imjnk} \qquad \forall (i,m,j,n)\in A, k\in K \tag{50}$$

$$e^{B,C,0}_{imk} + e^{B,D,0}_{imk} \leq x^0_{imk} \qquad \forall (i,m)\in N, k\in K \tag{51}$$

The bilinear energy constraints 38 are replaced by the linear constraints 52.

$$\begin{aligned} l_{jnk} \leq l_{imk} - \frac{1}{\mu^d_k}(T^S_{ij}u^{B,L}_{imjnk} + T^0_j u^{B,L,0}_{jnk}) + \mu^c_k(T^S_{ij}u^{D,B}_{imjnk} + T^0_j u^{D,B,0}_{jnk}) \\ + \overline{L}_k(1 - x_{imjnk}) \qquad \forall ((i,m),(j,n))\in A, k\in K \end{aligned} \tag{52}$$

Finally, the bilinear objective function 40 is replaced by the linear equation 53.

$$\min z = \sum_{k\in K} \sum_{((i,m),(j,n))\in A} \Big(T^S_{ij}(u^{D,B}_{imjnk} + u^{D,L}_{imjnk}) + T^0_i(u^{D,B,0}_{imk} + u^{D,L,0}_{imk})\Big) \tag{53}$$

The parameterization of the model produces a simplified linear formulation for the HS-SS-MIRP. However, since we force the speed levels in all arcs and for each ship to be the same, globally suboptimal results may be obtained.

4. Results

In this section, we present the results obtained by the experiments. We define three levels of speed for the ships, which result in $V = [v_1, v_2, v_3]$, with $v_1 < v_2 < v_3$. Consequently, the ships are required to travel at three power levels at different speeds. The power levels are contained in the set $W = [w_1, w_2, w_3]$, which is also ordered by $w_1 < w_2 < w_3$. We run the instances using Gurobi 10 solver on a Windows 10 machine with an Intel Xeon E5-2696v2 CPU and 16 GB of RAM.

4.1. Size of the instances

We test 14 instances, with their features varying from 1 ship and 2 ports, to 3 ships and 7 ports. The number of nodes in the graph increases with the maximum visits that we allow each port to have. An instance considering n ports, in which we allow one potential visit to each port and the origin is located on a port, would have n nodes. If we allowed two visits per port, the number of nodes would increase to $2n$. Generalizing, the graph would contain $\sum_{i \in P} \overline{m}_i$ nodes, with P being the set of all ports and $\overline{m}_i$ the maximum number of visits to the port $i \in P$. For our experimental setup, we consider the same number of potential visits to all ports, ranging from 2 to 5. Table 2 shows the size of the instances. The first column presents the 14 instances tested in this work, and the subsequent four columns display the number of rows of each model considering 2, 3, 4 or 5 visits, respectively. The final four columns display the number of columns of the models in the same fashion.

Table 2. Number of rows and columns of each instance.

	Number of rows				Number of columns			
Instance	2	3	4	5	2	3	4	5
1	557	1161	1981	3017	448	924	1568	2380
1b	557	1161	1981	3017	448	924	1568	2380
2	581	1196	2027	3074	492	990	1656	2490
2b	581	1196	2027	3074	492	990	1656	2490
3	1062	2242	3854	5898	880	1824	3104	4720
3b	1062	2242	3854	5898	880	1824	3104	4720
4	1687	3612	6257	9622	1380	2910	5000	7650
4b	1687	3612	6257	9622	1380	2910	5000	7650
5	2498	5368	9318	14348	2060	4350	7480	11450
5b	2498	5368	9318	14348	2060	4350	7480	11450
6	2467	5323	9236	14246	1992	4248	7344	11280
6b	2467	5323	9236	14246	1992	4248	7344	11280
7	5008	10916	19092	29536	4060	8736	15176	23380
7b	5008	10916	19092	29536	4060	8736	15176	23380

4.2. Experimental runs

To evaluate the performance of the model, we performed two sets of experiments with different time horizon lengths: 60 and 120 time units. Section 4.2.1 presents the results for the experiments considering $T = 60$, and Section 4.2.2 presents the results considering $T = 120$. For each time horizon, we solve each instance with three different speed levels, from low to high. The results are summarized in Tables 3 to 6 for $T = 60$, and Tables 7 to 10 for $T = 120$. The tables have the following structure: the first column shows the instance name; the second, third, and fourth columns report the computational time in seconds for each speed level; the fifth, sixth, and seventh columns indicate the optimality gap at the end of each run. A 0.0% gap means that the optimal solution was found. If the instance is unfeasible, we mark it as "UNF". If the time limit of 3600 seconds is reached without finding a gap or proving infeasibility, we mark it as inconclusive (INC). The last three columns show the best objective value obtained; if the instance is optimal, this is the optimal value. If a gap is obtained, this is the best feasible solution found. If the instance is unfeasible or inconclusive, we mark it as "UNF" or "INC", respectively.

4.2.1. Runs considering a time horizon length of 60 time units.

Table 3 shows the results obtained when we limit the number of visits per port to two. For the speed levels v_1 and v_2, we optimally solved all instances. For the speed level v_3, we proved infeasibility in instances 1 to 5b in up to 317.4 seconds, while we reached the time limit in instances 6 to 7b, which ended inconclusive. The objective value, which reflects the fuel use, is lower for the lower speed level v_1 than for the higher speed level v_2.

Table 3. Results for a time horizon value of $T = 60$ considering 2 maximum visits per port.

	Elapsed time [s]			% gap			Best objective value		
Instance	v_1	v_2	v_3	v_1	v_2	v_3	v_1	v_2	v_3
1	0.1	0.1	1.0	0.0	0.0	UNF	154.8	163.7	UNF
1b	0.2	0.1	1.1	0.0	0.0	UNF	154.8	163.7	UNF
2	0.2	0.3	0.7	0.0	0.0	UNF	186.8	197.9	UNF
2b	0.4	0.7	0.7	0.0	0.0	UNF	304.1	321.6	UNF
3	0.7	0.7	3.5	0.0	0.0	UNF	47.5	50.3	UNF
3b	0.7	0.9	3.2	0.0	0.0	UNF	47.5	50.3	UNF
4	3.6	3.4	76.1	0.0	0.0	UNF	50.5	53.5	UNF
4b	2.9	3.3	84.4	0.0	0.0	UNF	50.5	53.5	UNF
5	7.4	8.7	272.6	0.0	0.0	UNF	85.0	90.3	UNF
5b	7.6	13.9	317.4	0.0	0.0	UNF	85.5	90.9	UNF
6	236.4	292.6	3600.0	0.0	0.0	INC	191.5	202.9	INC
6b	8.9	5.8	3600.0	0.0	0.0	INC	99.3	105.4	INC
7	55.5	50.6	3600.0	0.0	0.0	INC	137.7	146.4	INC
7b	31.3	34.1	3600.0	0.0	0.0	INC	82.6	87.8	INC

Table 4 shows the results obtained with 3 maximum visits per port. For the speed levels v_1 and v_2, we optimally solved all instances except instance 6, where we obtained a gap of 19.1%. For the speed level v_3, we found infeasibility up to instance 3b, and reached the time limit for the remaining instances without conclusive results. As expected, the optimal objective values of all instances are the same as in Table 3. For instance 6, the best objective values found for the speed levels v_1 and v_2 are also the same as in Table 3, which were solved optimally. However, we could not prove the optimality for this instance.

Table 5 shows the results obtained considering four maximum visits per port. We observe that the slowest speed level v_1 allows us to optimally solve instances 1 up to 4b and instance 6b; while the remaining instances end with a gap with a maximum value of 34.5% in instance 6. With the medium speed level v_2, the performance improves and we optimally solve all instances except instances 5, 6 and 7b, where the maximum percentage gap is 31.0% in instance 6. The fastest speed level v_3 is the most challenging, as we can only prove the infeasibility in instances 1, 2 and 2b, and the rest of the instances are not solved within the time limit.

The results for five maximum visits per port are presented in Table 6. We solved instances up to 3b optimally for v_1 and v_2. Furthermore, instance 4 was optimal for v_1.

Table 4. Results for a time horizon value of $T = 60$ considering 3 maximum visits per port.

Instance	Elapsed time [s] v_1	Elapsed time [s] v_2	Elapsed time [s] v_3	% gap v_1	% gap v_2	% gap v_3	Best objective value v_1	Best objective value v_2	Best objective value v_3
1	1.0	1.1	13.6	0.0	0.0	UNF	154.8	163.7	UNF
1b	0.4	0.4	17.3	0.0	0.0	UNF	154.8	163.7	UNF
2	1.2	1.2	4.3	0.0	0.0	UNF	186.8	197.9	UNF
2b	1.5	1.4	7.4	0.0	0.0	UNF	304.1	321.6	UNF
3	3.2	3.3	955.4	0.0	0.0	UNF	47.5	50.3	UNF
3b	3.6	3.8	944.0	0.0	0.0	UNF	47.5	50.3	UNF
4	76.8	76.3	3600.0	0.0	0.0	INC	50.5	53.5	INC
4b	99.6	188.2	3600.0	0.0	0.0	INC	50.5	53.5	INC
5	207.0	369.8	3600.0	0.0	0.0	INC	85.0	90.3	INC
5b	161.7	227.5	3600.0	0.0	0.0	INC	85.5	90.9	INC
6	3600.0	3600.0	3600.0	19.9	19.1	INC	191.5	202.9	INC
6b	62.4	71.3	3600.0	0.0	0.0	INC	99.3	105.4	INC
7	741.8	540.9	3600.0	0.0	0.0	INC	137.7	146.4	INC
7b	837.8	428.6	3600.0	0.0	0.0	INC	82.6	87.8	INC

Table 5. Results for a time horizon value of $T = 60$ considering 4 maximum visits per port.

Instance	Elapsed time [s] v_1	Elapsed time [s] v_2	Elapsed time [s] v_3	% gap v_1	% gap v_2	% gap v_3	Best objective value v_1	Best objective value v_2	Best objective value v_3
1	1.8	2.1	946.1	0.0	0.0	UNF	154.8	163.7	UNF
1b	2.3	1.2	3600.0	0.0	0.0	INC	154.8	163.7	INC
2	3.4	4.3	101.4	0.0	0.0	UNF	186.8	197.9	UNF
2b	2.0	3.0	96.6	0.0	0.0	UNF	304.1	321.6	UNF
3	26.4	34.2	3600.0	0.0	0.0	INC	47.5	50.3	INC
3b	28.4	24.4	3600.0	0.0	0.0	INC	47.5	50.3	INC
4	457.8	518.3	3600.0	0.0	0.0	INC	50.5	53.5	INC
4b	700.5	1163.8	3600.0	0.0	0.0	INC	50.5	53.5	INC
5	3600.0	3600.0	3600.0	18.1	7.3	INC	85.0	90.3	INC
5b	3600.0	2611.2	3600.0	22.5	0.0	INC	85.5	90.9	INC
6	3600.0	3600.0	3600.0	34.5	31.0	INC	191.5	202.9	INC
6b	436.8	400.7	3600.0	0.0	0.0	INC	99.3	105.4	INC
7	3600.0	2864.1	3600.0	5.9	0.0	INC	137.7	146.6	INC
7b	3600.0	3600.0	3600.0	31.3	25.9	INC	82.6	87.8	INC

For larger instances, we obtained a gap within the time limit for v_1 and v_2, except for instance 6b, which was optimal for v_2. For v_3, none of the instances reached optimality. The largest gap was 48.5% for the instance 4b with v_1 and 37.9% for the instance 6 with v_2.

The optimization problem becomes more challenging at the highest speed level (v_3), as it imposes stricter timing constraints that reduce the feasible region of the solution space. Thus, it is harder to find optimal solutions that meet all operational and environmental criteria within the given time horizon. Most instances are unfeasible at this speed level, because the inventory constraints on the bases cannot be satisfied. The infeasibility is more difficult to prove for the model when a larger number of possible visits are considered.

We present the comparison of the elapsed times for each instance in Figure 3. The figure has three plots, each corresponding to a different speed level: v_1 (top), v_2 (middle), and v_3 (bottom). The instances are shown on the x-axis and the elapsed time in seconds is shown on the y-axis. The following symbols represent the number of maximum visits: circle (2), triangle (3), square (4), and pentagon (5). We observe that for v_1, the elapsed time is similar for 2 to 4 visits until instance 3b, but it increases sharply for 5 visits from instance 4 onward. For instances 4b and higher, 5 visits take

Table 6. Results for a time horizon value of $T = 60$ considering 5 maximum visits per port.

Instance	Elapsed time [s]			% gap			Best objective value		
	v_1	v_2	v_3	v_1	v_2	v_3	v_1	v_2	v_3
1	5.4	5.7	3600.0	0.0	0.0	INC	154.8	163.7	INC
1b	4.0	5.7	3600.0	0.0	0.0	INC	154.8	163.7	INC
2	10.6	9.6	3600.0	0.0	0.0	INC	186.8	197.9	INC
2b	6.5	6.2	3600.0	0.0	0.0	INC	304.1	321.6	INC
3	192.5	181.6	3600.0	0.0	0.0	INC	47.5	50.3	INC
3b	113.8	199.8	3600.0	0.0	0.0	INC	47.5	50.3	INC
4	3411.9	3600.0	3600.0	0.0	33.4	INC	50.5	53.5	INC
4b	3600.0	3600.0	3600.0	48.5	36.1	INC	50.5	53.5	INC
5	3600.0	3600.0	3600.0	36.4	28.1	INC	85.0	90.3	INC
5b	3600.0	3600.0	3600.0	32.1	35.6	INC	85.5	90.9	INC
6	3600.0	3600.0	3600.0	44.9	37.9	INC	191.5	202.9	INC
6b	3600.0	2349.1	3600.0	7.4	0.0	INC	99.3	105.4	INC
7	3600.0	3600.0	3600.0	15.6	12.5	INC	137.7	146.4	INC
7b	3600.0	3600.0	3600.0	45.3	34.4	INC	82.6	87.8	INC

much longer than the other options. We obtain optimal solutions for all instances with 2 and 3 visits. For v_2, the number of visits does not significantly affect the elapsed time for instances up to 3b. However, for instances 4 and higher, the elapsed time for 4 visits rises considerably, except for a sudden drop on instance 6b; and the elapsed time for 5 visits reaches the maximum limit of 3600 seconds, without finding an optimal solution. For v_3, instances up to 5b are proven to be unfeasible with 2 visits. From instance 3 onward, the instances are inconclusive after reaching the time limit with 4 and 5 visits. From instance 6 onward, the instances are inconclusive with all options of maximum visits.

4.2.2. Runs considering a time horizon length of 120 time units

In Table 7, we show the results for a time horizon of $T = 120$ with two maximum visits per port. The only instance that had feasible and optimal solutions for speed levels v_1 and v_2 was instance 4b. We also verified the infeasibility of the speed level v_3 in 10.3 seconds. For this longer time horizon, the other instances were unfeasible because they did not have enough potential visits to keep the port inventories within the acceptable ranges.

Table 7. Results for a time horizon value of $T = 120$ considering 2 maximum visits per port.

Instance	Elapsed time [s]			% gap			Best objective value		
	v_1	v_2	v_3	v_1	v_2	v_3	v_1	v_2	v_3
1	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
1b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
2	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
2b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
3	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
3b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
4	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
4b	5.9	4.9	10.3	0.0	0.0	UNF	323.3	336.7	UNF
5	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
5b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
6	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
6b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
7	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
7b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF

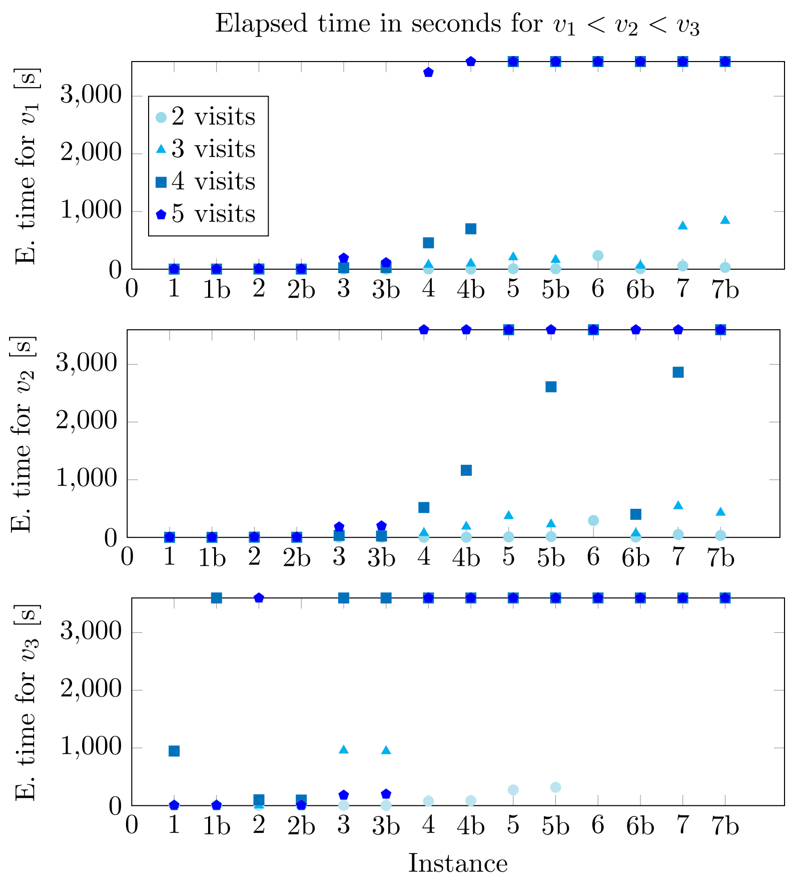

Figure 3. Elapsed time for each instance according to the number of visits considering all the speed levels

Table 8 shows the results for 3 maximum visits per port and a time horizon of $T = 120$. We obtained optimal solutions for instances 3 to 4b with speed levels v_1 and v_2. For the speed level v_3, we confirmed that instances 3 and 3b are unfeasible, but we could not reach a conclusion for instances 4 and 4b within the time limit. For instance 5b, we found a gap with speed levels v_1 and v_2, and the run was inconclusive with speed level v_3. Instances 1 to 2b, 5, and 6 to 7b were unfeasible for all speed levels.

The outcomes of our experiments with a time horizon of $T = 120$ and a maximum of 4 visits per port are illustrated in Table 9. We solved instances 3 and 3b optimally for speed levels v_1 and v_2, but we could not obtain conclusive results for speed level v_3. Instances 1 to 2b were unfeasible for all speed levels. Instances 6 to 7b did not reach a conclusive solution for any speed level. Instances 4 to 5b had a maximum gap of 34.7% and 30.1%, respectively, for speed levels v_1 and v_2. We could not solve instances 3 to 5b conclusively for the speed level v_3.

Table 10 shows the results obtained with a time horizon of $T = 120$, allowing for five maximum visits per port. For the speed levels v_1 and v_2, we optimally solved instances 2, 3 and 3b. Instance 2b delivered a gap for the speed level v_2 and ended inconclusively for the speed level v_1. We found maximum gaps of 45.5% and 44.2% for instances 4 to 5b considering speed levels v_1 and v_2. All instances ended inconclusively for the speed level v_3.

Table 8. Results for a time horizon value of $T = 120$ considering 3 maximum visits per port.

Instance	Elapsed time [s] v_1	v_2	v_3	% gap v_1	v_2	v_3	Best objective value v_1	v_2	v_3
1	0.0	0.0	0.1	UNF	UNF	UNF	UNF	UNF	UNF
1b	0.1	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
2	0.1	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
2b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
3	2.2	2.4	58.4	0.0	0.0	UNF	169.4	179.1	UNF
3b	4.1	3.7	105.9	0.0	0.0	UNF	297.7	314.8	UNF
4	682.4	1022.6	3600.0	0.0	0.0	INC	216.2	228.4	INC
4b	1707.0	1301.9	3600.0	0.0	0.0	INC	214.4	226.9	INC
5	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
5b	3600.0	3600.0	3600.0	34.5	38.9	INC	374.5	393.8	INC
6	0.4	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
6b	0.5	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
7	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
7b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF

Table 9. Results for a time horizon value of $T = 120$ considering 4 maximum visits per port.

Instance	Elapsed time [s] v_1	v_2	v_3	% gap v_1	v_2	v_3	Best objective value v_1	v_2	v_3
1	0.1	0.1	0.0	UNF	UNF	UNF	UNF	UNF	UNF
1b	0.0	0.0	0.0	UNF	UNF	UNF	UNF	UNF	UNF
2	0.1	0.2	0.2	UNF	UNF	UNF	UNF	UNF	UNF
2b	0.1	0.2	0.2	UNF	UNF	UNF	UNF	UNF	UNF
3	16.0	20.5	3600.0	0.0	0.0	INC	75.5	79.9	INC
3b	34.5	25.5	3600.0	0.0	0.0	INC	170.0	179.8	INC
4	3600.0	3600.0	3600.0	34.7	27.9	INC	145.2	150.1	INC
4b	3600.0	3600.0	3600.0	27.0	30.1	INC	144.6	150.1	INC
5	3600.0	3600.0	3600.0	8.2	7.7	INC	335.7	356.3	INC
5b	3600.0	3600.0	3600.0	9.0	7.8	INC	374.5	391.8	INC
6	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
6b	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
7	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
7b	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC

A significant observation from the results is that, for $T = 120$, increasing the maximum number of visits per port leads to a better objective value. This is attributed to the fact that more arcs can be traversed using electric propulsion instead of diesel. To illustrate this, we present instance 4b with 3 and 4 visits per port as an example. The routing of the two ships for instance 4b with 3 visits per port, is depicted in Figure 4, while the routing with 4 visits per port is shown in Figure 5. The dashed arrows indicate the arcs in which the ship used the battery as the power source. With three maximum visits, we notice that the ships only used electric propulsion on the first arc from the origin. With four maximum visits, we notice that ship 1 used electric propulsion on the first arc, while ship 2 used electric propulsion on six arcs, resulting in a lower objective value.

5. Discussion

We proposed a novel problem inspired by the operations of a fleet of heterogeneous ships that periodically visit scientific bases in Antarctica. Our problem extends the MIRP framework by incorporating hybrid electric ships that can vary in speed. We

Table 10. Results for a time horizon value of $T = 120$ considering 5 maximum visits per port.

	Elapsed time [s]			% gap			Best objective value		
Instance	v_1	v_2	v_3	v_1	v_2	v_3	v_1	v_2	v_3
1	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
1b	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
2	1545.7	3000.6	3600.0	0.0	0.0	INC	754.0	797.3	INC
2b	3600.0	3600.0	3600.0	INC	1.3	INC	INC	1045.0	INC
3	159.8	199.3	3600.0	0.0	0.0	INC	75.5	79.9	INC
3b	312.5	284.0	3600.0	0.0	0.0	INC	76.1	80.6	INC
4	3600.0	3600.0	3600.0	41.6	39.6	INC	143.4	150.8	INC
4b	3600.0	3600.0	3600.0	45.5	44.2	INC	142.2	149.5	INC
5	3600.0	3600.0	3600.0	40.5	20.9	INC	277.5	294.5	INC
5b	3600.0	3600.0	3600.0	39.3	38.9	INC	374.5	393.8	INC
6	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
6b	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
7	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC
7b	3600.0	3600.0	3600.0	INC	INC	INC	INC	INC	INC

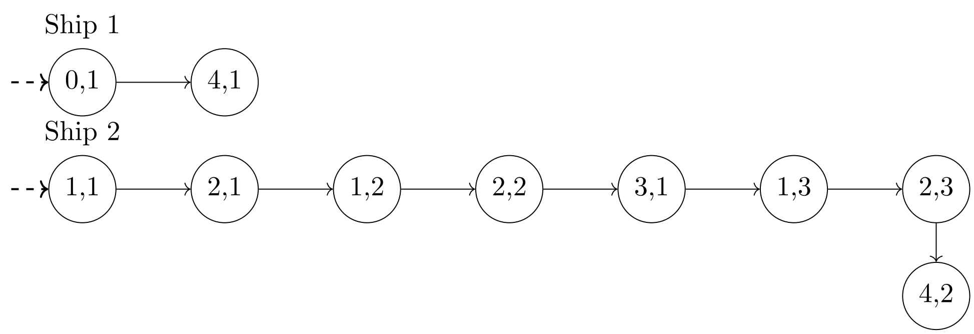

Figure 4. Routing of the ships for instance 4b considering 3 maximum visits per port

call this problem the Hybrid Ships with Speed Selection Maritime Inventory Routing Problem, and it is the first model that combines the MIRP with hybrid propulsion systems for ships to the best of our knowledge. This original model is a challenging mixed integer bilinear programming model due to the bilinear terms in the constraints and the objective function. To overcome this challenge, we parameterize the speed subindex, which linearizes the constraints and the objective function, giving rise to a mixed integer linear programming model with five distinct sets of constraints: routing, pickup and delivery, time, inventory, and energy.

To test our model, we used real-world data from 14 scientific bases in Antarctica, managed by the Antarctic Chilean Institute. We created 14 problem instances with different numbers of bases to visit (from 3 to 7), planning horizons (60 or 120 time units), speed levels (v1, v2, and v3), and maximum visits per port (from 2 to 5). We solved all instances with our model and analyzed the results under various scenarios. Our model can handle the complexity and uncertainty of the problem and provide feasible and efficient solutions for Antarctic logistics planning.

The results for $T = 60$ show that the optimal routing is independent of the maximum number of visits per port, as all instances achieved the same objective value regardless of this parameter. However, the feasibility and optimality of the solutions depend on the speed level and the size of the instance. For v1 and v2, we were able to solve most of the instances optimally, except for some larger ones where a gap remained. For v3, we could only prove infeasibility for a few small-size instances, while the rest were

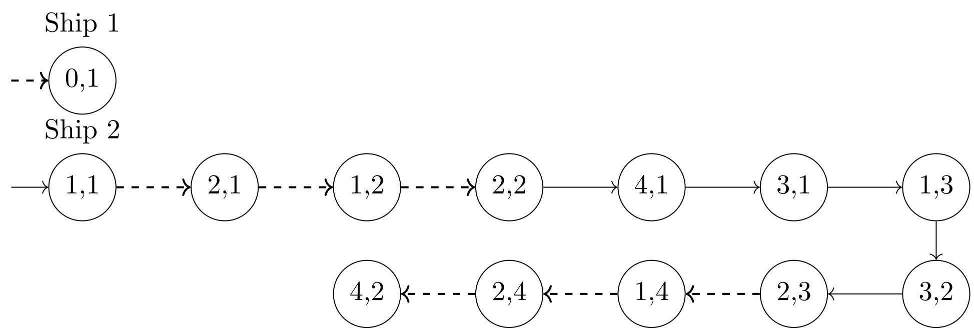

Figure 5. Routing of the ships for instance 4b considering 4 maximum visits per port

inconclusive. Tables 3 to 6 provide more details on the results for each instance and each maximum number of visits per port.

For a longer time horizon of $T = 120$, the maximum number of visits per port significantly affects the routing and the objective value. Some problem instances become unfeasible with few possible visits because the port inventory cannot remain within the acceptable range without frequent visits. Only one instance had feasible solutions for v1 and v2 with two maximum visits per port. This number gradually increased as we allowed more visits per port, reaching eight instances with a gap of five maximum visits per port. Furthermore, the best objective value depends on the maximum number of visits per port. Allowing more visits allows for more routes with the electric motor, reducing the diesel consumption and the objective value. This is because the objective function only accounts for fuel consumption, not for fixed costs per use or visit of the ship.

The results provide a comprehensive planning framework for government agencies to optimize logistics operations during a 4-month research season, using one day as the unit of time for the time horizon T. It also offers guidelines for estimating fuel and battery consumption for different hybrid electric ships throughout the season. However, there are some limitations to consider. First, it does not account for the stochastic nature of port operations and the weather conditions that may influence the ships' optimal speeds and power requirements. Second, it only considers hybrid electric ships with two energy sources: fuel and battery. Other hybrid configurations may include additional sources, such as solar panels and fuel cells.

To advance the research in this field and to face problem instances that are still computational challenges, future research should consider deriving valid inequalities that can improve the bilinear formulation. In this way, we could develop practical algorithms to obtain exact solutions for the bilinear problem, generalizing the model to accommodate different kinds of cargo, and adding other energy sources for the ships. These extensions would increase the relevance and value of the model for practical decision-making.

6. Conclusions

This paper presents a mixed integer linear programming model that optimizes routing, pickup, and delivery operations for a fleet of heterogeneous hybrid electric ships. The model incorporates various operational constraints, such as the ship's capacity, speed, and battery level. The model was tested in realistic instances derived from real-world data. The computational experiments demonstrate that the model can find optimal solutions for small- and medium-sized instances within a reasonable time limit of 3600 seconds and provide gaps for larger-size instances. The proposed model is a valuable tool to address complex problems in the maritime transportation sector from an academic and practical perspective. It also opens up new avenues for future research on uncertainty, dynamicity, climate change, etc.

Disclosure statement

The authors report there are no competing interests to declare.

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Data availability statement

The data that support the findings of this study are available from the corresponding author, Dagoberto Cifuentes-Lobos, upon reasonable request.

Notes on contributors

All authors contributed to the conception and design of the study. Conceptualization, formal analysis, investigation and methodology were performed by Dagoberto Cifuentes-Lobos, Lorena Pradenas, and Victor Parada. Data curation was performed by Dagoberto Cifuentes-Lobos and Victor Parada. Software, validation, and visualization were performed by Dagoberto Cifuentes-Lobos. The first draft of the manuscript was written by Dagoberto Cifuentes-Lobos, and the review and editing processes were performed by Lorena Pradenas and Victor Parada. All authors read and approved the final manuscript.

References

[1] Agra A, Christiansen M, Wolsey L. Improved models for a single vehicle continuous-time inventory routing problem with pickups and deliveries. European Journal of Operational Research. 2022 2;297:164–179.

[2] Duan G, Zhang K. Optimization on hybrid energy vessel routing and energy management for floating marine debris cleanup. Transportation Research Part C: Emerging Technologies. 2022;138:103649. Available from: https://www.sciencedirect.com/science/article/pii/S0968090X22000924.

[3] Fadda P, Mancini S, Serra P, et al. The heterogeneous fleet vehicle routing problem with draft limits. Computers & Operations Research. 2023;149:106024. Available from: https://www.sciencedirect.com/science/article/pii/S0305054822002544.
[4] Charalambopoulos N, Nearchou AC. Ship routing using genetic algorithms. Operations Research Forum. 2021;2(3). Cited by: 1; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85126319679&doi=10.1007%2fs43069-021-00093-w&partnerID=40&md5=e1511978a2691453a226d27668ad4abd.
[5] Hu Z. Model and algorithm for the large material distribution problem in maritime transportation. Journal of Coastal Research. 2018;82:294 – 306. Cited by: 4; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85056895146&doi=10.2112%2fSI82-042.1&partnerID=40&md5=5ce68bcae0916e426d3b14ea411059b8.
[6] Halvorsen-Weare EE, Fagerholt K, Nonås LM, et al. Optimal fleet composition and periodic routing of offshore supply vessels. Eur J Oper Res. 2012;223:508–517. Available from: https://api.semanticscholar.org/CorpusID:8602277.
[7] Vieira BS, Ribeiro GM, Bahiense L, et al. Exact and heuristic algorithms for the fleet composition and periodic routing problem of offshore supply vessels with berth allocation decisions. European Journal of Operational Research. 2021;295(3):908–923. Available from: https://www.sciencedirect.com/science/article/pii/S0377221721002484.
[8] Borthen T, Loennechen H, Wang X, et al. A genetic search-based heuristic for a fleet size and periodic routing problem with application to offshore supply planning. EURO Journal on Transportation and Logistics. 2018;7(2):121–150. Available from: https://www.sciencedirect.com/science/article/pii/S2192437620300704.
[9] Wang H, Lang X, Mao W. Voyage optimization combining genetic algorithm and dynamic programming for fuel/emissions reduction. Transportation Research Part D: Transport and Environment. 2021;90:102670. Available from: https://www.sciencedirect.com/science/article/pii/S1361920920308555.
[10] Lee H, Aydin N, Choi Y, et al. A decision support system for vessel speed decision in maritime logistics using weather archive big data. Computers & Operations Research. 2018;98:330–342. Available from: https://www.sciencedirect.com/science/article/pii/S0305054817301429.
[11] Brekkå I, Randøy S, Fagerholt K, et al. The fish feed production routing problem. Computers & Operations Research. 2022;144:105806. Available from: https://www.sciencedirect.com/science/article/pii/S0305054822000922.
[12] Chagas RDS, Soares JBCdO, Longhi RP, et al. A solution framework for the integrated periodic supply vessel planning and port scheduling in oil and gas supply logistics. Optimization and Engineering. 2023;24(3):1537 – 1575. Cited by: 1; All Open Access, Green Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85136973234&doi=10.1007%2fs11081-022-09742-3&partnerID=40&md5=6bb9118aec9c13975f65a4bff43876ad.
[13] Mor A, Speranza M. Vehicle routing problems over time: a survey. Annals of Operations Research. 2022;314:255–275.
[14] Fagerholt K, Hvattum LM, Papageorgiou DJ, et al. Maritime inventory routing: recent trends and future directions. International Transactions in Operational Research. 2023;30(6):3013 – 3056. Cited by: 1; All Open Access, Hybrid Gold Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85159634499&doi=10.1111%2fitor.13313&partnerID=40&md5=e1886b39ceb4915595655e0b15c7fab3.
[15] De A, Kumar SK, Gunasekaran A, et al. Sustainable maritime inventory routing problem with time window constraints. Engineering Applications of Artificial Intelligence. 2017;61:77–95. Available from: https://www.sciencedirect.com/science/article/pii/S0952197617300350.
[16] Agra A, Christiansen M, Delgado A. Discrete time and continuous time formulations for a short sea inventory routing problem. Optimization and Engineering. 2017;18(1):269 – 297. Cited by: 26; All Open Access, Green Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84964063347&doi=10.1007%

2fs11081-016-9319-0&partnerID=40&md5=62780b6b382af87631bc8cb742ffd72d.
[17] Shaabani H, Hoff A, Hvattum LM, et al. A matheuristic for the multi-product maritime inventory routing problem. Computers & Operations Research. 2023;154:106214. Available from: https://www.sciencedirect.com/science/article/pii/S0305054823000783.
[18] Nikolaisen JB, Vågen SS, Schütz P. Solving a maritime inventory routing problem under uncertainty using optimization and simulation. Computational Management Science. 2023;20(1). Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85160711294&doi=10.1007%2fs10287-023-00459-x&partnerID=40&md5=f605dee9dba381ffe06c2974c57b0d41.
[19] Agra A, Christiansen M, Delgado A, et al. A maritime inventory routing problem with stochastic sailing and port times. Computers and Operations Research. 2015;61:18 – 30. Cited by: 58; All Open Access, Bronze Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84925065056&doi=10.1016%2fj.cor.2015.01.008&partnerID=40&md5=1eed1c62d7f6eaaad1db2d22f68ee300.
[20] Zojaji A, Soltaniani K, Hvattum LM, et al. Cyclic solutions to a maritime inventory routing problem. Maritime Transport Research. 2022;3. Cited by: 1; All Open Access, Gold Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85138821605&doi=10.1016%2fj.martra.2022.100074&partnerID=40&md5=5c755adfa39e9dff2180533bde0c7dff.
[21] Algendi A, Urrutia S, Hvattum LM. Optimizing production levels in maritime inventory routing with load-dependent speed optimization. Flexible Services and Manufacturing Journal. 2023;35(1):111 – 141. Cited by: 2; All Open Access, Hybrid Gold Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85136526629&doi=10.1007%2fs10696-022-09460-z&partnerID=40&md5=8e1d03c9b5d04f017498ac77a369ea23.
[22] Diz GSdS, Oliveira F, Hamacher S. Improving maritime inventory routing: application to a brazilian petroleum case. Maritime Policy and Management. 2017;44(1):42 – 61. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84983356919&doi=10.1080%2f03088839.2016.1216622&partnerID=40&md5=cd3cb5f7228943461916ee3226296efc.
[23] Andersson H, Christiansen M, Desaulniers G, et al. Creating annual delivery programs of liquefied natural gas. Optimization and Engineering. 2017;18(1):299 – 316. Cited by: 16; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84951826806&doi=10.1007%2fs11081-015-9305-y&partnerID=40&md5=b14c65af48b6b9a08ab5a1225d2c22cd.
[24] Xu Xin ZC Xiaoli Wang, Chen K. Coastal shuttle tanker inventory routing model with a discrete loaded quantity. Applied Economics. 2021;53(53):6120–6137. Available from: https://doi.org/10.1080/00036846.2021.1918625.
[25] Sanghikian N, Martinelli R, Abu-Marrul V. A hybrid vns for the multi-product maritime inventory routing problem. In: Mladenovic N, Sleptchenko A, Sifaleras A, et al., editors. Variable Neighborhood Search; Cham. Springer International Publishing; 2021. p. 111–122.
[26] Adhi A, Siswanto N, Santosa B. Hybrid metaheuristic for solving maritime inventory routing problem in bulk product transportation. International Journal of Intelligent Engineering and Systems. 2023;16(2):361 – 374. Cited by: 0; All Open Access, Bronze Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85153329961&doi=10.22266%2fIJIES2023.0430.29&partnerID=40&md5=c89af97efb20ad9163ad0f57910475b2.
[27] Stanzani AdL, Pureza V, Morabito R, et al. Optimizing multiship routing and scheduling with constraints on inventory levels in a brazilian oil company. International Transactions in Operational Research. 2018;25(4):1163 – 1198. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85032975717&doi=10.1111%2fitor.12478&partnerID=40&md5=800ee4bbb6243a71a65d1142ca307c32.
[28] Radan F, Fatemi Ghomi S, Mirzapour Al-e hashem S, et al. Maritime in-

ventory routing problem considering weather conditions and tide at ports. Transportation Research Record. 2023;2677(5):934 – 950. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85163116484&doi=10.1177%2f03611981221138565&partnerID=40&md5=a4e5b008d36795f1c2fa1c87b76382ff.

[29] Uggen KT, Fodstad M, Nørstebø VS. Using and extending fix-and-relax to solve maritime inventory routing problems. TOP. 2013;21(2):355 – 377. Cited by: 41; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84878959111&doi=10.1007%2fs11750-011-0174-z&partnerID=40&md5=d52299ec480be5f92b02f0b5a665ec68.

[30] Friske MW, Buriol LS, Camponogara E. A relax-and-fix and fix-and-optimize algorithm for a maritime inventory routing problem. Computers and Operations Research. 2022;137. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85122496367&doi=10.1016%2fj.cor.2021.105520&partnerID=40&md5=cac865a6dfb0434a15a36bb1c689649f.

[31] Heidari N, Hemmati A. An alns-based matheuristic algorithm for a multi-product many-to-many maritime inventory routing problem. Computational Management Science. 2023;20(1). All Open Access, Hybrid Gold Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85173603741&doi=10.1007%2fs10287-023-00478-8&partnerID=40&md5=eaeae8f62b9b45c52b361c743145d34f.

[32] Eide L, Årdal GCH, Evsikova N, et al. Load-dependent speed optimization in maritime inventory routing. Computers & Operations Research. 2020;123:105051. Available from: https://www.sciencedirect.com/science/article/pii/S0305054820301684.

[33] Hvattum LM, Norstad I, Fagerholt K, et al. Analysis of an exact algorithm for the vessel speed optimization problem. Networks. 2013;62(2):132 – 135. Cited by: 89; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84882449865&doi=10.1002%2fnet.21503&partnerID=40&md5=d6671f0b4c4469a85311cb1375e177e0.

[34] Andersson H, Fagerholt K, Hobbesland K. Integrated maritime fleet deployment and speed optimization: Case study from roro shipping. Computers and Operations Research. 2015;55:233 – 240. Cited by: 100; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84918810326&doi=10.1016%2fj.cor.2014.03.017&partnerID=40&md5=02a3bdb4f95a956ae2cc1b1dec450228.

[35] Wen M, Ropke S, Petersen H, et al. Full-shipload tramp ship routing and scheduling with variable speeds. Computers and Operations Research. 2016; 70:1 – 8. Cited by: 44; All Open Access, Green Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84954113351&doi=10.1016%2fj.cor.2015.10.002&partnerID=40&md5=6e493061d745d0d9d393b82118272498.

[36] Psaraftis HN, Kontovas CA. Speed models for energy-efficient maritime transportation: A taxonomy and survey. Transportation Research Part C: Emerging Technologies. 2013;26:331 – 351. Cited by: 369; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84872716875&doi=10.1016%2fj.trc.2012.09.012&partnerID=40&md5=3d1e29539d2c9476ad3f68b2ef582f32.

[37] Psaraftis HN, Kontovas CA. Ship speed optimization: Concepts, models and combined speed-routing scenarios. Transportation Research Part C: Emerging Technologies. 2014;44:52 – 69. Cited by: 214; All Open Access, Green Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84899125114&doi=10.1016%2fj.trc.2014.03.001&partnerID=40&md5=21c0ab722e54167fd76b3538615d26f9.

[38] Pan X, Zhu X, Zhao F. More environmental sustainability routing and energy management for all electric ships. Frontiers in Energy Research. 2022;9. All Open Access, Gold Open Access; Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85125092521&doi=10.3389%2ffenrg.2021.821236&partnerID=40&md5=2d18c9eb75fb14a5687183382a2505ba.

[39] Ritari A, Spoof-Tuomi K, Huotari J, et al. Emission abatement technology selection, routing and speed optimization of hybrid ships. Journal of Marine Science and Engineering. 2021;9(9). Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85114226984&doi=10.3390%2fjmse9090944&partnerID=40&md5=

051bcfbb20e4c794b0978169cddd38b8.

[40] Domachowski Z. Minimizing greenhouse gas emissions from ships using a pareto multi-objective optimization approach. Polish Maritime Research. 2021;28(2):96 – 101. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85111131636&doi=10.2478%2fpomr-2021-0026&partnerID=40&md5=e83bec5063b8800152acbaffe4df06e5.

[41] McCormick GP. Computability of global solutions to factorable nonconvex programs: Part i - convex underestimating problems. Mathematical Programming. 1976;10(1):147 – 175. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-34250381644&doi=10.1007%2fBF01580665&partnerID=40&md5=cd454cc76045e11371eaf619bcf2f0ad.

[42] Assis LS, Camponogara E, Menezes BC, et al. An minlp formulation for integrating the operational management of crude oil supply. Computers & Chemical Engineering. 2019;123:110–125. Available from: https://www.sciencedirect.com/science/article/pii/S0098135418306355.

[43] Li C, Ding T, Liu X, et al. An electric vehicle routing optimization model with hybrid plug-in and wireless charging systems. IEEE Access. 2018;6:27569 – 27578. Available from: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85047004393&doi=10.1109%2fACCESS.2018.2832187&partnerID=40&md5=b1fe2a92fb54af41b76d6d71fe3572bd.

Capítulo 4

Discusión

En este trabajo se presentaron dos formulaciones innovadoras basadas en programación entera mixta, orientadas a optimizar la planificación logística de una flota heterogénea que abastece periódicamente las bases científicas chilenas en la Antártica. La diversidad de la flota y las condiciones extremas del entorno antártico impulsaron el desarrollo de dos modelos específicos: el MV-MIRP, enfocado en múltiples barcos con cargas homogéneas, y el HS-SS-MIRP, que además incorpora la gestión energética mediante propulsión híbrida con velocidades adaptativas.

El modelo MV-MIRP, formulado como un problema MILP, proporcionó soluciones exactas para horizontes superiores a 60 días al integrar técnicas avanzadas como inecuaciones válidas, mejorando significativamente la eficiencia computacional. Al evaluarlo con 14 instancias realistas construidas a partir de datos operativos, se comprobó que, sin la inclusión de estas inecuaciones, sólo 12 instancias fueron resueltas de forma óptima para el horizonte de 60 días. Sin embargo, al agregar estas técnicas, se logró resolver óptimamente la totalidad de los escenarios, demostrando claramente que, bajo estas condiciones, el horizonte de planificación de 60 días es completamente abordable desde un punto de vista tecnológico.

No obstante, al extender el horizonte a 120 días, el desafío computacional aumentó considerablemente. Inicialmente, sólo 4 instancias alcanzaron soluciones óptimas con la formulación original. Tras integrar las inecuaciones válidas, esta cifra ascendió a 7 instancias resueltas de manera óptima. Aun así, la resolución completa del horizonte de 120 días permanece como un desafío significativo, evidenciado por la presencia de gaps considerables en las soluciones finales, alcanzando hasta un 29.1 % en la instancia más compleja (instancia 5b). Esto enfatiza la necesidad de futuras investigaciones que exploren técnicas aún más robustas o enfoques heurísticos complementarios para mejorar la escalabilidad en horizontes extendidos.

Por otra parte, el modelo HS-SS-MIRP, formulado como un problema bilineal entero mixto (MIBLP), introdujo una dimensión adicional al integrar la gestión energética y la velocidad adaptativa en barcos híbridos. En el horizonte de 60 días, todas las instancias fueron resueltas de manera óptima al limitar las visitas máximas por puerto a dos, confirmando nuevamente la factibilidad tecnológica del planteamiento. Sin embargo, se observó que aumentar el número de visitas no aportó beneficios significativos en este horizonte, sino que incrementó innecesariamente la complejidad del problema, derivando en soluciones subóptimas con gaps.

En cambio, para el horizonte más extenso de 120 días, sólo una instancia se resolvió óptimamente cuando se restringió a dos visitas por puerto. Curiosamente, al incrementar el número máximo de visitas, aumentó también la cantidad de escenarios que alcanzaron soluciones óptimas, generando simultáneamente mejoras en el valor objetivo. Este fenómeno se explica por la estructura de la función objetivo, que persigue explícitamente la minimización del consumo de diésel. De este modo, un mayor número de visitas permite flexibilizar el aprovechamiento de la propulsión híbrida y las velocidades adaptativas, generando rutas energéticamente más eficientes.

En conjunto, estos resultados no sólo validan el potencial de las formulaciones propuestas para mejorar la sostenibilidad operativa en regiones logísticas extremas, sino que también señalan claramente los límites computacionales y áreas de mejora futuras, estableciendo un puente sólido hacia nuevas investigaciones que profundicen en técnicas avanzadas de optimización combinatoria aplicada a la logística antártica.

Capítulo 5

Conclusiones

La presente tesis doctoral abordó la logística antártica desde una perspectiva de *Maritime Inventory Routing Problem* (MIRP) extendida con dos aportes fundamentales: la planificación de rutas y niveles de inventario en flotas múltiples para horizontes de tiempo grandes, y la incorporación de propulsión híbrida con selección de velocidad. Dos artículos científicos enviados y uno publicado, proponen modelos de Programación Entera Mixta (MILP y MIBLP) que combinan: sostenibilidad energética con la eficiencia logística en el entorno extremo de la Antártica.

La tesis formula y valida un modelo unificado de MIRP con múltiples embarcaciones y horizonte de planificación de hasta 120 días, con aporte cualitativo significativo a la literatura previa, que considera horizontes menores o una sola embarcación. Posteriormente, se incorporaron variables de propulsión eléctrica y velocidades adaptativas, con resultados de reducción en el consumo de combustible y, por ende, en emisiones contaminantes. Este hallazgo es de especial relevancia en zonas tan sensibles como la Antártica, donde la sostenibilidad de las operaciones marítimas es prioritaria.

Los resultados computacionales evidenciaron que las comparaciones con formulaciones previas y el uso de inecuaciones válidas pueden reducir los tiempos computacionales en instancias pequeñas y medianas (hasta 60 días). Sin embargo, para horizontes mayores (120 días) y redes logísticas superiores, se detectaron *gaps* considerables en algunas instancias, que confirmaron la complejidad de escalar estos modelos. Aun así, las estrategias de resolución exacta ofrecieron soluciones cercanas al óptimo en tiempos aceptables.

Sobre la sostenibilidad, las simulaciones con embarcaciones híbridas y velocidad adaptativa demostraron ahorros de combustible diésel en comparación con naves convencionales, impactando tanto en la reducción de costos como en las emisiones de CO_2 y otros contaminantes. Adicionalmente, se generaron instancias realistas con datos de bases chilenas en la Antártica, reforzando la aplicabilidad práctica de los modelos. Aunque el foco se centró en la región antártica, la metodología propuesta es transferible a otros contextos con restricciones ambientales severas, siempre que se ajusten parámetros como: tamaño de flota, de puertos y de horizontes de planificación.

No obstante, la tesis presenta ciertas limitaciones. Se asumió un entorno determinista en relación a demanda y condiciones climáticas, sin contemplar eventos estocásticos que podrían impactar los planes de navegación. Asimismo, la discretización de velocidades y la introducción de términos bilineales imponen un mayor desafío computacional, que se atenúa con diferentes estrategias de parametrización, podría penalizar la escalabilidad en escenarios de gran magnitud.

Posibles estudios futuros, pueden incorporar modelos robustos o estocásticos para capturar la incertidumbre en la demanda o el clima, explorar enfoques multiobjetivo que equilibren: costos, emisiones y seguridad operacional. También sería prometedor aplicar técnicas de descomposición y metaheurísticas híbridas para manejar casos aún más extensos.

Bibliografía

[ant, 1959] (1959). The antarctic treaty. Accessed: 2025-01-22.

[mar, 1973] (1973). International convention for the prevention of pollution from ships (marpol). Accessed: 2025-01-22.

[Cifuentes-Lobos et al., 2025] Cifuentes-Lobos, D., Pradenas, L., and Parada, V. (2025). Enhanced milp approach for long-term multi-vessel maritime inventory routing with application to antarctic logistics. *Journal of Marine Science and Engineering*, 13(2).

[Instituto Antártico Chileno, 2010] Instituto Antártico Chileno (2010). Operaciones logísticas. Accessed: 2025-01-22.

[Instituto Antártico Chileno, 2025] Instituto Antártico Chileno (2025). Instituto antártico chileno. Accessed: 2025-01-22.