



UNIVERSIDAD DE CONCEPCIÓN
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS

SMBHB Candidate Search Using 46-Year OVRO+UMRAO Light Curves

By: Brian Claudio Molina Santana

Tesis presentada a la Facultad de Ciencias Físicas y Matemáticas de la
Universidad de Concepción para optar al grado académico de Doctor en
Ciencias Físicas

September 2026
Concepción, Chile

Advisor: Dr. Rodrigo Reeves Díaz
Co-advisor: Dr. Anthony Readhead

© 2026, Brian Claudio Molina Santana

Se autoriza la reproducción total o parcial, con fines académicos, por cualquier medio o procedimiento, incluyendo la cita bibliográfica del documento

Dedicated to Viviana and Dante.

ACKNOWLEDGMENTS

I am grateful to all the people and institutions whose contributions made this research possible. In particular, I gratefully acknowledge the support of the Office of the Vice-Rector for Research and Development of the Universidad de Concepción (VRID), the National Agency for Research and Development (ANID), the Center for Excellence in Astrophysics and Associated Technologies (CATA), the Millennium Nucleus TITANS, the Center for Astronomical Instrumentation (CePIA), the Department of Astronomy, and the Faculty of Physical and Mathematical Sciences of the Universidad de Concepción.

I am grateful to my advisor and co-advisor, Rodrigo Reeves and Anthony Readhead, for giving me the opportunity to participate in the collaboration between Caltech and the Universidad de Concepción, and for guiding me throughout this research. I thank Rodrigo for introducing me to the field of astronomical instrumentation and enabling me to bring together two of my main interests: observational astrophysics and technological development, particularly cutting-edge analog electronics. I thank Anthony for introducing me to the study of AGN and their periodicities, and above all for inviting me to join his research group. His curiosity and enthusiasm were a constant source of motivation. I especially admire his intellectual openness and his approach to science: freely formulating ideas, subjecting them to rigorous testing, and recognizing that revising or correcting a hypothesis in light of new evidence does not represent failure, but rather an essential part of learning and scientific progress, particularly when working at the frontiers of knowledge.

I am grateful to Caltech and the Owens Valley Radio Observatory (OVRO), as well as to the University of Michigan and the University of Michigan Radio Astronomy Observatory (UMRAO), for providing the observations, infrastructure, and scientific collaboration that supported this work. I am especially grateful to Margo Aller for calibrating and providing the UMRAO data used in this research, and for reviewing and commenting on both this thesis and the publications arising from it. I also extend my gratitude to everyone involved in the OVRO collaboration, both those who work directly with the data and those who make their acquisition possible through the technical, operational, and logistical support of the observatory. In particular, I thank Mark Hodges and Philippe Vergara as representatives of this collective effort.

Finally, I thank all those who have been and continue to be part of CePIA, who have taken on the extraordinary challenge of developing cutting-edge astronomical instrumentation in a national context where this type of work often faces considerable difficulties. Despite these challenges, they have demonstrated that projects that may seem almost impossible can be transformed into tangible scientific and technological achievements. Their work has been a constant source of admiration and inspiration for me.

Resumen

Los agujeros negros supermasivos, alojados en el centro de la mayoría de las galaxias, impulsan algunos de los fenómenos más energéticos del universo: al acretar materia y formar jets relativistas pueden dar origen a núcleos galácticos activos (*active galactic nuclei*, AGN) y a su marcada variabilidad. En este contexto, la búsqueda de periodicidades asociadas a sistemas binarios de agujeros negros supermasivos (*supermassive black hole binaries*, SMBHB) es especialmente atractiva, pero metodológicamente exigente, ya que sesgos sutiles pueden conducir a interpretaciones erróneas. Por ello, esta tesis adopta un enfoque deliberadamente robusto, apoyado en una serie temporal excepcionalmente larga y densamente muestreada, sin precedentes en la historia de la astronomía observacional.

Para llevar a cabo esta investigación se utilizó la combinación de datos de dos programas de monitoreo en ondas de radio, que abarca ~ 46 años, con una cadencia aproximadamente semanal. La muestra se construye a partir de observaciones del radiotelescopio de 40 m de diámetro del Owens Valley Radio Observatory (OVRO; 15 GHz) y del radiotelescopio de 26 m de diámetro del University of Michigan Radio Astronomy Observatory (UMRAO; 14.5 GHz), ambos en Estados Unidos, e incluye 83 AGN, en su mayoría blázares. El análisis combina una minuciosa inspección y fusión de las curvas de luz OVRO+UMRAO con herramientas de series temporales, incluyendo el periodograma de Lomb–Scargle generalizado (GLSP), análisis tiempo–frecuencia (p.ej., WWZ), ajustes sinusoidales y la utilización de imágenes de alta resolución, que permiten realizar análisis morfológicos y el seguimiento del origen de la periodicidad detectada. Además, se realizó un estudio de relaciones armónicas entre periodicidades, lo que permite aproximarse a un escenario realista de la cantidad de periodicidades reales que podrían ser detectadas.

Como resultado principal se identifica un candidato destacado a SMBHB: el blazar J1309+1154, que exhibe un patrón sinusoidal con período de 17.9 años, observado durante ~ 2.5 ciclos, y evidencia adicional de un armónico cercano a 9.2 años.

Abstract

Supermassive black holes, hosted at the centers of most galaxies, drive some of the most energetic phenomena in the Universe: by accreting matter and launching relativistic jets they can produce active galactic nuclei (AGN) and their pronounced variability. In this context, the search for periodicities associated with supermassive black hole binaries (SMBHBs) is particularly appealing, yet methodologically demanding, since subtle biases can lead to erroneous interpretations. For this reason, this thesis adopts a deliberately robust approach, supported by an exceptionally long and densely sampled time series, unprecedented in the history of observational astronomy.

To carry out this research, we combined data from two radio monitoring programs spanning ~ 46 years with an approximately weekly cadence. The sample is built from observations with the 40 m telescope of the Owens Valley Radio Observatory (OVRO; 15 GHz) and the 26 m telescope of the University of Michigan Radio Astronomy Observatory (UMRAO; 14.5 GHz), both in the United States, and includes 83 active galactic nuclei (AGN) dominated by blazars. The analysis combines the homogenization and merging of the OVRO+UMRAO light curves with time-series tools, including the generalized Lomb–Scargle periodogram (GLSP), time–frequency analysis (e.g., WWZ), sinusoidal fitting, and the use of high-resolution imaging, which enables morphological analyses and tracing the origin of the detected periodicity. In addition, we conducted a study of harmonic relationships among periodicities, providing a realistic assessment of how reliably genuine periodic signals can be identified.

As the main result, we identify a prominent SMBHB candidate: the blazar J1309+1154, which exhibits a sinusoidal pattern with a period of 17.9 years, observed over ~ 2.5 cycles, and additional evidence for a harmonic near 9.2 years.

Contents

ACKNOWLEDGMENTS	I
Resumen	III
Abstract	IV
1 Introduction	1
1.1 Scientific context	3
1.1.1 Hypothesis	4
1.2 Research Objectives	5
1.2.1 General Objective	5
1.2.2 Specific Objectives	5
2 AGN Physics: Unified Model, Emission, and Variability	7
2.1 Why AGN vary across timescales	11
2.1.1 Orientation-based unification schemes	12
2.1.2 Relativistic jets and emission processes	12
2.1.3 Jet launching and collimation: an MHD perspective	15
2.1.4 Synchrotron Radiation	16
2.1.5 Blazars and compact radio sources	18
2.2 Multiwavelength Variability: Key Physical Mechanisms by Band	19
2.2.1 Radio domain: key drivers of flux-density variability (S_ν)	20
2.2.2 Optical/UV domain: key drivers of variability (hybrid jet–disk regime)	20
2.2.3 X-ray and γ -ray domains: key drivers of extreme variability	21
2.3 Potential physical mechanisms responsible for periodic variability	22
2.3.1 Historical background and physical context	23
2.3.2 Relativistic Doppler modulation	24
2.3.3 The kinematic orbital model	25
2.3.4 PKS 2131-021 as an Application of the KO Model	28
2.3.5 The Modified Kinetic Orbital Model	30
3 Radio Monitoring with OVRO and UMRAO	35
3.0.1 Radio monitoring facilities: OVRO and UMRAO	36
3.1 The UMRAO 26-m monitoring program	37

3.2	The OVRO 40-m monitoring program	38
3.3	Data Calibration	39
3.3.1	Low level calibration	39
3.3.2	High level calibration	40
3.3.3	Absolute Calibration	41
3.3.4	Filtering, Post-filtering and Inspection	41
4	The Extended OVRO+UMRAO Dataset	43
4.1	Light curves for the OVRO+UMRAO dataset	44
4.2	OVRO and UMRAO Data Combination	46
4.3	Time systems, units, and flux-density scales	48
4.4	Classification and redshift distribution of the OVRO+UMRAO sample	52
4.5	Periodicity Analysis Methods Applied to the OVRO and UMRAO Data	55
4.6	Preliminary Periodicity Diagnostics and Results	56
4.7	Global Preliminary Results	64
4.8	Introduction of the Source J1309+1154	66
5	Research Programme: Searching for SMBHB Candidates	68
5.1	Evidence of periodicity in J1309+1154	69
5.1.1	Harmonic in PKS J1309+1154	72
5.1.2	VLBA observations of PKS J1309+1154	74
5.1.3	ALMA Observations of PKS J1309+1154	77
5.1.4	Metsähovi observations of PKS J1309+1154	78
5.2	Multifrequency radio light curves of J1309+1154	80
5.3	Stochastic Modeling of Blazar Light Curves	81
5.3.1	Power Spectral Density and PSD Slope	82
5.3.2	Probability Density Function	86
5.3.3	Timmer and Koenig (TK)	88
5.3.4	Emmanoulopoulos method (EMP)	89
5.3.5	Statistical significance analysis	92
5.3.6	Local significance	92
5.3.7	Global significance	93
5.3.8	Significance for J1309+1154: Results	94
5.3.9	Discussion: Statistical significance in J1309+1154	95
6	Harmonic relations between GLSP peaks	97
6.1	Controlled Tests of the GLSP	98
6.1.1	Effect of the ζ parameter in the GLSP.	99
6.1.2	Transient Flares as a Source of Spurious GLSP Peaks	101
6.1.3	Effect of cadence on apparent harmonic relations	102
6.1.4	Inspection of special cases in real light curve GLSPs	103
6.2	Synthetic light curves	105
6.2.1	Test 1: Same amplitude, different periods, random phases.	105
6.2.2	Summary of Synthetic Light Curve Tests	109

6.2.3	Qualitative Interpretation of the Tests	110
6.3	Harmonic relations in blazar light curves	113
6.4	Harmonic counting algorithm	115
6.5	Results of the harmonic relation counts	120
7	Summary, Conclusions, and Outlook	124
7.1	Summary and Discussion	124
7.1.1	Periodicity search and candidate selection	124
7.1.2	Evidence supporting the periodicity of J1309+1154	125
7.1.3	Statistical significance and harmonic relations	126
7.1.4	PSD estimation and limitations	126
7.1.5	Light-curve simulations and shape effects	127
7.2	Conclusions	128
7.2.1	Publications arising from this research	130
7.3	Outlook and Future Work	131
	References	133
A	GLSP and WWZ analysis for the OVRO+UMRAO sample	140
B	GLSP Comparison for OVRO, UMRAO, and Combined Data	224
C	Synthetic Light Curves	246
C.0.1	Test 2: Same amplitude, different periods, random phases for each iteration.	246
C.0.2	Test 3: Different amplitudes, periods, and random phases.	248
C.0.3	Test 4: Same amplitude, different periods (5/7 factor), random phases.	251
C.0.4	Test 5: Single sinusoidal signal with red noise.	253
C.0.5	Test 6: Sum of two sinusoidal signals with varying amplitudes, periods, phases, and red noise.	255
D	Observed PSDs and Slopes (β)	257
E	Sine Wave Fitting Results	264
F	Proposed WWZ Metric as a Visual Guide for Inspecting a Sample of Light Curves	267
G	Periodicity Analysis Methods	272
G.0.1	Generalized Lomb–Scargle periodogram	272
G.0.2	Sinusoidal least squares fitting	276
G.0.3	Weighted Wavelet Z-Transform (WWZ)	277
H	A hint of a harmonic in PKS J1309+1154	283
I	Optical and infrared observations of PKS J1309+1154	286

List of Tables

4.1	Summary of temporal coverage, sampling, and flux-density statistics for the UMRAO, OVRO, and combined OVRO+UMRAO light curves.	51
4.2	Source Properties and Redshifts for the OVRO+UMRAO Sample.	52
4.3	Summary of the dominant GLSP period, its associated power, and their comparison with the combined light curves.	60
5.1	Period estimates for J1309+1154 from SWF, GLSP, and WWZ analyses.	70
5.2	Intrinsic power spectral density slopes for the OVRO+UMRAO sample, ordered by right ascension.	83
5.3	Summary of the statistical significance analysis for J1309+1154.	95
6.1	Harmonic counts for observed and simulated light curves.	122
6.2	Global harmonic counts for observed and simulated light curves.	123
E.1	Best-fit SWF parameters for the combined light curve of J1309+1154 using the sine-plus-trend model.	265
E.2	Best-fit Periods from Sine-wave Fits	266

List of Figures

2.1	Schematic of the main structures of an AGN. Redrawn from Ramos Almeida & Ricci (2017)	9
2.2	AGN unified scheme and the role of orientation. Schematic representation of the unified model for active galactic nuclei (AGN), illustrating the central engine (accretion disk and broad-line region), obscuration by circumnuclear dust, and the presence of a relativistic jet. The observed AGN class depends primarily on viewing angle and obscuration; blazars correspond to cases where the jet is closely aligned with the line of sight, producing strongly beamed emission. Redrawn from Beckmann & Shrader (2013) ; original graphic credit to Marie-Luise Menzel (MPE).	13
2.3	Schematic structure of a radio-loud quasar, illustrating the magnetic-field geometry and shock structures along its relativistic jet. Adapted from a figure by A. Marscher, https://www.bu.edu/blazars/	14
2.4	Schematic distribution of emission regions in a radio-loud AGN, including the millimeter-wave core and downstream jet. Adapted from a figure by A. Marscher, https://www.bu.edu/blazars/	14
2.5	Proposed model for the inner jet of BL Lacertae. Distances from the black hole are shown on a logarithmic scale in units of the Schwarzschild radius, R_s . Redrawn from Fig. 3 of Marscher et al. (2008)	15
2.6	Synchrotron radiation: a relativistic electron in a magnetic field follows a helical trajectory and emits radiation. The characteristic emission frequency increases with the electron energy (γ) and with the component of the magnetic field perpendicular to its velocity ($B_\perp$). Adapted from Wilhelm (2021)	17
2.7	Flux-density variability (Jy) comparison between the blazar PKS 1413+135 (black) and the compact symmetric object OQ 208 (purple), shown without uncertainty bars. OQ 208 remains substantially more stable during the time window shown (Kiehlmann et al., 2024).	19
2.8	Relativistic Doppler factor D as a function of viewing angle θ for fixed flow speeds (left) and Lorentz factor Γ for fixed viewing angles (right).	26

2.9	Simplified schematic of the kinematic orbital model for a supermassive black hole binary capable of producing sinusoidal flux-density variations in a blazar. The orbital motion of the jet-producing black hole changes the Doppler factor of the emitting material as measured by the observer. The figure was redrawn and adapted from O'Neill et al. (2022) and Kiehlmann et al. (2025). The purple and yellow colors are used solely to distinguish the orbits of the two binary components.	30
2.10	Schematic two-dimensional representation of the MKO model. The relativistic emitting plasma travels through a helical channel defined by a slower wind–jet interface whose direction is modified by the orbital motion. The regions associated with emission at 225 and 15 GHz are shown for illustrative purposes only. The figure was redrawn and adapted from Hincks et al. (2026) and Sullivan et al. (2026).	31
2.11	Schematic representation of the binary system, accretion disk, wind, and jet corresponding to the top panel of Figure 1 in Sullivan et al. (2026). The system consists of a primary black hole, H, and a secondary black hole, S. The primary is accompanied by an accretion disk, D, and launches a relativistic jet, J, approximately along the orbital angular momentum direction, L. The disk produces a wind, W, that collimates the jet. The purple curves represent the orbits of the binary components and are included solely as visual aids. The figure was redrawn and adapted from the original.	32
2.12	Geometric representation of the wind-collimated jet. The jet-producing primary black hole, with mass M_1 , moves with dimensionless orbital speed β_H , while M_2 is the mass of the secondary black hole. The jet is confined within a cone of opening angle θ_j centered on the $\hat{z}$ axis. The effective propagation speed of the wind–jet interface is represented by β_w , $\hat{r}$ denotes the local flow direction at angular position (χ, ψ) , and $\hat{n}$ points toward the observer, forming an angle i with the cone axis. The figure was redrawn and adapted from the middle panel of Figure 1 in Sullivan et al. (2026).	33
3.1	Radio telescopes providing the long-term monitoring used in this thesis. Left: the OVRO 40-m telescope (15 GHz) located in Owens Valley, California (USA). Right: the UMRAO 26-m telescope (14.5 GHz) located in Dexter, Michigan (USA).	35
3.2	Beam-switching integration states and beam positions. Redrawn from Readhead et al. (1989), following the conventions of Richards et al. (2011).	39

3.3	Block diagram of the Ku-band receiver used on the Owens Valley Radio Observatory (OVRO) 40 m telescope until 2014, based on Richards et al. (2011). The instrument employed a dual-beam, Dicke-switched radiometer architecture, in which a ferrite radio-frequency switch operating at 500 Hz alternately directed the signals from the antenna (<i>ant</i>) and reference (<i>ref</i>) beams to a common receiver chain.	42
3.4	Simplified schematic of the KuPol receiver installed on the OVRO 40 m telescope in 2014. The receiver uses a correlation architecture implemented with 180° hybrids and operates as a dual-beam continuous-comparison radiometer for total-intensity measurements. Signals with the same circular polarization from the antenna and reference horns are combined to form sum and difference outputs. The noise-injection diodes provide calibration signals and enable receiver-gain variations to be corrected, while the square-law detector diodes convert the received radio-frequency power into output voltages. Only the beam-differenced total-intensity measurements, corresponding to Stokes I , are used in this work. .	42
4.1	Skymap of the OVRO+UMRAO sample in Aitoff projection. The 83 combined sources are shown in red and numbered, while the OVRO-only sample is shown in gray. The purple dotted lines delineate the Galactic latitude band $ b < 20^\circ$	44
4.2	It corresponds to the full OVRO+UMRAO sample and allows us to visualize the number of observations over time and the range of flux densities in which they are distributed. Each hexagon represents a two-dimensional bin in the (MJD, flux) plane and indicates how many observations fall within it; the color of each hexagon is determined by that number of observations. The figure illustrates the long historical baseline provided by UMRAO and the denser sampling contributed by OVRO after 2008.	45
4.3	Example of a multifrequency radio light curve constructed from OVRO and UMRAO observations.	46
4.4	Histogram of the per-source median cadence. The red and blue, curves correspond to UMRAO, OVRO, respectively.	49
4.5	Start OVRO+UMRAO sample	50
4.6	End OVRO+UMRAO sample	50
4.7	Temporal baseline in Years	50
4.8	Number of observations	51
4.9	Mean flux density [Jy]	51
4.10	Mean flux uncertainties (σ_s) [Jy]	51
4.11	Distribution of redshifts for the OVRO+UMRAO sample. The green histogram shows the number of sources per redshift bin, and the black dashed line marks the sample median.	54
4.12	GLSP example. The detected main period, corresponding to the highest peak, is 11.59 years, with a power of 0.59.	55

4.13 Preliminary sinusoidal least-squares visualization of the GLSP periods for the source J1256–0547, also known as 3C 279. Left: GLSP. Right: light curve with fits for P_1 , P_2 , and P_3 shown in green, purple, and orange, respectively; red and blue points denote UMRAO and OVRO data.	56
4.14 Example of the Weighted Wavelet Z-transform (WWZ) analysis for the source J0818+4222. Left: three-dimensional representation of the WWZ spectrum. Right: conventional time and period representation with superimposed contours. The color scale indicates the WWZ amplitude. A persistent periodicity of approximately 12 years is observed, with increased power from ~ 2009 onward. This feature is fully consistent with the Generalized Lomb–Scargle (GLS) result shown in Figure 4.12. The WWZ was computed using $c = 0.0125$, closely approximating the value $c = 1/(8\pi^2)$ suggested by Foster (1996); this dimensionless decay parameter controls the Gaussian weighting around each central time τ and thus the trade-off between time and frequency resolution. A detailed description of the GLSP and WWZ methods is provided in Appendix G.	56
4.15 Example diagnostic panels for J0019+7327. The green, purple, and orange curves show the sinusoidal fits for the GLSP periods P_1 , P_2 , and P_3 , respectively. The complete set of panels for all sources is provided in Appendix A.	57
4.16 Distribution of the dominant period P_1 obtained from the GLSP analysis for UMRAO, OVRO, and OVRO+UMRAO, expressed in years. The columns show the three data configurations. The mean and median of each distribution are indicated in each panel. It can be observed that the UMRAO and OVRO light-curve distributions differ markedly, indicating that most sources do not show stable periodicity.	58
4.17 Distribution of the GLSP power $\mathcal{P}_1$ associated with the dominant period P_1 for UMRAO, OVRO, and OVRO+UMRAO. The columns show the three data configurations. The text boxes indicate the mean and median of each distribution. The similar power of the dominant peak across the individual and combined datasets indicates consistency, suggesting that differences in cadence and temporal baseline do not systematically affect the detected GLSP power.	58
4.18 Source-by-source comparison between UMRAO and OVRO+UMRAO for the dominant period P_1 and its associated GLSP power $\mathcal{P}_1$. The dashed line indicates the one-to-one relation. The complete absence of clustering of data along the dotted line shows very clearly that the periodicities identified by the GLSP method in these 83 blazars are not reliable and should be treated with great caution and skepticism.	59

4.19	GLSP power as a function of the number of cycles for the main peak, P_1 , for each sample: UMRAO-only in red, OVRO-only in blue, and the combined sample in green. The outline colors are included only as visual references. Three sources are shown as examples to illustrate how the GLSP power can vary depending on the observing epoch and/or the radio telescope used.	60
4.20	WWZ mosaic of the OVRO+UMRAO sample, ordered by right ascension. Peak values in the left-hand curves and the color scale are normalized to the sample's global maximum WWZ power. Although J1613+3412 exhibits the highest WWZ power, its lack of temporal persistence does not support a plausible periodic interpretation; therefore, the mosaic is intended primarily for comparative reference. Superscript "d" denotes the detrended light curve of J1309+1154. The complete set of panels is provided in Appendix A.	62
4.21	Global WWZ mosaic for OVRO+UMRAO. The curve on the left uses a proposed diagnostic metric conceived solely as a global visual guide. This metric considers WWZ power, period resolution, and temporal persistence to illustrate that these parameters are not independent when assessing a plausible periodicity. In this example, the metric was calibrated using the detrended light curve of J1309+1154, and the highest peaks are observed for J2134-0153 (PKS 2131-021) and J0818+4222. Further details are provided in Appendix F.	63
4.22	Global distributions derived from the WWZ analysis of the combined OVRO+UMRAO light curves. The panels show the maximum WWZ power normalized by the global maximum of the sample (top left), the absolute maximum WWZ power (top right), the period associated with this maximum (bottom left), and the year in which it occurs (bottom right). In the top-right panel, the shaded area identifies sources with $W_j^{\max}/W_{\text{global}}^{\max} \geq 0.30$	64
4.23	Illustrative multiwavelength view of J1309+1154, combining LoTSS/LOFAR emission at 144 MHz, the optical Pan-STARRS image, and the high-resolution VLBA/MOJAVE structure at 15 GHz. This comparison is intended as a visual guide and does not represent a strict angular-scale matching between the panels.	66
4.24	Multifrequency OVRO+UMRAO radio light curves of J1309+1154. The present analysis uses the high-frequency light curve, combining OVRO 15 GHz and UMRAO 14.5 GHz data. The curve also shows a noticeable long-term declining linear trend that must be considered in the periodicity analysis.	67
5.1	Unified GLSP, with red, blue, and green curves corresponding to UMRAO, OVRO, and the combined dataset, respectively.	70
5.2	Preliminary detrended analyses of J1309+1154 for UMRAO, OVRO, and the combined OVRO+UMRAO data. The sinusoidal fits use the three strongest GLSP peaks. The harmonic near ~ 9 yr is clearly visible in the UMRAO WWZ. The UMRAO WWZ is displayed using a square-root power scale, whereas the OVRO and combined OVRO+UMRAO WWZ plots are displayed using a linear power scale. See Appendix A for the full sample.	71

5.3	Top: Least-squares sinusoidal fit (green line) obtained using the SWF model described by Molina et al. (2026) (Equation E.3; Table E.1). Bottom: Sinusoidal fit to the residuals (purple line), with a period of approximately 9 years.	72
5.4	WWZ maps of PKS J1309+1154. Panel (a) shows the WWZ spectrum computed from the full UMRAO+OVRO dataset on a linear power scale, where the fundamental periodicity dominates and the harmonic is not clearly resolved. Panel (b) shows the UMRAO data alone from 1979 to 2012. Panels (c) and (d) show the UMRAO data with OVRO observations added up to 2015 and 2019, respectively. Panels (b), (c), and (d) are displayed using a square-root power scale, which enhances the weaker feature near ~ 9 yr, consistent with a first harmonic in a 2:1 relation with the fundamental period.	73
5.5	VLBA component for J1309+1154 (Lister et al., 2021).	75
5.6	Top: component separation from the core as a function of time (Lister et al., 2021). Bottom: UMRAO/OVRO light curve of J1309+1154 compared with MOJAVE total flux and VLBA component flux densities from DIFMAP model fitting. The core and nearby component 5 broadly follow the single-dish modulation, unlike the more distant jet components. (Molina et al., 2026) . . .	76
5.7	OVRO 15 GHz (blue) and ALMA Band 3 (purple) light curves of PKS J1309+1154. Solid curves show the least-squares sinusoidal fit to OVRO ($P = 6201$ days) and the fit to ALMA with this period held fixed. The ALMA fit has a temporal offset of -300^{+260}_{-200} days relative to OVRO, indicating that ALMA leads OVRO (Molina et al., 2026).	78
5.8	Preliminary periodicity analysis of J1309+1154 using the Metsähovi dataset: (a) GLSP; (b) light curve with sinusoidal fits corresponding to the three highest-power GLSP peaks (17.64, 8.87, and 1.96 yr, shown in green, purple, and orange, respectively); and (c) WWZ map. A secondary WWZ feature near 9 yr is consistent with a possible harmonic of the approximately 18 yr modulation.	79
5.9	Multifrequency radio light curves of J1309+1154 combining single-dish total-flux-density measurements from UMRAO (26-m telescope; 4.8, 8.0, and 14.5 GHz), OVRO (40-m telescope; 15 GHz), and Metsähovi (14-m telescope; 37 GHz), together with MOJAVE/VLBA measurements near 15 GHz and ALMA Band 3 observations at 91.5 and 103.5 GHz.	80
5.10	Illustrative approximation to the observed PSD of PKS J1309+1154, obtained using the Lomb–Scargle periodogram with Astropy’s <code>psd</code> normalization. This estimate remains affected by the finite duration and irregular sampling of the light curve and does not directly represent the intrinsic PSD.	85

5.11	Observed PSD of PKS J1309+1154, shown in green, obtained using the procedure described by O’Neill et al. (2022). The intrinsic PSD slope, β , was estimated through a forward Monte Carlo analysis using the data available through 2024. The PSDs corresponding to the 1000 simulations are shown as gray lines. For the significance analyses, a value of $\beta = 1.66$, estimated using data available through 2023, was adopted. Details of the method and corresponding plots for all 83 sources in the OVRO+UMRAO sample are presented in Appendix D.	85
5.12	Distribution of the intrinsic PSD slopes for the OVRO+UMRAO sample. The mean and median are indicated.	86
5.13	Empirical flux-density distribution of PKS J1309+1154 from the combined OVRO+UMRAO measurements.	87
5.14	Visual comparison of one observed light curve and three simulated realizations generated with the EMP algorithm. The observed panel is not identified.	90
5.15	Stacked light curves and corresponding probability density functions (PDFs) for J2134–0153 (upper panels) and J1309+1154 (lower panels), using TK (left) and EMP (right) simulations. Observed light curves and PDFs are shown in black, and simulations in color. The displayed TK realizations show approximately Gaussian PDFs, most clearly for J1309+1154. For J2134–0153, the TK realizations do not reach flux densities comparable to the isolated flare near the end of the observed light curve, whereas EMP closely reproduces the observed PDF and allows events of comparable amplitude throughout the simulated time baseline.	91
5.16	Stacked GLSPs of the EMP simulations (black), overlaid with the observed GLSP (green). The comparison illustrates the tests used to determine the local and global significance.	95
6.1	Numerical convergence of the dominant GLSP period of J1309+1154 with increasing oversampling parameter ζ , using the combined OVRO+UMRAO light curve.	99
6.2	Heatmap of the dominant GLSP period as a function of ζ for the 83 sources in the combined OVRO+UMRAO sample.	100
6.3	Light curve of AO 0235+164 using OVRO 40 m data. The orange box highlights a flare.	101
6.4	GLSP of AO 0235+164 computed using the full OVRO data set spanning 2008–2020, including the flare.	101
6.5	GLSP of AO 0235+164 computed after excluding the flare. This example illustrates that similar GLSP shapes in different epochs are not conclusive by themselves, given that the GLSP shape is highly sensitive to transient structures such as flares.	101

6.6	Left: GLSPs computed at different cadences. The green curve corresponds to simulation #1 of AO 0235+164 resampled at the observed cadence. Right: the corresponding light curves. Only marginal differences are observed between the GLSPs.	103
6.7	GLSP of J0019+7327 in frequency and period. OVRO-only is shown in blue, UMRAO-only in red, and OVRO+UMRAO in green dotted lines. The full set of sources is shown in Appendix B. . . .	104
6.8	Test 1: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 3000, e) 6000.	108
6.9	Test 6.1 ($A_2 = A_1$), for two values of the imposed primary period (5000 and 6000 days).	111
6.10	Test 6.2 ($A_2 = 2A_1$), for two values of the imposed primary period (5000 and 6000 days).	112
6.11	Mean powers detected in the six strongest GLSP peaks of the observed light curves, shown in green, compared with those obtained from the simulated light curves, shown in gray, for different PSD slopes. Panels correspond to: (a) $\beta = 1.5$, which shows the closest agreement between the distributions; (b) $\beta = 1.9$; and (c) $\beta = 2.5$	114
6.12	Flow diagram of the harmonic counting algorithm. For each light curve, the GLSP is computed with $\zeta = 29$ and $P > 100$ days, the six strongest peaks are selected, all 15 period ratios are tested against the adopted harmonic families, and the accepted independent relations are assigned to canonical classes.	119
A.1	J0010+1058, (RA = 2.629°, DEC = 10.975°), $z = 0.089$	141
A.2	J0019+7327, (RA = 4.941°, DEC = 73.458°), $z = 1.781$	142
A.3	J0050-0929, (RA = 12.672°, DEC = -9.485°), $z = 0.635$	143
A.4	J0102+5824, (RA = 15.691°, DEC = 58.403°), $z = 0.644$	144
A.5	J0108+0135, (RA = 17.162°, DEC = 1.583°), $z = 2.109$	145
A.6	J0111+3906, (RA = 17.905°, DEC = 39.108°), $z = 0.668$	146
A.7	J0112+2244, (RA = 18.024°, DEC = 22.744°), $z = 0.265$	147
A.8	J0136+4751, (RA = 24.244°, DEC = 47.858°), $z = 0.859$	148
A.9	J0204+1514, (RA = 31.210°, DEC = 15.236°), $z = 0.405$	149
A.10	J0217+7349, (RA = 34.378°, DEC = 73.826°), $z = 2.346$	150
A.11	J0217+0144, (RA = 34.454°, DEC = 1.747°), $z = 1.715$	151
A.12	J0228+6721, (RA = 37.209°, DEC = 67.351°), $z = 0.523$	152
A.13	J0237+2848, (RA = 39.468°, DEC = 28.802°), $z = 1.206$	153
A.14	J0238+1636, (RA = 39.662°, DEC = 16.616°), $z = 0.940$	154
A.15	J0259+0747, (RA = 44.863°, DEC = 7.794°), $z = 0.893$	155
A.16	J0303+4716, (RA = 45.897°, DEC = 47.271°), $z = \text{U}$	156
A.17	J0309+1029, (RA = 47.265°, DEC = 10.488°), $z = 0.862$	157
A.18	J0319+4130, (RA = 49.951°, DEC = 41.512°), $z = 0.018$	158
A.19	J0336+3218, (RA = 54.125°, DEC = 32.308°), $z = 1.259$	159
A.20	J0339-0146, (RA = 54.879°, DEC = -1.776°), $z = 0.847$	160
A.21	J0418+3801, (RA = 64.589°, DEC = 38.027°), $z = 0.049$	161
A.22	J0423-0120, (RA = 65.816°, DEC = -1.343°), $z = 0.913$	162

A.23 J0424+0036, (RA = 66.195°, DEC = 0.602°), $z = 0.268$	163
A.24 J0433+0521, (RA = 68.296°, DEC = 5.354°), $z = 0.033$	164
A.25 J0501-0159, (RA = 75.303°, DEC = -1.987°), $z = 2.286$	165
A.26 J0530+1331, (RA = 82.735°, DEC = 13.532°), $z = 2.070$	166
A.27 J0607-0834, (RA = 91.999°, DEC = -8.581°), $z = 0.872$	167
A.28 J0609-1542, (RA = 92.421°, DEC = -15.711°), $z = 0.324$	168
A.29 J0721+7120, (RA = 110.473°, DEC = 71.343°), $z = \text{U}$	169
A.30 J0730-1141, (RA = 112.580°, DEC = -11.687°), $z = 1.591$	170
A.31 J0738+1742, (RA = 114.531°, DEC = 17.705°), $z = \text{U}$	171
A.32 J0739+0137, (RA = 114.825°, DEC = 1.618°), $z = 0.189$	172
A.33 J0757+0956, (RA = 119.278°, DEC = 9.943°), $z = 0.266$	173
A.34 J0808+4950, (RA = 122.165°, DEC = 49.843°), $z = 1.435$	174
A.35 J0818+4222, (RA = 124.567°, DEC = 42.379°), $z = \text{U}$	175
A.36 J0831+0429, (RA = 127.954°, DEC = 4.494°), $z = 0.175$	176
A.37 J0854+2006, (RA = 133.704°, DEC = 20.108°), $z = 0.306$	177
A.38 J0958+6533, (RA = 149.697°, DEC = 65.565°), $z = 0.367$	178
A.39 J1058+0133, (RA = 164.623°, DEC = 1.566°), $z = 0.891$	179
A.40 J1104+3812, (RA = 166.114°, DEC = 38.209°), $z = 0.030$	180
A.41 J1150+2417, (RA = 177.580°, DEC = 24.298°), $z = 0.209$	181
A.42 J1159+2914, (RA = 179.883°, DEC = 29.245°), $z = 0.725$	182
A.43 J1217+3007, (RA = 184.467°, DEC = 30.117°), $z = 0.130$	183
A.44 J1221+2813, (RA = 185.382°, DEC = 28.233°), $z = 0.102$	184
A.45 J1224+2122, (RA = 186.227°, DEC = 21.380°), $z = 0.433$	185
A.46 J1229+0203, (RA = 187.278°, DEC = 2.052°), $z = 0.158$	186
A.47 J1256-0547, (RA = 194.047°, DEC = -5.789°), $z = 0.536$	187
A.48 J1305-1033, (RA = 196.388°, DEC = -10.555°), $z = 0.278$	188
A.49 J1309+1154, (RA = 197.391°, DEC = 11.907°), $z = \text{U}$	189
A.50 J1310+3220, (RA = 197.619°, DEC = 32.345°), $z = 0.995$	190
A.51 J1337-1257, (RA = 204.416°, DEC = -12.957°), $z = 0.539$	191
A.52 J1415+1320, (RA = 213.995°, DEC = 13.340°), $z = 0.247$	192
A.53 J1419+5423, (RA = 214.944°, DEC = 54.387°), $z = 0.152$	193
A.54 J1512-0905, (RA = 228.211°, DEC = -9.100°), $z = 0.360$	194
A.55 J1540+1447, (RA = 235.206°, DEC = 14.796°), $z = 0.605$	195
A.56 J1555+1111, (RA = 238.929°, DEC = 11.190°), $z = 0.433$	196
A.57 J1613+3412, (RA = 243.421°, DEC = 34.213°), $z = 1.399$	197
A.58 J1635+3808, (RA = 248.815°, DEC = 38.134°), $z = 1.815$	198
A.59 J1642+6856, (RA = 250.533°, DEC = 68.944°), $z = 0.751$	199
A.60 J1642+3948, (RA = 250.745°, DEC = 39.810°), $z = 0.593$	200
A.61 J1653+3945, (RA = 253.468°, DEC = 39.760°), $z = 0.034$	201
A.62 J1719+1745, (RA = 259.804°, DEC = 17.752°), $z = 0.137$	202
A.63 J1733-1304, (RA = 263.261°, DEC = -13.080°), $z = 0.902$	203
A.64 J1740+5211, (RA = 265.154°, DEC = 52.195°), $z = 1.379$	204
A.65 J1743-0350, (RA = 265.995°, DEC = -3.834°), $z = 1.054$	205
A.66 J1748+7005, (RA = 267.137°, DEC = 70.097°), $z = 0.770$	206
A.67 J1751+0939, (RA = 267.887°, DEC = 9.650°), $z = 0.320$	207

A.68	J1800+7828, (RA = 270.190°, DEC = 78.468°), $z = 0.680$	208
A.69	J1806+6949, (RA = 271.711°, DEC = 69.824°), $z = 0.051$	209
A.70	J1824+5651, (RA = 276.029°, DEC = 56.850°), $z = 0.663$	210
A.71	J1927+7358, (RA = 291.952°, DEC = 73.967°), $z = 0.302$	211
A.72	J2005+7752, (RA = 301.379°, DEC = 77.879°), $z = 0.342$	212
A.73	J2007+4029, (RA = 301.937°, DEC = 40.497°), $z = 1.736$	213
A.74	J2022+6136, (RA = 305.528°, DEC = 61.616°), $z = 0.227$	214
A.75	J2123+0535, (RA = 320.935°, DEC = 5.589°), $z = 1.941$	215
A.76	J2134-0153, (RA = 323.543°, DEC = -1.888°), $z = 1.283$	216
A.77	J2158-1501, (RA = 329.526°, DEC = -15.019°), $z = 0.672$	217
A.78	J2225-0457, (RA = 336.447°, DEC = -4.950°), $z = 1.404$	218
A.79	J2232+1143, (RA = 338.152°, DEC = 11.731°), $z = 1.037$	219
A.80	J2236+2828, (RA = 339.094°, DEC = 28.483°), $z = 0.795$	220
A.81	J2253+1608, (RA = 343.491°, DEC = 16.148°), $z = 0.859$	221
A.82	J2257+0743, (RA = 344.322°, DEC = 7.720°), $z = 0.190$	222
A.83	J2348-1631, (RA = 357.011°, DEC = -16.520°), $z = 0.576$	223
B.1	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0010+1058, J0019+7327, J0050-0929, and 0059+581.	225
B.2	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0108+0135, J0111+3906, J0112+2244, and J0136+4751.	226
B.3	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0204+1514, J0217+7349, J0217+0144, and 0224+671.	227
B.4	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0237+2848, J0238+1636, J0259+0747, and 0300+471.	228

B.5	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0309+1029, J0319+4130, 0333+321, and J0339–0146.	229
B.6	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: 0415+379, J0423–0120, C0424+0036, and J0433+0521.	230
B.7	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0501–0159, J0530+1331, J0607–0834, and J0609–1542.	231
B.8	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0721+7120, PKS0727–115, J0738+1742, and J0739+0137.	232
B.9	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0757+0956, J0808+4950, J0818+4222, and J0831+0429.	233
B.10	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0854+2006, J0958+6533, J1058+0133, and J1104+3812.	234
B.11	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1150+2417, J1159+2914, J1217+3007, and J1221+2813.	235

B.12 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: C1224+2122, J1229+0203, J1256-0547, and J1305-1033.	236
B.13 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1309+1154, J1310+3220, J1337-1257, and J1415+1320.	237
B.14 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1419+5423, PKS1510-089, J1540+1447, and J1555+1111.	238
B.15 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1613+3412, J1635+3808, J1642+6856, and J1642+3948.	239
B.16 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1653+3945, J1719+1745, J1733-1304, and J1740+5211.	240
B.17 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1743-0350, J1748+7005, J1751+0939, and J1800+7828.	241
B.18 Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1806+6949, J1824+5651, J1927+7358, and J2005+7752.	242

B.19	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: 2005+403, J2022+6136, J2123+0535, and J2134–0153.	243
B.20	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J2158–1501, C2225–0457, 2230+114, and J2236+2828.	244
B.21	Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J2253+1608, 2254+074, and J2348–1631.	245
C.1	Test 2: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 3000, e) 6000.	247
C.2	Test 3: For a simulated period (in days): a) 100, b) 1000.	249
C.3	Test 3: For a simulated period (in days): c) 2000, d) 3000, e) 4000, f) 6000.	250
C.4	Test 4: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 4000, e) 6000.	252
C.5	Test 5: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 4000, e) 6000.	254
C.6	Test 6: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 4000, e) 6000.	256
D.1	Observed PSDs and estimated β values for sources 1–15 of the 83-source sample.	258
D.2	Observed PSDs and estimated β values for sources 16–30 of the 83-source sample.	259
D.3	Observed PSDs and estimated β values for sources 31–45 of the 83-source sample.	260
D.4	Observed PSDs and estimated β values for sources 46–60 of the 83-source sample.	261
D.5	Observed PSDs and estimated β values for sources 61–75 of the 83-source sample.	262
D.6	Observed PSDs and estimated β values for sources 76–83 of the 83-source sample.	263
G.1	GLSP example. The detected main period, corresponding to the highest peak, is 22.42 years, with a power of 0.7.	275

G.2	3D views of the WWZ results. Panel (a), corresponding to the source J1058+0133, shows a single peak near 1997 and a very low-power periodic feature at a period of approximately 15 years. Panel (b), corresponding to J1309+1154, shows a clear periodic feature near 18 years that persists throughout the temporal baseline. Panels (c) and (d) correspond to PKS 2131–021 (J2134–0153): panel (c) covers the interval from July 2003 to November 2023, while panel (d) shows the full temporal range.	281
G.3	Example of a Weighted Wavelet Z-transform (WWZ) map of J0818+4222 with contours, computed using a decay parameter of $c = 0.003$. The map exhibits limited temporal localization but good frequency (or period) resolution.	282
G.4	Example of a Weighted Wavelet Z-transform (WWZ) map of J0818+4222 with contours, showing a periodic signal that has persisted over the past ~ 11.1 years. The adopted decay parameter is $c = 0.0125$, a commonly used value originally adopted by Foster (1996).	282
G.5	Example of a Weighted Wavelet Z-transform (WWZ) map of J0818+4222 with contours, computed using a decay parameter of $c = 0.1$. The map exhibits better temporal localization but poorer frequency (or period) resolution.	282
H.1	Schematic model of the SMBHB system proposed by Readhead et al. (2026), in which both black holes contribute relativistic jets of different intensities. The dominant jet produces the main periodic modulation, while the weaker component can introduce an additional modulation compatible with the observed harmonic structure. Reproduced from Readhead et al. (2026).	284
I.1	Optical and near-infrared light curves of PKS J1309+1154. The optical data include CRTS and ZTF observations in the <i>gri</i> bands, while the infrared data correspond to WISE W1 and W2 photometry. The W1 and W2 magnitudes are shifted upward by 3 mag for visual comparison. Although long-term variability is present, the current optical and infrared data do not show a clear coherent modulation at the 17.9 yr radio period Molina et al. (2026).	287

Chapter 1

Introduction

Active galactic nuclei (AGN) are among the most luminous and variable sources in the Universe. Their observed behaviour is ultimately powered by accretion onto supermassive black holes and, in many systems, shaped by relativistic outflows. In jet-dominated objects such as blazars, variability becomes a central observable: it carries information about the dynamics and geometry of the emitting region and about physical processes operating at scales that are otherwise extremely difficult to access. In particular, long-term periodic or quasi-periodic patterns in AGN light curves are of special interest because they can encode characteristic time-scales associated with sub-parsec structure near the central engine.

Time-domain signatures are especially valuable when contrasted with the limits of direct imaging. Even very long baseline interferometry (VLBI) at (sub-)millimeter wavelengths achieves angular resolutions of order $\sim 20 \mu\text{as}$, as demonstrated by the Event Horizon Telescope, yet the relevant emitting regions and dynamical scales for many variability processes can remain unresolved ([Event Horizon Telescope Collaboration et al., 2019](#)). Consequently, detecting and validating periodicity in monitoring data offers a complementary route to probe the innermost AGN environment.

A major motivation for searching for long-term periodicity in AGN is the possibility that some signals are linked to supermassive black hole binaries (SMBHBs). In the context of hierarchical galaxy evolution, SMBHBs are expected to form following galaxy mergers and to spend appreciable time in stages where they can modulate accretion flows and/or jet behaviour, making them plausible drivers of quasi-

periodic variability in at least a subset of sources (Begelman et al., 1980). In parallel, SMBHB populations are widely regarded as prime contributors to a stochastic nanohertz gravitational wave (GW) background, with characteristic time-scales of months to years that are now being probed by pulsar timing arrays (Agazie et al., 2023; EPTA Collaboration, 2023; Miles et al., 2023; Agarwal et al., 2026).

While periodic or quasi-periodic behaviour can also arise from alternative mechanisms such as variability induced by warped or precessing accretion flows (Britzen et al., 2018; Abraham, 2018) or jet precession associated with misalignment between the black hole spin axis and the accretion disc (Caproni et al., 2004), these possibilities are not explored further in this thesis. Instead, motivated by the theoretical expectation for SMBHBs and their potential relevance to the GW background, the working assumption adopted here is that any robust periodicities identified in our study are associated with SMBHB-related processes.

Moreover, if AGN light curves are often dominated by red-noise-like stochastic variability and are observed with irregular sampling, then, particularly when the time baseline is short, the cadence is poor, or the uncertainties are not well controlled, periodicity searches may yield false positives. For this reason, this thesis adopts a deliberately rigorous, and when necessary conservative, strategy to evaluate any detected periodicity.

The approach exploits an exceptionally long and densely sampled radio monitoring data set formed by combining measurements from the OVRO 40 m telescope at 15 GHz and the UMRAO 26 m telescope at 14.5 GHz Richards et al. (2011); Aller et al. (1985, 2017). This unique time baseline enables a population-level search for periodicity and harmonic structure in a well-defined sample of blazars, supported by complementary time-series tools, simulation-based significance testing tailored to stochastic variability, and high-resolution VLBA imaging to connect candidate periodicities with parsec-scale morphology and the most plausible origin of the radio emission.

Within this scientific framework, it is important to define the scope of the present research and distinguish its original contributions from the supporting material. Chapters 4, 5, 6 and 7 present the original research conducted as part of this doctoral thesis. Whenever methods developed by third parties are employed,

their sources are identified through the appropriate citations throughout the text. The supporting scientific background provides the conceptual framework and methodological foundation required for this research. Although this material draws on existing knowledge, it has been selected, organized and written specifically to address the objectives of this thesis.

1.1. Scientific context

Long-term periodicity searches in AGN sit at the intersection of accretion physics, jet dynamics, rigorous time-series inference, and multi-messenger constraints on supermassive black hole growth.

From a physical standpoint, the extraordinary luminosities of AGN arise because accretion can convert gravitational binding energy into radiation with high efficiency: angular momentum transport and viscous dissipation power bright continuum emission, while collimated, relativistic jets provide an additional channel for energy release and for transferring variability outward into compact radio-emitting regions. In blazars, Doppler beaming further amplifies both flux and variability, making them natural laboratories for time-domain studies of the central engine.

In X-ray binary systems with stellar-mass black holes, temporal phenomenology (including QPOs and state-dependent noise components) has been used to link the shape of the light curve and the power spectrum with changes in accretion geometry and dynamic timescales in strong gravity (Remillard & McClintock, 2006). Extending this approach to AGN is conceptually natural (McHardy et al., 2006), but observationally demanding: the characteristic timescales associated with SMBHs can span from years to decades, and the disk, corona, and jet can contribute differently depending on the wavelength and the observing epoch.

Within this long-timescale regime, persistent AGN periodicity is of particular interest as it may signal an SMBHB, a natural consequence of galaxy mergers (Begelman et al., 1980). This search is strongly motivated by recent Pulsar Timing Array (PTA) evidence for a nanohertz gravitational-wave background from inspiraling SMBHBs (Agazie et al., 2023). If confirmed, electromagnetic periodicities would offer crucial complementary constraints on binary, dynamics

and demographics and their influence on accretion and jets.

Despite this strong motivation, robustly identifying AGN periodicity remains challenging. Light curves are typically dominated by stochastic red noise, which, combined with uneven sampling and multi-trial statistical penalties, can easily produce false periodogram peaks (Lobanov & Roland, 2005). Furthermore, physical interpretation is non-unique: a true periodic signal could arise from an SMBHB, but also from jet precession, disk warping, or quasi-periodic instabilities. Because direct sub-parsec imaging across large samples is not routinely feasible, even with modern VLBI capabilities (Event Horizon Telescope Collaboration et al., 2019), distinguishing these scenarios relies heavily on rigorous time-domain analysis.

Decades-long radio monitoring programs provide the essential baselines needed to overcome these ambiguities. By combining these extensive datasets with stringent red-noise controls, it becomes possible to systematically evaluate how common coherent multi-year signals actually are, test their phase stability over multiple cycles, and cross-reference candidates with contemporaneous VLBI kinematics (Richards et al., 2011). This systematic approach is crucial to translate candidate periodicities into reliable constraints on SMBHB, dynamics, as well as demographics and AGN physics.

1.1.1 Hypothesis

Searches for periodicity in blazars have historically been methodologically delicate: in the presence of red-noise-dominated variability and irregular sampling, time series with limited duration or cadence can produce apparent peaks and incomplete cycles that invite ambiguous interpretations. For this reason, a sensible starting point is to assume that if long-term, physically-driven periodic modulation exists, it emerges only in a small fraction of sources once stringent robustness criteria are applied.

Under this framework, the working hypothesis of this thesis is that long-term periodic modulation, potentially driven by SMBHB dynamics, is a genuine but rare phenomenon among blazars. Consequently, applying a deliberately conservative time-series analysis to the combined, multi-decade OVRO and UMRAO datasets will successfully decouple true year-scale periodicities from red-noise artifacts,

enabling us to:

- (i) isolate a small, robust sample of candidates exhibiting year-scale periodic or quasi-periodic modulation; and
- (ii) constrain the population-level incidence (and place upper limits) of such behaviour, including any associated harmonic structure.

1.2. Research Objectives

1.2.1 General Objective

Develop and apply a robust time-domain analysis framework to the combined OVRO and UMRAO datasets to identify plausible SMBHB candidates, decouple them from stochastic variability, and quantify their population-level incidence.

1.2.2 Specific Objectives

1. Construct a homogeneous OVRO+UMRAO light-curve data set for the full sample, ensuring consistent time systems, flux-density scales, uncertainties, and quality control.
2. Perform a systematic search for long-term periodic or quasi-periodic modulation in the light curves using complementary time-series approaches (e.g., sinusoidal modelling, GLSP, and time-frequency methods such as WWZ).
3. Use high-resolution VLBA imaging, when available, to provide parsec-scale morphological and kinematic context for key candidates, helping to assess whether the modulation is plausibly linked to the nuclear region and/or jet structure.
4. Assess the robustness of candidate periodicities by testing their persistence across time baselines and by comparing results obtained from OVRO-only, UMRAO-only, and combined OVRO+UMRAO data products.

5. Establish a conservative statistical significance framework tailored to red-noise variability, explicitly accounting for sampling, trial factors, and the potential for spurious periodogram features.
6. Develop and apply a harmonic scanning algorithm, informed by per-source simulation ensembles, to evaluate the prevalence and stability of harmonic structure and to estimate realistic rates of spurious harmonic relations.
7. Summarize population-level constraints by quantifying the frequency (or upper limits) of long-term periodic or quasi-periodic behaviour in the monitored blazar sample, together with any associated harmonic structure.

Chapter 2

AGN Physics: Unified Model, Emission, and Variability

A black hole is a region of spacetime where gravity is so extreme that nothing, not even light, can escape once it crosses the boundary known as the event horizon. Physically, a black hole is characterized by three classical parameters: mass, electric charge and angular momentum. Depending on their mass, black holes are broadly classified into three categories: stellar-mass black holes ($\sim 3\text{--}100 M_\odot$), which are the end products of massive stellar evolution; intermediate-mass black holes ($\sim 10^2\text{--}10^5 M_\odot$), whose formation channels remain an open question; and supermassive black holes ($\gtrsim 10^6 M_\odot$), which reside at the centers of virtually all massive galaxies and whose origin is thought to be linked to the early assembly of large-scale structure in the Universe. It is this last class that lies at the heart of the phenomena discussed in this chapter.

The unified model of an AGN consists of a stratified architecture that can be divided into two concentric zones: the central engine and the surrounding structures (see Figure 2.1).

The central domain includes the supermassive black hole, the accretion disk, and the corona. The black hole represents the gravitational potential well around which the entire system is organized. Gas falling toward it forms an accretion disk, where viscous dissipation raises the temperature and converts gravitational energy into radiation, mainly in the ultraviolet and X-ray bands, with very high efficiency compared with other astrophysical processes. Above this disk, an ultrahot plasma

corona, with temperatures of the order of 10^9 K, scatters disk photons through inverse Compton emission, producing the characteristic X-ray component of the AGN spectrum. At typical distances of 0.01 to 0.1 pc, dense and high-velocity gas clouds produce broad spectral lines. This region, known as the broad-line region or BLR, acts as a kinematic tracer of the innermost gravitational field.

The second domain begins with the dusty torus, a thick structure of gas and dust surrounding the central system on scales from parsecs to tens of parsecs. Its role is crucial from an observational point of view: depending on the viewing angle, the torus can obscure the direct emission from the accretion disk and the BLR. This naturally explains the dichotomy between type 1 AGN, such as quasars and Seyfert I galaxies, where broad lines are observed, and type 2 AGN, such as Seyfert II galaxies, where the BLR is hidden and narrow lines dominate. These lines are produced by lower-density gas ionized by photons escaping from the central system, forming the narrow-line region or NLR, which can extend to hundreds of parsecs and can be detected with conventional optical telescopes.

In many AGN, a relativistic jet of collimated plasma is launched approximately perpendicular to the disk, reaching velocities close to c . This jet transports mechanical energy to kiloparsec or even megaparsec scales, where it can deposit that energy into large radio lobes through its interaction with the intergalactic medium. Across both domains, multiphase winds and outflows connect the active nucleus with the host galaxy, regulating both the gas reservoir available for accretion and the stellar evolution of the host system.

From the standpoint of radiative transfer, the structures of an AGN are not equally transparent. The accretion disk is usually modeled as optically thick, whereas the hot corona is comparatively optically thin and allows inverse Compton scattering of disk photons. The BLR contains dense gas that may be optically thick in certain lines or against ionizing radiation, yet does not form a continuous envelope. By contrast, the dust torus is the primary optically thick structure in the unified model, capable of obscuring both the disk and the BLR depending on the viewing angle (see Figure 2.2). The NLR, being more extended and of lower density, is comparatively optically thin, while jets can transition from optically thick to optically thin depending on the observed frequency and on the distance from the nucleus.

An Active Galactic Nucleus (AGN) is the extraordinarily compact and luminous central region found in a fraction of galaxies across the Universe. Unlike the radiation produced by stellar populations, the immense luminosity of an AGN originates in the release of gravitational potential energy as matter spirals inward onto a supermassive black hole (SMBH, 10^6 – $10^{10} M_\odot$) sitting at the base of the host galaxy’s gravitational well. This energy-conversion mechanism is so efficient that AGN rank among the most persistent and luminous electromagnetic sources known.

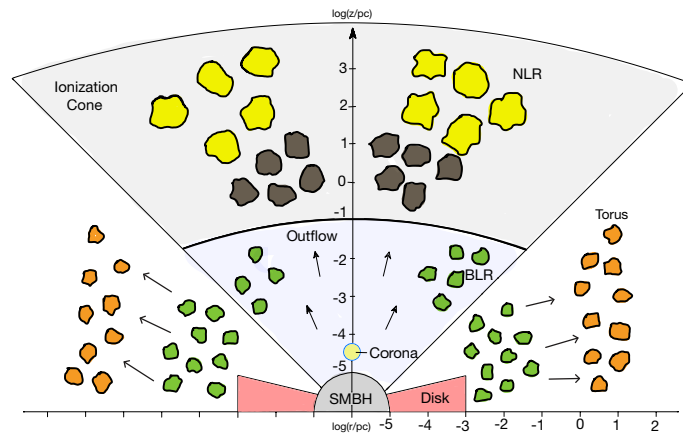


Figure 2.1: Schematic of the main structures of an AGN. Redrawn from [Ramos Almeida & Ricci \(2017\)](#)

Their emission is, however, far from steady. Flux density variations spanning timescales from minutes to decades are a defining observational hallmark of these objects, and they arise as a natural consequence of the stochastic nature of accretion. Magnetohydrodynamic instabilities within the disk, density contrasts in the surrounding medium, and sporadic tidal disruption events all conspire to produce a fluctuating mass inflow rate [Padovani et al. \(2017\)](#), which is then imprinted directly onto the observed radiative output.

This intrinsic variability is, in turn, one of the most physically informative windows into AGN physics. The processes that drive flux changes unfold on sub-parsec scales that remain beyond the reach of direct imaging even with microarcsecond resolution instruments such as the *Event Horizon Telescope*. Reconstructing the causal chain that links the morphological structure of the central engine with its photometric behavior therefore demands temporal inference and a rigorous statistical framework. Characterizing how, how much, and on what timescales AGN vary is thus an essential prerequisite for identifying and interpreting persistent

quasi-periodic signals or apparent periodicities in observed light curves (Ulrich et al., 1997; Vaughan, 2005).

Among AGN subtypes, blazars present a regime in which this temporal reading becomes exceptionally powerful. In these sources the continuum is overwhelmingly dominated by a relativistic plasma jet oriented within a small angle of the observer’s line of sight, a geometry that brings Doppler boosting to the fore. As a result, modest intrinsic fluctuations are amplified into large and rapid excursions in the observed flux, while the associated timescales contract correspondingly. For a flow described by the Doppler boosting factor:

$$\delta = \frac{1}{\Gamma (1 - \beta \cos \theta)} \quad (2.1)$$

where Γ is the bulk Lorentz factor, β the plasma speed in units of c , and θ the viewing angle, observer-frame timescales transform as $t_{\text{obs}} \simeq (1 + z) t' / \delta$, while the flux scales as $S_\nu \propto \delta^p S'_{\nu'}$, with the index p set by the emission geometry and the intrinsic spectral slope.

The interplay between viewing angle, internal jet dynamics, and the thermodynamic evolution of the emitting plasma can therefore account for much of the observed variability across the entire blazar spectral energy distribution, from radio wavelengths to gamma rays (Urry & Padovani, 1995; Wilkinson et al., 1979; Lister et al., 2009).

AGN can reach extreme luminosities owing to their high efficiency in converting gravitational energy into radiation. In its most compact form, the luminosity can be written as:

$$L \simeq \eta \dot{M} c^2 \quad (2.2)$$

where $\dot{M}$ is the mass accretion rate and η is the radiative efficiency, which is typically of order 0.1 for a thin disk, with its precise value depending on the black hole spin (Shakura & Sunyaev, 1973; Rees, 1984). To contextualize this radiative power relative to the SMBH mass, the Eddington limit provides a fundamental scale. It corresponds to the maximum luminosity at which the outward force produced by radiation pressure on free electrons transmitted to the ionized gas through electrostatic coupling between electrons and ions balances the inward gravitational force. Under the assumptions of steady, spherically symmetric accretion and opacity dominated by Thomson scattering in fully ionized hydrogen,

the Eddington luminosity is:

$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T} \approx 1.26 \times 10^{38} \left(\frac{M}{M_\odot} \right) \text{ erg s}^{-1} \quad (2.3)$$

where G is the gravitational constant, M is the mass of the central object, m_p is the proton mass, c is the speed of light, σ_T is the Thomson scattering cross section for electrons, and $M_\odot$ is the solar mass. The ratio L/L_{Edd} , known as the *Eddington ratio*, summarizes the accretion regime and the apparent radiative efficiency of the system. On cosmological scales, this dimensionless parameter becomes crucial: evidence that SMBHs grow primarily via accretion and release energy over the history of the Universe connects directly with integrated energy budget arguments (Sołtan, 1982), reinforcing the paradigm that accretion and its associated feedback are central to the coevolution of SMBHs and their host galaxies.

2.1. Why AGN vary across timescales

AGN emission is rarely constant; its flux varies on timescales ranging from minutes to decades. This variability does not have a single cause, but rather emerges from the superposition of physical processes with different characteristic timescales and stochastic nature.

In the accretion disk, the characteristic scales can be organized in terms of (i) a dynamical time $t_{\text{dyn}} \sim \Omega^{-1}$, (ii) a thermal time associated with pressure relaxation and dissipation, and (iii) a viscous time linked to angular momentum transport. The rate at which the black hole captures material fluctuates due to magnetorotational instabilities or variations in the density of the environment (Padovani et al., 2017). These nonlinear couplings often produce “red noise” power spectra, where more power resides at low frequencies (long timescales) (Ulrich et al., 1997; Vaughan, 2005).

Because of this, variability encodes physical processes on sub-parsec scales that can rarely be resolved directly. In systems dominated by a relativistic jet oriented toward the observer (blazars), this temporal reading is even more powerful. Doppler boosting amplifies the flux and compresses timescales, as shown by equation 2.1.

A fundamental methodological challenge in this context is to distinguish between stochastic fluctuations consistent with red noise and genuine quasi-periodic signals. Periodograms applied to red-noise light curves with irregular sampling often display apparent peaks that do not represent physical periodicities; therefore, the use of realistic stochastic null hypotheses and control of the trial factor (the *look-elsewhere effect*) are imperative (O’Neill et al., 2022).

2.1.1 Orientation-based unification schemes

In orientation-based unification schemes, the main difference lies in whether the line of sight intersects the circumnuclear dusty torus or not (separating Type 1 from Type 2 AGN). For the *radio-loud* subpopulation, orientation with respect to the jet is critical: when the jet points almost directly toward the observer, Doppler boosting dominates and produces the blazar phenotype; at larger inclination angles, the object is observed as a steep-spectrum radio quasar or a radio galaxy (FR I or FR II) (see Fig. 2.2).

This perspective is fundamental for variability studies. Since t_{obs} is compressed and the amplitude is amplified by the factor δ , any population-level inference about long-term periodicities must consider that blazars are a population strongly biased toward high observed variability.

2.1.2 Relativistic jets and emission processes

Relativistic jets are a characteristic feature of radio-loud AGN (Fig. 2.2) and dominate the observed non-thermal emission of blazars through relativistic Doppler boosting. Their collimated structure is resolved with very long baseline interferometry (VLBI), while variability and polarization provide information on the emitting plasma.

Figure 2.3 illustrates the central regions of a radio-loud quasar and its jet. The accretion disk emits mainly at optical and ultraviolet wavelengths, while inverse Compton scattering in the hot corona can produce X-rays. In magnetically driven jet models, an ordered, approximately helical magnetic field contributes to the acceleration and collimation of the flow. Farther downstream, shocks

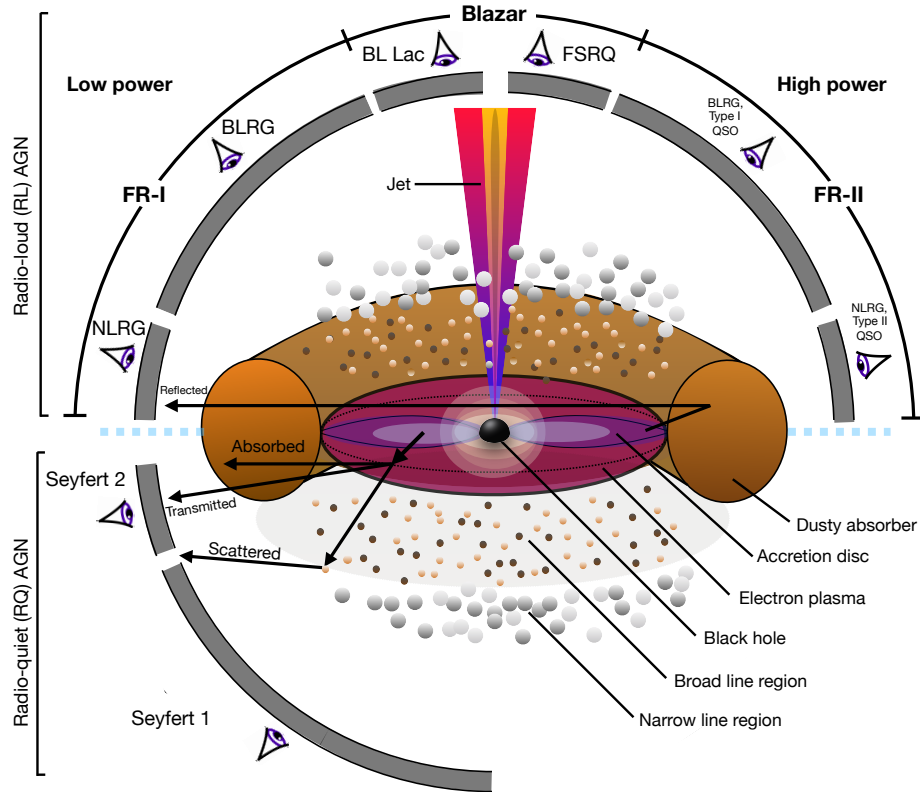


Figure 2.2: AGN unified scheme and the role of orientation. Schematic representation of the unified model for active galactic nuclei (AGN), illustrating the central engine (accretion disk and broad-line region), obscuration by circumnuclear dust, and the presence of a relativistic jet. The observed AGN class depends primarily on viewing angle and obscuration; blazars correspond to cases where the jet is closely aligned with the line of sight, producing strongly beamed emission. Redrawn from Beckmann & Shrader (2013); original graphic credit to Marie-Luise Menzel (MPE).

can accelerate particles and enhance non-thermal emission, while turbulence can reduce the magnetic-field order.

The jet emission can originate from several regions with different physical conditions (Fig. 2.4). High-frequency synchrotron emission tends to be concentrated near particle-acceleration sites because the most energetic electrons cool rapidly. At radio frequencies, synchrotron self-absorption can obscure the inner jet (see Section 2.1.4). When opacity determines the apparent core position, the core lies near the optical-depth transition, $\tau_\nu \sim 1$, and generally appears farther downstream at lower frequencies. At millimeter wavelengths, reduced opacity may reveal a standing shock that dominates the observed core emission. Together, these effects help interpret the locations of emitting regions and time delays between observing bands.

An observationally motivated example is the model proposed by Marscher et al.

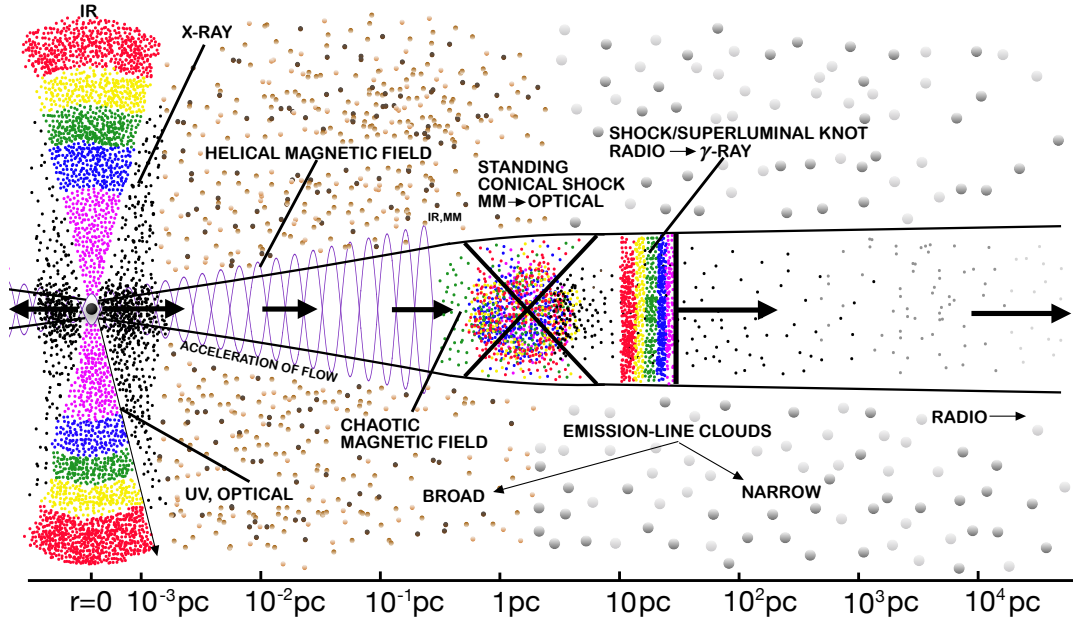


Figure 2.3: Schematic structure of a radio-loud quasar, illustrating the magnetic-field geometry and shock structures along its relativistic jet. Adapted from a figure by A. Marscher, <https://www.bu.edu/blazars/>.

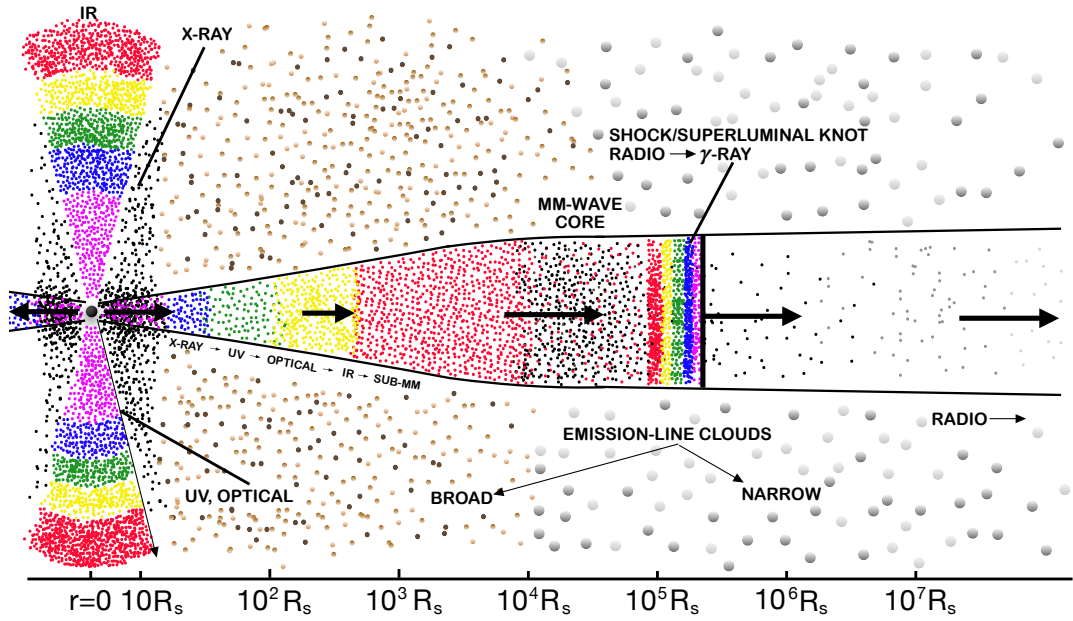


Figure 2.4: Schematic distribution of emission regions in a radio-loud AGN, including the millimeter-wave core and downstream jet. Adapted from a figure by A. Marscher, <https://www.bu.edu/blazars/>.

(2008) for BL Lacertae (Fig. 2.5). In this model, a disturbance follows a spiral streamline through a region with a helical magnetic field, producing synchrotron and inverse Compton emission. The polarization evolution and two flaring episodes were interpreted in terms of its passage through the acceleration and collimation zone and, subsequently, a standing conical shock associated with the millimeter-wave core. This scenario illustrates how related flares can originate at different locations along the jet.

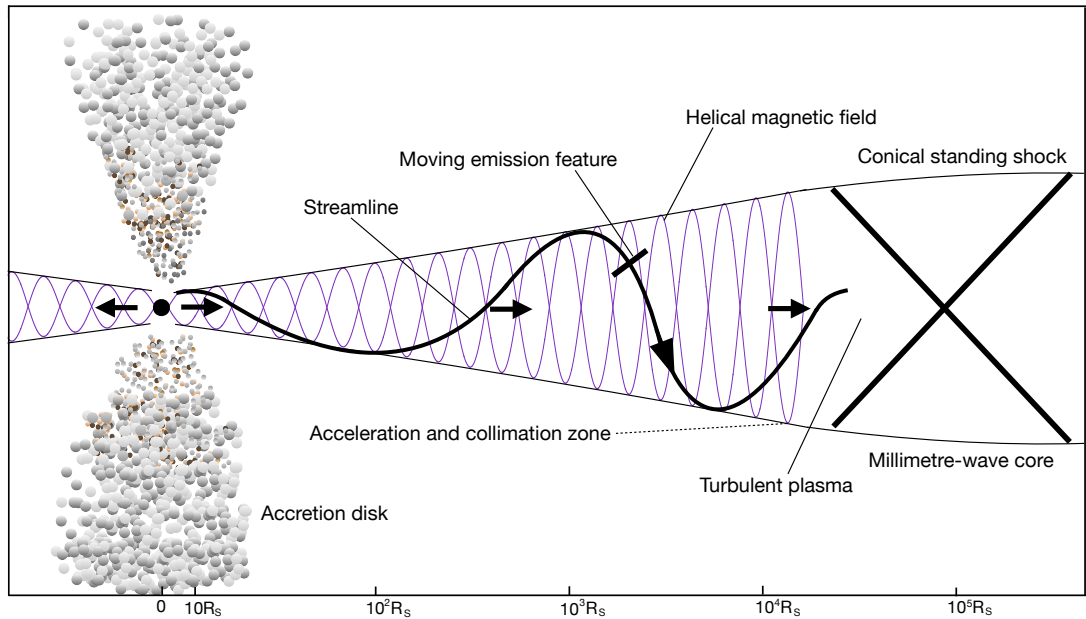


Figure 2.5: Proposed model for the inner jet of BL Lacertae. Distances from the black hole are shown on a logarithmic scale in units of the Schwarzschild radius, R_s . Redrawn from Fig. 3 of Marscher et al. (2008).

2.1.3 Jet launching and collimation: an MHD perspective

The theoretical consensus holds that jet launching and collimation require magnetohydrodynamic (MHD) processes in the accretion environment. Two canonical mechanisms are: (i) extraction of the black hole's rotational energy via magnetic fields anchored at the event horizon (the mechanism of Blandford & Znajek (1977)), and (ii) magnetocentrifugal launching from the accretion disk itself (the mechanism of Blandford & Payne (1982)).

Modern relativistic MHD simulations confirm that jet efficiency can be extraordinarily high in regimes where magnetic flux accumulates near the black hole (magnetically arrested disk states, or MAD). On parsec scales, the final

collimation and stability of the jet depend on the surrounding medium and on magnetohydrodynamic instabilities (such as Kelvin–Helmholtz or *kink*), providing natural routes for structural and photometric variability [Tchekhovskoy et al. \(2011\)](#).

2.1.4 Synchrotron Radiation

At radio frequencies, the observed emission from relativistic AGN jets is generally dominated by non-thermal synchrotron radiation produced by relativistic electrons moving through magnetic fields. As illustrated in Fig. 2.6, an electron with velocity components parallel and perpendicular to a locally uniform magnetic field follows a helical trajectory. The magnetic Lorentz force continuously deflects its motion, and the resulting transverse acceleration produces synchrotron radiation. In the plasma rest frame, the characteristic emission frequency scales as $\nu_c \propto \gamma^2 B_\perp$, where γ is the electron Lorentz factor and $B_\perp$ is the magnetic-field component perpendicular to its velocity. Thus, more energetic electrons and stronger perpendicular magnetic fields produce emission at higher characteristic frequencies. The same electron population can also generate inverse-Compton emission by scattering seed photons to higher energies. In the Thomson regime, assuming isotropic electron and photon distributions in the plasma rest frame, the relative radiative powers approximately satisfy

$$\frac{P_{\text{IC}}}{P_{\text{syn}}} \simeq \frac{U_{\text{ph}}}{U_B}, \quad (2.4)$$

where U_{ph} is the photon energy density and $U_B = B^2/(8\pi)$ is the magnetic energy density. Therefore, inverse-Compton emission may dominate the total radiative output of some AGN jets even when synchrotron radiation dominates their radio emission.

For an electron energy distribution $N(\gamma) \propto \gamma^{-p}$, the optically thin synchrotron spectrum follows

$$S_\nu \propto \nu^\alpha, \quad \alpha = -\frac{p-1}{2} < 0. \quad (2.5)$$

Thus, when the observed bands sample the optically thin part of the spectrum, the flux density decreases toward higher radio frequencies.

At lower frequencies, the same plasma can absorb part of its own synchrotron emission. This process is known as synchrotron self-absorption, or SSA, and

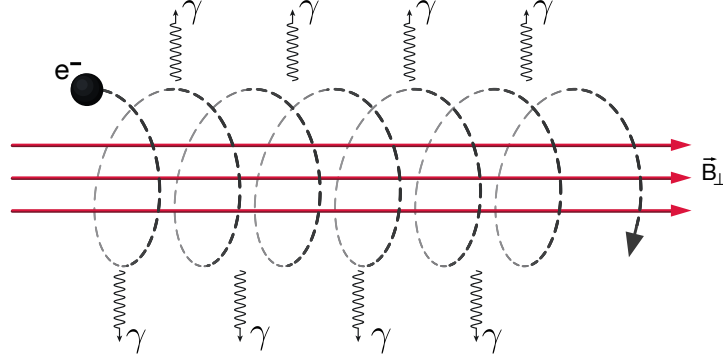


Figure 2.6: Synchrotron radiation: a relativistic electron in a magnetic field follows a helical trajectory and emits radiation. The characteristic emission frequency increases with the electron energy (γ) and with the component of the magnetic field perpendicular to its velocity ($B_{\perp}$). Adapted from [Wilhelm \(2021\)](#).

becomes important when the optical depth is large,

$$\tau_{\nu} \propto \nu^{-(p+4)/2}. \quad (2.6)$$

The turnover frequency ν_m is defined by $\tau_{\nu_m} = 1$ and separates the optically thick regime from the optically thin regime. Below this frequency, the radiation is partially absorbed before escaping, whereas above it the emission can leave the source more directly. For a homogeneous source, the optically thick synchrotron spectrum follows approximately $S_{\nu} \propto \nu^{5/2}$.

In a stratified relativistic jet, the surface from which the radiation escapes approximately satisfies $\tau_{\nu} \sim 1$. This surface is located closer to the central engine at higher frequencies and farther downstream at lower frequencies. This opacity effect defines the apparent VLBI core and produces the observed core shift. More generally, the frequency-dependent core position can be expressed as

$$r_{\tau=1}(\nu) \propto \nu^{-1/k_r}, \quad (2.7)$$

where k_r depends on the gradients of the magnetic field and particle density along the jet. For a conical jet close to equipartition, $k_r \simeq 1$, giving

$$r_{\tau=1}(\nu) \propto \nu^{-1}. \quad (2.8)$$

Therefore, a disturbance moving along the jet, such as a shock or a compact

plasma feature, is detected first at high frequency and later at lower frequency. For two observing frequencies $\nu_1 > \nu_2$, and assuming $k_r \simeq 1$, the expected delay is proportional to

$$\Delta t \propto \frac{\nu_2^{-1} - \nu_1^{-1}}{\beta_{\text{app}} c}. \quad (2.9)$$

Using the core-shift coefficient Ω_r , this relation may be written schematically as

$$\Delta t \propto \frac{\Omega_r}{\beta_{\text{app}} c} \left(\frac{1}{\nu_2} - \frac{1}{\nu_1} \right), \quad (2.10)$$

with the exact normalization depending on whether the core distances are projected or deprojected and on whether the delay is expressed in the source or observer frame. Since $\nu_1 > \nu_2$, the delay is positive: the high-frequency signal leads the low-frequency signal. The lower flux density at higher radio frequencies, when the emission is optically thin, follows from the negative spectral index, whereas the time delay arises from the frequency-dependent position of the synchrotron photosphere along the jet (Kellermann & Pauliny-Toth, 1969; Blandford & Königl, 1979; Lobanov, 1998; Hughes et al., 1992).

2.1.5 Blazars and compact radio sources

Blazars are traditionally subdivided into FSRQs (*Flat-Spectrum Radio Quasars*), which show strong and broad optical emission lines, and BL Lac objects, whose spectra lack prominent lines and are dominated almost entirely by the jet’s non-thermal continuum. In both cases, their architecture makes them natural laboratories to study dynamical (e.g., periodic plasma injection) and geometric (e.g., jet precession or helicity) processes.

In contrast, Compact Symmetric Objects (CSOs) are sub-kpc radio sources often interpreted as young radio galaxies. Unlike blazars, their variability is less extreme and less dominated by orientation, which enables studies of early activity phases and the intrinsic kinematic evolution of the jet as it interacts with the circumnuclear medium. Comparing CSOs with blazars helps isolate purely relativistic phenomena from those tied to environment and source evolution.

Figure 2.7 illustrates this contrast by comparing the radio light curves of the blazar PKS 1413+135 and the CSO OQ 208. PKS 1413+135 exhibits pronounced

flares and relatively rapid changes in flux density, whereas OQ 208 shows a smoother, gradual decline over the observing interval, with smaller short-term fluctuations. This comparison highlights the importance of considering both the amplitude and the timescale of variability, since CSOs can also undergo substantial changes over sufficiently long intervals. As discussed by [Kiehlmann et al. \(2024\)](#), variability therefore provides a complementary diagnostic to radio morphology for distinguishing strongly beamed sources from bona fide CSOs.

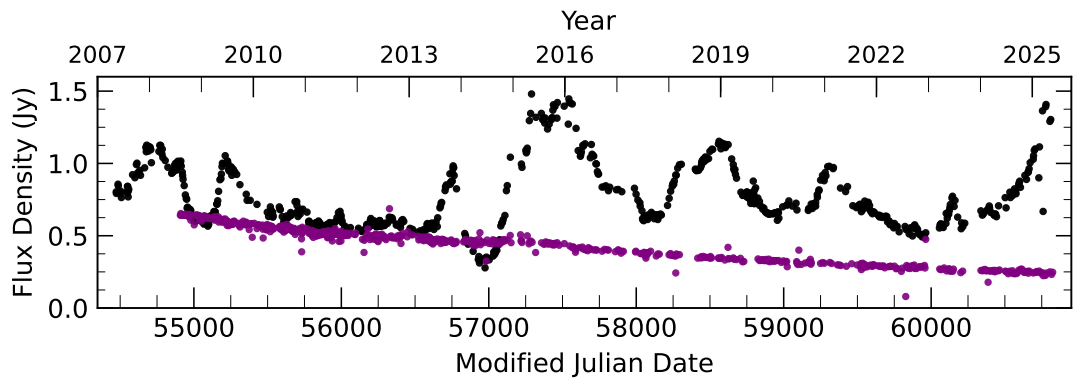


Figure 2.7: Flux-density variability (Jy) comparison between the blazar PKS 1413+135 (black) and the compact symmetric object OQ 208 (purple), shown without uncertainty bars. OQ 208 remains substantially more stable during the time window shown ([Kiehlmann et al., 2024](#)).

2.2. Multiwavelength Variability: Key Physical Mechanisms by Band

AGN variability does not arise from a single process, but from the superposition of distinct mechanisms that tend to dominate at different wavelengths: opacity and propagation effects in the radio, jet–disk competition in the optical/UV, and Compton and coronal processes at high energies. The following subsections summarize the key mechanisms operating in each spectral domain, with emphasis on their typical contribution to the observed flux-density variability. This overview provides a concise roadmap linking observational signatures, such as amplitude, timescale, and multi-band correlations, to their most plausible physical interpretations.

2.2.1 Radio domain: key drivers of flux-density variability (S_ν)

1. Doppler boosting: Geometric amplification (δ^3 or $\delta^{2+\alpha}$). Dominant in blazars: $S_\nu \propto \delta^{3+\alpha}$, so small viewing-angle changes can yield large amplitudes and apparent superluminal motion (Cohen et al., 1977; Blandford & Königl, 1979).
2. Propagating shocks (shock-in-jet): Disturbances within a relativistic jet produce shocks that accelerate particles and compress the magnetic field, generating synchrotron flares whose spectral and temporal evolution can be traced through the total flux density (Marscher & Gear, 1985). Contemporaneous UMRAO work independently developed a shock-in-jet model constrained primarily by the temporal evolution of the linear polarization (Aller et al., 1985; Hughes et al., 1985).
3. Interstellar scintillation (ISS) and Extreme Scattering Events (ESEs): Propagation through the Galactic ISM can produce extrinsic radio variability. ISS commonly causes rapid variations (< 2 days) at low frequencies ($\nu < 5$, GHz), whereas ESEs produce stronger, frequency-dependent changes lasting weeks to months.
4. Synchrotron self-absorption (SSA) and free-free absorption (FFA): Frequency dependent opacity can produce chromatic delays, with variability appearing first at high frequencies and later at lower frequencies. SSA arises within the synchrotron-emitting plasma, whereas FFA is caused by intervening ionized gas.
5. Blandford–Znajek power: Long-term jet power supplied by black-hole spin extraction; GRMHD simulations support spin-dependent jet efficiencies and long-timescale modulation of the jet output. (Blandford & Znajek, 1977).

2.2.2 Optical/UV domain: key drivers of variability (hybrid jet–disk regime)

1. Jet synchrotron (blazars): High-frequency synchrotron emission can dominate the optical continuum, often highly polarized, producing large and

rapid variability and correlations with γ -rays (Urry & Padovani, 1995).

2. Accretion disk instabilities: Thermal emission (multi-temperature blackbody) with stochastic temperature fluctuations yields month-to-year variability, particularly in radio-quiet quasars (Schmidt et al., 2010).
3. Reverberation: X-ray reprocessing by the disk: UV/optical responds with a lag $\tau = R/c$, enabling size-scale constraints and black-hole mass inference in conjunction with line reverberation (Peterson, 1993; Cackett et al., 2007).
4. Gravitational microlensing: Extrinsic magnification by stars in intervening galaxies can differentially amplify disk regions, effectively mapping disk structure via time-dependent magnification patterns (Morgan et al., 2010; Chartas et al., 2002).

2.2.3 X-ray and γ -ray domains: key drivers of extreme variability

1. External Compton (EC): Inelastic scattering (upscattering) of disk/BLR seed photons by relativistic jet electrons; typical of FSRQs, with variability driven by jet changes and/or seed-photon field variations (Sikora et al., 1994; Ackermann et al., 2015).
2. Synchrotron self-Compton (SSC): Inelastic scattering of the jet's own synchrotron photons; often relevant for BL Lacs and can yield near-simultaneous optical/X-ray and γ variability (Maraschi et al., 1992; Bloom & Marscher, 1996).
3. Coronal variability (Seyferts): Thermal Comptonization in a hot corona; geometric/compactness changes can reshape X-ray spectra on hour-like timescales (Haardt & Maraschi, 1991; Wilkins & Fabian, 2012).
4. Pair injection / magnetic reconnection: Rapid magnetic dissipation and mini-jets within the main flow can explain minute-scale TeV variability, challenging simple causality constraints (Aharonian et al., 2007).

2.3. Potential physical mechanisms responsible for periodic variability

The presence of an approximately sinusoidal modulation in the light curve of an active galactic nucleus does not allow the responsible physical mechanism to be identified directly. Blazar light curves exhibit stochastic variability, steep power spectral densities, and, in many cases, non gaussian flux distributions and possible changes in stationarity.

For this reason, interpretation through a physical model must be complemented by rigorous statistical analyses of periodicity, including the persistence of the phase and the coherence of the modulation, together with multifrequency observational corroboration and even morphological studies of the source.

This distinction is particularly important in the context of the present thesis, whose main objective is not to assign each periodogram maximum in advance to a physical scenario, but rather to determine under which conditions an observed periodicity can be regarded as an intrinsic and reproducible signal.

When sufficiently strong evidence of periodicity exists, its interpretation can be organized into three components:

1. A physical mechanism that establishes the fundamental period;
2. A mechanism that transforms the former into an observable variation;
3. An emitting region that transmits or modifies the signal before it is detected.

In the family of models considered in this section, the physical mechanism corresponds to the orbital motion of a supermassive black hole binary system. The observed variation is produced mainly through periodic changes in the Doppler factor of a relativistic jet oriented close to the line of sight. The synchrotron emission from the jet therefore acts as the electromagnetic tracer of the orbital motion.

2.3.1 Historical background and physical context

The foundations of this interpretation date back to the development of relativistic models of radio sources. Rees (1966) showed that relativistic motion significantly modifies the temporal and spatial appearance of an observed source. This work provided part of the framework required to interpret apparent superluminal motions and other effects produced by velocities close to the speed of light.

Subsequently, Blandford & Rees (1974) developed the *twin exhaust* model, in which the plasma generated in an active galactic nucleus is collimated into two oppositely directed relativistic flows. This work constitutes one of the fundamental precursors of modern relativistic jet models, although it does not yet represent a formulation of the orbital model considered in this thesis.

The scenario of binary supermassive black holes in active galactic nuclei was systematically developed by Begelman et al. (1980). Within this framework, a galaxy merger can leave two gravitationally bound supermassive black holes. The orbit of the binary can evolve through interactions with stars and gas and, at sufficiently small separations, through the emission of gravitational waves.

The orbital motion of a binary provides a potentially stable physical mechanism. However, the orbit is not observed directly in a radio light curve. A mechanism is required to transform the orbital motion into a detectable modulation of the emission.

An orbital model was later employed by O’Neill et al. (2022) to interpret the sinusoidal modulation of PKS 2131–021. Kiehlmann et al. (2025) explicitly describe this scenario as a model due to Blandford and adopt the designation *Kinematic Orbital model*, abbreviated as KO.

Kiehlmann et al. (2025) also note that a physically equivalent mechanism had been independently developed by Sobacchi et al. (2017), in which the orbital motion of the jet producing black hole periodically modifies the effective direction of the relativistic flow and, consequently, its Doppler factor. Subsequently, the *jet–wind* model developed by Sullivan et al. (2026) proposes that a wind associated with the disk forms a helical channel that confines and redirects the relativistic jet.

The historical sequence described above does not imply that every periodic modulation observed in a blazar is produced by a binary system. It summarizes

only the physical interpretation proposed within the KO model, the modified Kinetic Orbital (MKO) model, and the jet–wind family of models.

2.3.2 Relativistic Doppler modulation

Consider an emitting region moving with velocity v . The dimensionless velocity is defined as

$$\beta = \frac{v}{c}, \quad (2.11)$$

where c is the speed of light. The corresponding Lorentz factor is

$$\Gamma = \frac{1}{\sqrt{1 - \beta^2}}. \quad (2.12)$$

If θ represents the angle between the direction of motion of the plasma and the line of sight, the Doppler factor is given by

$$D = \frac{1}{\Gamma (1 - \beta \cos \theta)}. \quad (2.13)$$

The Doppler factor depends strongly on the angle θ when the plasma moves at a relativistic velocity. In a blazar, the jet is oriented close to the line of sight, so even a small angular variation can produce an appreciable change in D .

The observed flux density can be expressed in general form as

$$S_{\nu,\text{obs}} = D^p S_{\nu,\text{int}}, \quad (2.14)$$

where $S_{\nu,\text{int}}$ represents the intrinsic contribution of the emitting region and p is the Doppler boosting index (Urry & Padovani, 1995).

If the spectrum is defined by

$$S_\nu \propto \nu^\alpha, \quad (2.15)$$

the exponent commonly used for a continuous jet is

$$p = 2 - \alpha. \quad (2.16)$$

For a discrete moving component, the following is frequently used:

$$p = 3 - \alpha. \quad (2.17)$$

For this reason, when comparing different formulations, it is necessary to specify both the convention used for the spectral index and the geometry of the emitting region (Lind & Blandford, 1985; Urry & Padovani, 1995).

A small variation in the Doppler factor produces approximately

$$\frac{\Delta S_\nu}{S_\nu} \simeq p \frac{\Delta D}{D}. \quad (2.18)$$

This relation shows that a small geometric variation can produce a significant variation in the observed flux density. The period does not need to correspond to a periodic variation in the intrinsic luminosity of the jet. It can be produced by a periodic change in its effective orientation relative to the line of sight (Sobacchi et al., 2017).

Figure 2.8 illustrates how the viewing angle and flow speed determine the Doppler factor. The left panel shows D as a function of θ for selected values of β and their corresponding Lorentz factors Γ . The vertical lines mark the reference angles $\theta_{\text{ref}} = 1/\Gamma$, defined in radians and displayed in degrees. The right panel shows D as a function of Γ at fixed viewing angles chosen from these reference values, with $\beta = (1 - \Gamma^{-2})^{1/2}$. Matching colors and line styles associate each curve in the right panel with the corresponding reference angle in the left panel.

2.3.3 The kinematic orbital model

The KO model considers a binary system formed by two supermassive black holes with masses M_1 and M_2 . One of them produces a relativistic jet and, simultaneously, moves around the center of mass of the binary.

For a circular orbit, the separation between the two black holes can be estimated

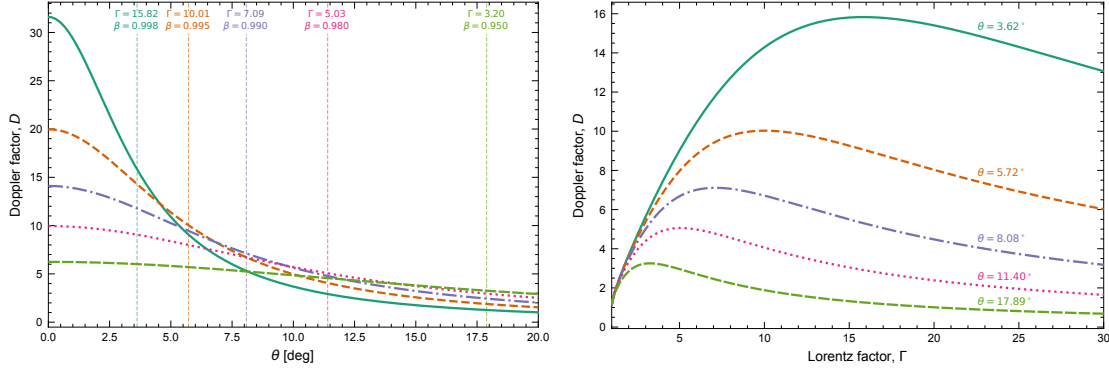


Figure 2.8: Relativistic Doppler factor D as a function of viewing angle θ for fixed flow speeds (left) and Lorentz factor Γ for fixed viewing angles (right).

using Kepler's third law:

$$a^3 = \frac{G(M_1 + M_2)P_{\text{orb}}^2}{4\pi^2}, \quad (2.19)$$

where a is the separation of the binary and P_{orb} is the orbital period in the source frame.

If the black hole of mass M_1 produces the jet, its distance from the center of mass is

$$a_1 = a \frac{M_2}{M_1 + M_2}. \quad (2.20)$$

Its orbital velocity is then

$$v_{\text{orb}} = \frac{2\pi a_1}{P_{\text{orb}}}. \quad (2.21)$$

The orbital velocity is much lower than the internal velocity of the jet. However, owing to the sensitivity of the Doppler factor to the viewing angle, the orbital component can appreciably modify the effective direction of the relativistic motion.

In a simplified representation, the angle between the jet and the line of sight can be written as

$$\theta(t) = \theta_0 + \Delta\theta \cos\left(\frac{2\pi t}{P_{\text{orb}}} + \phi_0\right), \quad (2.22)$$

where θ_0 is the mean viewing angle, $\Delta\theta$ is the amplitude of the angular variation, and ϕ_0 is the initial phase.

The Doppler factor then varies as

$$D(t) = \frac{1}{\Gamma [1 - \beta \cos \theta(t)]}. \quad (2.23)$$

The observed flux density is given by

$$S_\nu(t) = S_{\nu,0} D(t)^p, \quad (2.24)$$

where $S_{\nu,0}$ represents a normalization of the intrinsic emission.

For an approximately circular orbit and a small angular variation, the observed flux can be approximated by

$$S_\nu(t) \simeq S_0 + A_1 \cos \left(\frac{2\pi t}{P_{\text{orb}}} + \phi_1 \right). \quad (2.25)$$

This expression shows why the KO model produces, to first order, an approximately sinusoidal signal.

The observed period is affected by the cosmological redshift:

$$P_{\text{obs}} = (1 + z) P_{\text{orb}}, \quad (2.26)$$

where z is the redshift of the source.

An important characteristic of the KO model is that it does not require precession of the jet axis. In its basic formulation, the launching axis can remain fixed in the reference frame of the black hole. The modulation originates from the combination of the jet velocity and the orbital velocity of the black hole.

Consequently, the KO model must be distinguished from a precession model. Although both mechanisms can produce periodic variations in the Doppler factor, they differ in their underlying physical driver and associated geometry.

2.3.4 PKS 2131-021 as an Application of the KO Model

The 45.1-year radio light curve analyzed by O’Neill et al. (2022) exhibited two epochs of strong sinusoidal variability separated by approximately two decades during which the oscillation was not detected. The periods measured in the two active epochs agreed to within $\lesssim 2\%$, while their phases agreed to within approximately 10% of a cycle. This agreement indicates that the underlying periodic process remained coherent even during the interval in which it was not manifested in the observed light curve.

In a supermassive black hole binary, both components orbit their common center of mass. In the KO model, shown schematically in Fig. 2.9, one of these black holes produces a relativistic jet launched along its spin axis with a fixed velocity relative to the black hole. However, the black hole itself also has an orbital velocity. The relativistic combination of the orbital and jet velocities therefore causes the velocity of the emitting material relative to the observer to vary periodically. Because the jet moves at nearly the speed of light and is directed approximately toward the observer, even small changes in its relative velocity can produce appreciable changes in the Doppler factor and, consequently, in the observed flux density. Since the orbital velocity β_1 is small compared with the jet velocity, its effect can be expanded in powers of β_1 . Retaining only the leading term, which corresponds to the first-order or linear approximation, produces a sinusoidal modulation, whereas quadratic and higher-order terms may contribute harmonics. O’Neill et al. (2022) showed that the observed amplitude, $\delta \ln S \sim 0.2$, is compatible with component masses of order $10^8 M_\odot$ and typical blazar values $\gamma \sim \theta^{-1} \sim 10$. The intermittent visibility of the signal was explained through the superposition of a periodic component originating in a region of the jet directly affected by the SMBHB and a nonperiodic, predominantly stochastic component produced over a wide range of locations along the jet.

Subsequently, Kiehlmann et al. (2025) extended the analysis across a broad frequency range and found coherent sinusoidal variations from 2.7 GHz to the optical regime. The phase lag changes systematically with frequency and increases toward longer wavelengths. This frequency dependence was interpreted in terms of synchrotron self-absorption in the jet: higher-frequency emission becomes observable closer to the central engine, whereas lower-frequency emission emerges

at greater distances along the jet. Consequently, different observing frequencies preferentially sample regions located at different distances from the SMBHB and can exhibit different phases of the same orbital modulation.

To quantify this relationship, [Kiehlmann et al. \(2025\)](#) assumed that the emission observed at a frequency ν originates at a characteristic distance $r'(\nu)$ along the jet, measured in the SMBHB rest frame. Accounting for light-travel-time effects and assuming a jet directed approximately toward the observer, they obtained

$$\delta r'(\nu) = \beta(1 + \beta)\gamma^2 c P_0 \delta\phi \simeq 2\gamma^2 c P_0 \delta\phi, \quad (2.27)$$

where $\delta r'(\nu)$ is the difference between the emission distances corresponding to the measured phase difference, $\beta = v_{\text{jet}}/c$ is the dimensionless jet speed, $\gamma = (1 - \beta^2)^{-1/2}$ is the corresponding Lorentz factor, P_0 is the orbital period in the rest frame of the host galaxy, and $\delta\phi$ is the phase difference expressed as a fraction of a cycle. The final approximation assumes $\beta \simeq 1$. As an illustrative special case, the authors considered a configuration in which the orbital plane and the jet axis are orthogonal; in this geometry, the locus of the jet is a helix on the surface of a cone. From the observed phase–frequency relation, which spans a factor of approximately 128 in radio frequency, they inferred a lower limit $r' \gtrsim 32.1$ pc and a projected scale $r'_{\text{proj}} \gtrsim 2.13$ pc for their adopted parameters. Although these scales are large, they do not constitute an inconsistency of the model: the jet moves at nearly the speed of light through the characteristic emission surface associated with each frequency. On this basis, [Kiehlmann et al. \(2025\)](#) concluded that the multifrequency observations of PKS 2131–021 are consistent with the KO model.

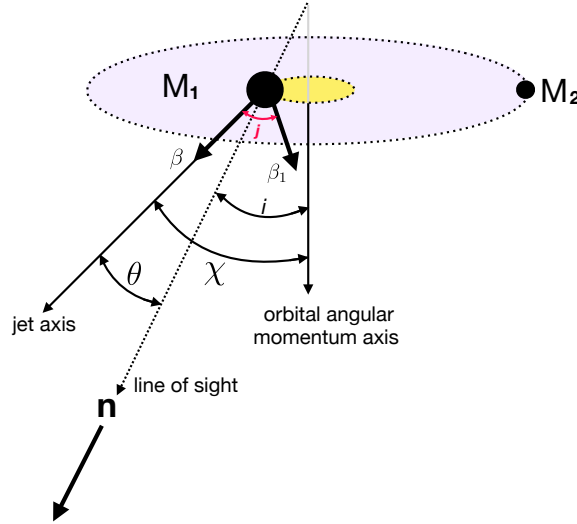


Figure 2.9: Simplified schematic of the kinematic orbital model for a supermassive black hole binary capable of producing sinusoidal flux-density variations in a blazar. The orbital motion of the jet-producing black hole changes the Doppler factor of the emitting material as measured by the observer. The figure was redrawn and adapted from O’Neill et al. (2022) and Kiehlmann et al. (2025). The purple and yellow colors are used solely to distinguish the orbits of the two binary components.

2.3.5 The Modified Kinetic Orbital Model

The modified Kinetic Orbital (MKO) model (see Figure 2.10) seeks to explain the sinusoidal modulation, the approximately constant fractional amplitude across frequencies, and the systematic frequency-dependent phase shift observed in PKS 2131–021 and PKS J0805–0111 (Hincks et al., 2026; Sullivan et al., 2026). Unlike the ballistic KO model described in the previous section, in the MKO model the relativistic emitting plasma does not itself transport the helical pattern. Instead, it travels within a channel defined by a slower wind–jet interface, possibly associated with a hydromagnetic wind from the accretion disk or with a jet sheath. The orbital motion periodically changes the direction of this interface, producing a conical-helical channel. The essential modification introduced by the MKO model is therefore the separation between the propagation of the pattern, determined by the wind–jet interface, and the motion of the plasma that produces the radiation.

The characteristic wavelength of the helix is

$$\lambda_h = \beta_w c P_{\text{orb}}, \quad (2.28)$$

where $\beta_w c$ is the effective propagation speed of the interface and P_{orb} is the orbital period. When the modulation is evaluated at the observer time, the

distance-dependent phase is

$$\phi(r) = \frac{2\pi r}{\lambda_h} (1 - \beta_w \cos \theta), \quad (2.29)$$

where r is the radial distance from the black hole and $\cos \theta = \hat{r} \cdot \hat{n}$, with $\hat{r}$ denoting the local flow direction and $\hat{n}$ the observer direction (Sullivan et al., 2026, Equation 30). Because the interface propagates more slowly than the relativistic plasma, a given phase difference can be produced over a shorter distance and with fewer helical turns than in the ballistic KO model.

The MKO model therefore distinguishes three velocities: the orbital velocity of the black hole, β_{HC} ; the propagation speed of the helical pattern, $\beta_w c$; and the velocity of the emitting plasma, $\beta_j c$, associated with the Lorentz factor $\Gamma_j = (1 - \beta_j^2)^{-1/2}$. The orbit sets the modulation period, the interface determines the propagation of the helix, and the relativistic plasma produces the Doppler boosting. If higher-frequency emission originates closer to the black hole and lower-frequency emission at greater distances, each frequency can sample a different phase of the pattern. Applications to PKS 2131–021 and PKS J0805–0111 reproduce the main amplitude and phase trends, including the ACT observations at 95, 147, and 225 GHz. The intermittency of the periodic component does not arise directly from the helical channel. As an additional hypothesis, disturbances or precession of the accretion disk may temporarily interrupt the launching of the jet. If the jet is subsequently reestablished, the modulation can recover the expected phase because the orbital motion continued during the interruption.

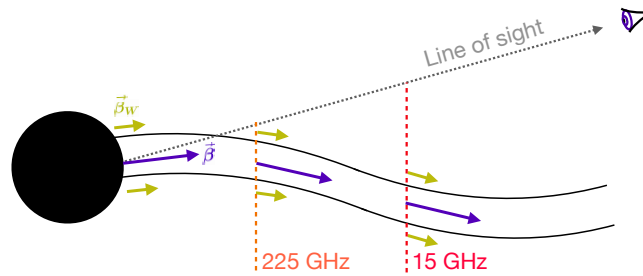


Figure 2.10: Schematic two-dimensional representation of the MKO model. The relativistic emitting plasma travels through a helical channel defined by a slower wind–jet interface whose direction is modified by the orbital motion. The regions associated with emission at 225 and 15 GHz are shown for illustrative purposes only. The figure was redrawn and adapted from Hincks et al. (2026) and Sullivan et al. (2026).

The Role of the Wind–Jet Interface

Within the MKO framework, [Sullivan et al. \(2026\)](#) developed a physical realization in which a subrelativistic wind collimates and determines the direction of the jet (Fig. 2.11). In this formulation, the wind may correspond to a hydromagnetic flow produced by an extended accretion disk or to a sheath surrounding the jet. The wind is assumed not to contribute to the radiation and to carry most of the inertia of the wind–jet system, whereas the synchrotron emission originates in the ultrarelativistic plasma flowing within it. Consequently, the instantaneous direction of the emitting flow is determined by the wind–jet interface rather than by the ballistic propagation of the plasma.

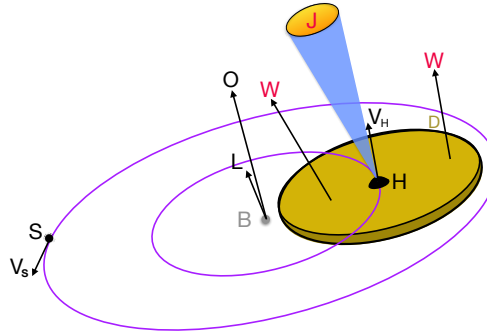


Figure 2.11: Schematic representation of the binary system, accretion disk, wind, and jet corresponding to the top panel of Figure 1 in [Sullivan et al. \(2026\)](#). The system consists of a primary black hole, H, and a secondary black hole, S. The primary is accompanied by an accretion disk, D, and launches a relativistic jet, J, approximately along the orbital angular momentum direction, L. The disk produces a wind, W, that collimates the jet. The purple curves represent the orbits of the binary components and are included solely as visual aids. The figure was redrawn and adapted from the original.

The effective propagation speed of the wind–jet interface is written as $\beta_w c$, where c is the speed of light and $\beta_w = v_w/c$, with v_w denoting the wind velocity in the schematic formulation. Sullivan et al. assume $\beta_w \lesssim 0.9$. The dimensionless orbital speed of the jet-producing black hole is $\beta_H = v_H/c$, where v_H is its orbital speed in the binary center-of-mass frame. This orbital motion periodically changes the effective direction of the interface and generates a conical-helical channel. The emitting plasma follows this channel with dimensionless speed $\beta_j = v_j/c$, where v_j is the plasma speed, and has Lorentz factor $\Gamma_j = (1 - \beta_j^2)^{-1/2}$.

The geometrical configuration of the binary system is illustrated in Fig. 2.12, where M_1 and M_2 denote the masses of the primary and secondary black holes, respectively. The unit directions $\hat{x}$, $\hat{y}$, and $\hat{z}$ define a Cartesian coordinate system

in which $\hat{z}$ is the axis of the conical jet. The jet has opening angle θ_j , and $\hat{r}$ denotes the local radial direction of the emitting flow. The angular position of $\hat{r}$ within the cone is specified by the polar angle χ and the azimuthal angle ψ . The unit direction $\hat{n}$ points toward the observer, and i is the angle between $\hat{n}$ and the cone axis $\hat{z}$. The periodic change in the direction of $\hat{r}$ relative to $\hat{n}$ modifies the Doppler factor and produces the sinusoidal flux-density modulation. The ratio β_H/β_w influences the local amplitude of this variation, although the integrated amplitude also depends on the extent of the emitting region.

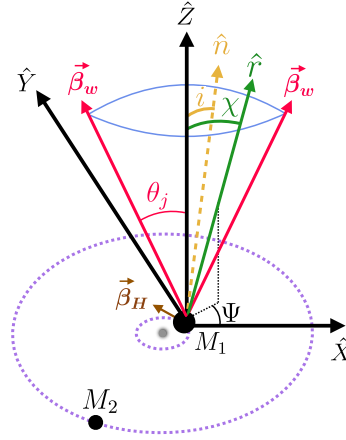


Figure 2.12: Geometric representation of the wind-collimated jet. The jet-producing primary black hole, with mass M_1 , moves with dimensionless orbital speed β_H , while M_2 is the mass of the secondary black hole. The jet is confined within a cone of opening angle θ_j centered on the $\hat{z}$ axis. The effective propagation speed of the wind-jet interface is represented by β_w , $\hat{r}$ denotes the local flow direction at angular position (χ, ψ) , and $\hat{n}$ points toward the observer, forming an angle i with the cone axis. The figure was redrawn and adapted from the middle panel of Figure 1 in [Sullivan et al. \(2026\)](#).

The helical wavelength, λ_h , is determined by the effective speed of the interface, as expressed in Equation 2.28, rather than by the speed of the emitting plasma. At a given observing frequency ν , the radiation originates over a finite radial interval, $r_{\min}(\nu) \leq r \leq r_{\max}(\nu)$, where r is the radial distance from the jet-producing black hole. The synchrotron emission and absorption must therefore be integrated over this interval. The inner boundary, $r_{\min}(\nu)$, is determined by the radius of the photosphere that is optically thick at frequency ν , whereas the outer boundary, $r_{\max}(\nu)$, may be related to the end of wind collimation or to the maximum distance at which emission at that frequency is produced. Higher frequencies can receive contributions from smaller radii than lower frequencies and can therefore sample different phases of the helical pattern. If the emitting region spans an excessively broad range of phases, the contributions overlap and reduce the amplitude of the

modulation.

In their applications to PKS 2131–021 and PKS J0805–0111, Sullivan et al. adopt the model-dependent value $\beta_w = 0.9$. This value prevents the sinusoidal variation from being strongly smeared out when the emission is integrated over regions extending beyond 10 light-years and suggests a transrelativistic wind–jet interface. For PKS 2131–021, the adopted realization places the average radius of the 15 GHz emission at approximately 20 light-years, corresponding to about ten helical wavelengths. Without the wind, the same phase shifts would require an average radius of approximately 100–500 light-years, depending on Γ_j . More generally, a purely ballistic interpretation may require separations of approximately 100 pc and coherence over more than one hundred helical wavelengths, whereas the slower interface allows the separation to be reduced to fewer than approximately ten wavelengths. The authors also note that nonperiodic large-scale components observed through very long baseline interferometry (VLBI) may dominate the emission when the helical structure loses coherence, thereby diluting the periodic signal.

The variable flow direction also produces polarimetric predictions. Under the assumption of a predominantly transverse magnetic field, both the polarization degree and the electric vector position angle may vary periodically. These predictions initially describe local regions of the jet; integrating regions with different phases can reduce the polarimetric signal through decoherence. The model also predicts changes in the position of the emission centroid. For the reference realization of PKS 2131–021, the calculated centroid displacement is approximately 2 microarcseconds ($2 \mu\text{as}$), probably too small to be detected with the Event Horizon Telescope (EHT), although it could be observable in closer sources or in sources with more extended millimeter-emitting regions.

Finally, Sullivan et al. present an illustrative model in which cold, dense clouds associated with the broad-line region are injected into the wind. Their motion may produce periodic variations in the central wavelength and width of the emission lines. These predictions depend on the assumed distribution and kinematics of the clouds and are not a unique consequence of the model. Nevertheless, polarization, centroid motion, interferometric imaging, and spectroscopic variability provide complementary tests that do not rely exclusively on periodicity in the light curves.

Chapter 3

Radio Monitoring with OVRO and UMRAO

The observational basis of this thesis will be built from two long-term radio monitoring programs: (i) the Owens Valley Radio Observatory (OVRO) 40 m telescope at 15 GHz ([Richards et al., 2011](#)) and (ii) the University of Michigan Radio Astronomy Observatory (UMRAO) 26 m telescope at 14.5 GHz ([Aller et al., 1985](#)). Both instruments use broadband receivers with overlapping frequency coverage, facilitating the direct comparison and combination of their measurements. The working sample is defined as the subset of sources monitored by both programs, which currently includes 83 blazars in common. For each source we will use time series of flux measurements with their reported uncertainties and timestamps, preserving full traceability between single-telescope data sets and the combined OVRO+UMRAO light curve.



Figure 3.1: Radio telescopes providing the long-term monitoring used in this thesis. Left: the OVRO 40-m telescope (15 GHz) located in Owens Valley, California (USA). Right: the UMRAO 26-m telescope (14.5 GHz) located in Dexter, Michigan (USA).

3.0.1 Radio monitoring facilities: OVRO and UMRAO

The radio variability analysis is based on long-term single-dish monitoring at 14.5–15 GHz, combining historical coverage from the University of Michigan Radio Astronomy Observatory (UMRAO) with the high-cadence Owens Valley Radio Observatory (OVRO) program.

UMRAO observations at 14.5 GHz were obtained with the 26 m equatorially mounted, prime-focus paraboloid as part of the long-running UMRAO monitoring program, which operated from the mid-1960s until 2012.5 and included total flux density and linear polarization monitoring for hundreds of AGN (Aller et al., 1985). At 14.5 GHz, UMRAO employed dual, rotating, linearly polarized feed horns placed symmetrically about the prime focus and alternating beams on the source; the receiver chain used a broadband uncooled HEMT amplifier with a bandwidth of 1.68 GHz. Individual measurements were constructed from a series of on-on observations alternating between feed horns over 30–40 minutes, with typical weekly cadence. The adopted flux-density scale follows Baars et al. (1977), using Cassiopeia A (3C 461) as the primary calibrator including its gradual decay, with secondary calibrators interleaved through each observing run to track pointing and gain variations.

OVRO observations are drawn from the OVRO 40 m Telescope Monitoring Program (Richards et al., 2011). The telescope uses off-axis dual-beam optics, (see Figure 3.3), with a beamwidth (FWHM) of 157 arcsec. Since 2014 May, observations have been performed with a cryogenic correlation receiver centered at 15 GHz, in which a 180° phase switch is used to correct rapid gain variations. Before this upgrade, the two beams were alternated using a Dicke switch. The observations follow a double-switching strategy in which one beam remains on-source, reducing gain fluctuations and atmospheric and ground contributions (Readhead et al., 1989). Relative calibration is provided by a temperature-stable noise diode (see Figure 3.4), while the absolute flux-density scale is established using 3C 286, assuming a flux density of 3.44 Jy on the Baars et al. (1977) scale, with DR 21 serving as a secondary calibrator (Richards et al., 2011).

3.1. The UMRAO 26-m monitoring program

For the present OVRO+UMRAO study, we use the total-intensity measurements, that is, the Stokes parameter I , obtained with the UMRAO 26 m telescope at 14.5 GHz. In this program, the observed signal initially corresponds to antenna temperatures measured through differential observations of the source and nearby reference positions on the sky. These instrumental measurements are then transformed into calibrated flux densities, expressed in janskys, using an absolute scale based on Cassiopeia A as the primary calibrator, including its secular decay at centimeter wavelengths. This scale is complemented by 11 secondary calibrators, whose relative flux densities were determined from long-term averages of antenna temperatures measured at Michigan ([Aller et al., 1985](#)).

The calibration first corrects temporal variations in the system gain, produced mainly by changes in aperture efficiency associated with insolation and ambient temperature. To control this effect, calibrators were typically observed every third source, corresponding to an approximate cadence of 1–2 h. At 14.5 GHz, these variations reach a typical daily peak-to-peak amplitude of approximately 15%. In addition to this temporal correction, the position-dependent gain of the telescope is also corrected. The main term is modeled as a function of the zenith distance z as

$$G_A = G_0 [1 - A(1 - \cos z)],$$

where G_0 is the zenith gain and, for 14.5 GHz, $A = 0.24$. This correction describes the gain loss associated with gravitational deformation of the paraboloid structure. An additional correction accounts for the transverse displacement of the focus system relative to the electrical axis of the antenna, parametrized as

$$G = G_A (1 - BX^2),$$

where X is the displacement in centimeters and, for 14.5 GHz, $B = 9.77 \times 10^{-3} \text{ cm}^{-2}$. To minimize these systematic effects, the observations were restricted to within 3 h of the meridian.

Finally, the calibrated antenna temperatures are corrected for pointing errors, assuming a Gaussian beam response. At 14.5 GHz, the characteristic Gaussian width for point sources is $\sigma \simeq 81.4''$, and observations requiring a pointing

correction factor larger than 1.5 are automatically rejected. The reported uncertainties in flux density include measurement noise, errors associated with the pointing correction, and the uncertainty in the temporal determination of the gain. They do not include, however, the absolute uncertainty of the flux scale or the uncertainty in the gain coefficient with zenith distance; therefore, when comparing these data with measurements from other instruments, an additional estimated uncertainty of $\sim 3\%$ should be considered. The final product used in this study corresponds to weighted monthly averages, where the weights depend on the daily uncertainties and the adopted monthly error is the larger of the scatter of the daily measurements around the mean and the propagated internal uncertainty.

3.2. The OVRO 40-m monitoring program

The 40-m radio telescope is located at Owens Valley Radio Observatory ($37^{\circ}13'53.0''$ N, $118^{\circ}16'57.9''$ W), California, United States, at an approximate altitude of 1220 m above sea level. Together with the favorable local climate, these conditions enable successful radio-astronomical observations. The 40-m radio telescope is equipped with a receiver that, in its analog mode, detects total intensity centered at a frequency of 15 GHz, observing more than two thousand sources of interest approximately twice per week.

This radio telescope uses the beam-switching technique to reduce unwanted effects produced by variations in the local atmospheric conditions ([Readhead et al., 1989](#)).

The injection of the NOISE and CAL diodes provides internal reference signals with known amplitudes: the former at a level comparable to the system temperature, and the latter at a level comparable to the astronomical sources being observed. In particular, the cal diode is used to monitor and correct slow variations in the receiver gain, serving as the basis for the relative calibration of each flux-density measurement.

3.3. Data Calibration

The calibration pipeline consists of three main stages:

3.3.1 Low level calibration

1. FLUX procedure: This stage is responsible for obtaining the relative flux density after beam switching, still in digital units. The value of κ , which corresponds to a calibration factor, is updated in later steps. The corresponding beam-switching sequence is illustrated in Figure 3.2, which shows the four integration states, A – D , and the associated beam positions. For example, during integration A , the beam associated with the horn labelled Ant points toward the reference position R_1 , while the Ref beam points at the source S . The calibration signal labelled Cal is activated during integrations B and C of the separate CAL procedure described below, with the telescope held at an off-source position. Further details of these measurements are provided by [Richards et al. \(2011\)](#).

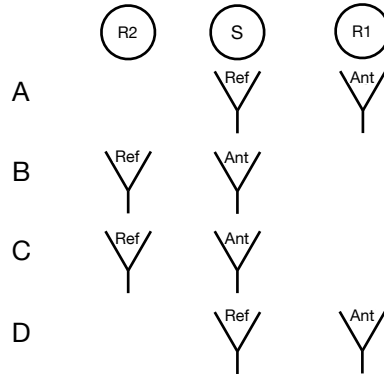


Figure 3.2: Beam-switching integration states and beam positions. Redrawn from [Readhead et al. \(1989\)](#), following the conventions of [Richards et al. \(2011\)](#).

$$S_{15} = \frac{\kappa}{2} (\xi_B + \xi_C - \xi_A - \xi_D) \quad (3.1)$$

and the rms error is given by:

$$\sigma_{15} = \frac{\kappa}{2} \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2} \quad (3.2)$$

2. CAL procedure: This procedure is very similar to the Flux procedure, but in this case the calibration diodes are activated.

$$S_{CAL} = \frac{1}{2} (\xi_B + \xi_C - \xi_A - \xi_D) \quad (3.3)$$

$$\sigma_{CAL} = \frac{1}{2} \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2} \quad (3.4)$$

3. Point procedure: A POINT procedure is a calibration observation used to measure how much the telescope deviates from the expected position on the sky. At the OVRO 40 m telescope, a bright source is observed, the real position of the beam maximum is estimated, and the pointing offset is calculated. This offset makes it possible to correct the pointing model and to ensure that flux measurements do not lose gain because the source is slightly off the center of the beam.

3.3.2 High level calibration

1. Gain Curve: The effective gain of the antenna changes with zenith angle due to gravitational deformation of the telescope and the increased atmospheric attenuation at high air mass. To correct for this effect, a polynomial gain curve is used, constructed from observations of bright calibrators at different zenith angles. Daily variations in atmospheric opacity are below 1% rms, so they are incorporated into the error model rather than corrected explicitly (Richards et al., 2011).
2. Focus Correction: This correction compensates for changes in the optimal focus position of the OVRO 40 m telescope, produced mainly by gravitational deformation as elevation changes, day-night thermal effects, and the geometry relative to the Sun. The receiver system can be displaced along the optical axis, and its optimal position is modeled using polynomial functions that depend on elevation and solar elongation. During data processing, the “focus miss”, that is, the relative error between the focus used and the ideal focus,

is estimated, and the associated gain loss is corrected in order to improve the accuracy of the measured flux densities.

3. Error Models: As part of the high level calibration, the error models quantify the actual uncertainty of each flux-density measurement by incorporating not only the measured thermal noise, but also systematic contributions associated with pointing errors, tracking errors, atmospheric variations, rapid weather changes, and flux-dependent effects. In this scheme, the total uncertainty is modeled by combining the direct instrumental error with additional terms proportional to the flux density and to the switched power, using empirically calibrated parameters. In addition, when the error bars are overestimated for certain sources, a source-dependent scale factor is applied based on the residual behavior of the light curve, with the goal of reducing artificially large errors without masking real astrophysical variability.

3.3.3 Absolute Calibration

Absolute calibration is the stage of the pipeline that fixes the final flux-density scale in physical units, transforming the relative instrumental measurements from digital units (DU) into janskys (Jy). In practice, this step updates the calibration coefficient by combining the relative response measured during the activation of the calibration noise diodes with observations of stable primary calibrators. In the OVRO 40 m program, the observing period is divided into epochs over which the relation between the calibration diode signal and the receiver input is assumed to remain stable. For each epoch, an absolute factor is determined from regular observations of primary calibrators, with 3C 286 and DR 21 among the most frequently used. In particular, the flux-density scale adopts for 3C 286 a reference value of 3.44 Jy at 15 GHz, following the model of Baars et al. Thus, the total factor applied to each measurement combines the relative calibration of each individual observation with the epoch-dependent absolute factor, placing all light curves on a common flux-density scale.

3.3.4 Filtering, Post-filtering and Inspection

After the standard calibration, the OVRO 40 m pipeline applies post-filtering and inspection procedures to remove corrupted or unreliable flux-density measurements.

This process combines automatic cuts based on observing conditions, pointing or calibration failures, anomalous uncertainties, and switched-difference diagnostics, together with manual inspection of the total power time series during problematic meteorological or instrumental conditions.

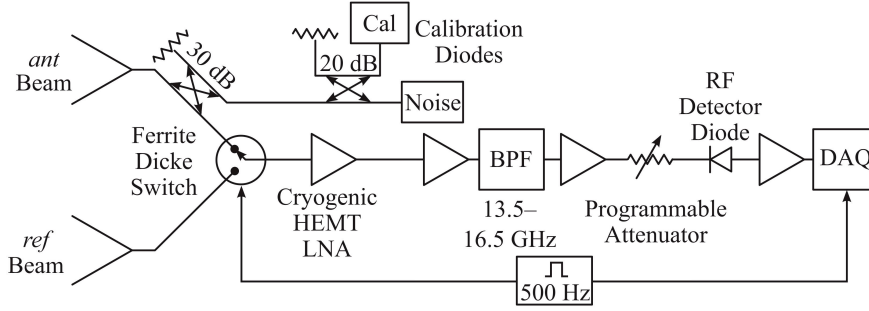


Figure 3.3: Block diagram of the Ku-band receiver used on the Owens Valley Radio Observatory (OVRO) 40 m telescope until 2014, based on [Richards et al. \(2011\)](#). The instrument employed a dual-beam, Dicke-switched radiometer architecture, in which a ferrite radio-frequency switch operating at 500 Hz alternately directed the signals from the antenna (*ant*) and reference (*ref*) beams to a common receiver chain.

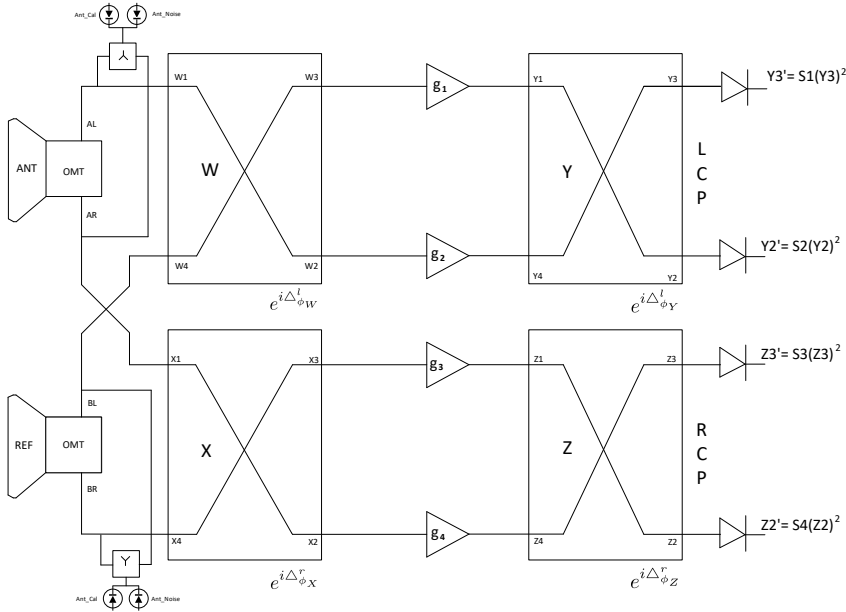


Figure 3.4: Simplified schematic of the KuPol receiver installed on the OVRO 40 m telescope in 2014. The receiver uses a correlation architecture implemented with 180° hybrids and operates as a dual-beam continuous-comparison radiometer for total-intensity measurements. Signals with the same circular polarization from the antenna and reference horns are combined to form sum and difference outputs. The noise-injection diodes provide calibration signals and enable receiver-gain variations to be corrected, while the square-law detector diodes convert the received radio-frequency power into output voltages. Only the beam-differenced total-intensity measurements, corresponding to Stokes I , are used in this work.

Chapter 4

The Extended OVRO+UMRAO Dataset

The OVRO+UMRAO (UO) sample consists of 83 sources selected on the basis of adequate temporal sampling in both monitoring programs. The sample covers the northern sky down to a declination limit of $\delta = -20^\circ$. Most of the UO sources, 75 out of 83, are included in the *MOJAVE 1.5 jy quarter century flux-limited agn sample*, while the remaining sources are included in the *MOJAVE hard spectrum Fermi AGN sample*. These MOJAVE-based samples provide several of the ancillary classifications used in this work, including optical identification, SED class, and radio spectral type.

As shown in Figure 4.1, both monitoring programs preferentially avoid the Galactic plane. This selection reduces potential contamination from Galactic foreground emission, source confusion in crowded low-latitude regions, and uncertainties in source identification at optical and other ancillary wavelengths. Nevertheless, four sources in the UO sample have Galactic latitudes $|b| < 20^\circ$; when relevant, this property is considered explicitly in the interpretation of their variability and classification.

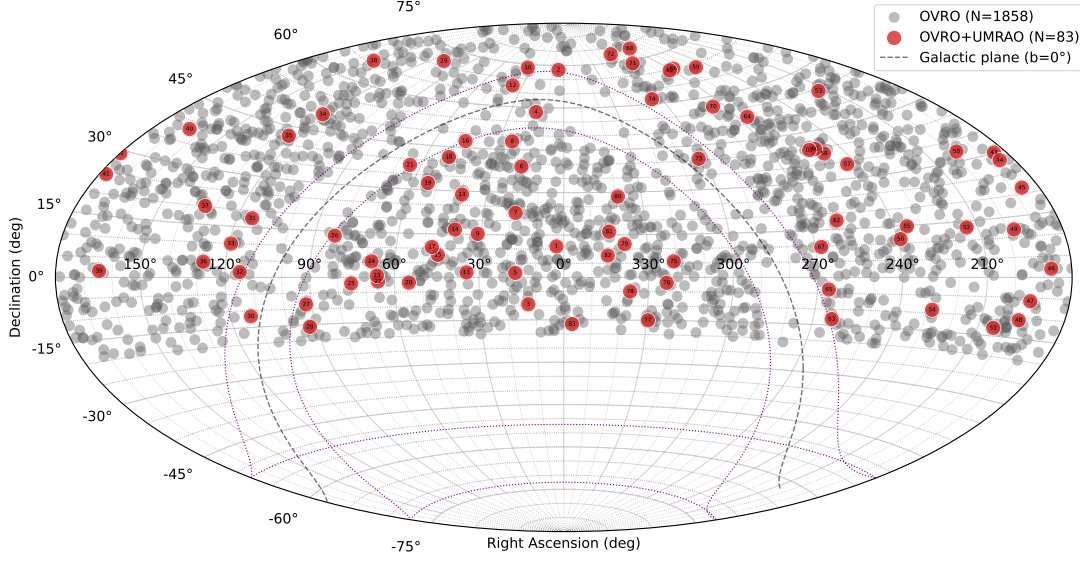


Figure 4.1: Sky map of the OVRO+UMRAO sample in Aitoff projection. The 83 combined sources are shown in red and numbered, while the OVRO-only sample is shown in gray. The purple dotted lines delineate the Galactic latitude band $|b| < 20^\circ$.

4.1. Light curves for the OVRO+UMRAO dataset

In this work, a light curve is defined as a time series of flux density, $S_\nu(t)$, measured at an observing frequency ν . These curves constitute the observational basis for characterizing source variability, searching for dominant time scales, and evaluating possible periodic or quasi periodic behavior. For the OVRO+UMRAO sample, we use total intensity measurements obtained at 15 GHz with OVRO and at 14.5 GHz with UMRAO.

Flux density is expressed in janskys, where

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}. \quad (4.1)$$

Therefore, S_ν represents the received flux per unit area and per unit frequency. In this context, the light curves describe the temporal evolution of the radio emission from the observed source.

Figure 4.2 summarizes the distribution of the sample observations in the time versus flux density plane. The lower axis shows the Modified Julian Date (MJD), while the upper axis indicates the approximate calendar year. The vertical axis

corresponds to the flux density in Jy, and the color scale represents the number of observations on a logarithmic scale. The figure shows that the sample covers a broad range of flux densities, from sub Jy sources to sources of several tens of Jy, with the highest concentration of measurements in the approximate range 0.3–10 Jy.

The change in point density along the time axis reflects the differences between the monitoring programs. UMRAO provides the longest historical baseline, starting in the 1970s, while OVRO contributes a denser sampling from 2008 onward. The OVRO+UMRAO combination allows the construction of light curves with a longer temporal baseline, although the differences in cadence, coverage, and calibration between both programs must be considered in the interpretation. For this reason, in addition to the combined light curves, the UMRAO only and OVRO only light curves are analyzed separately when necessary.

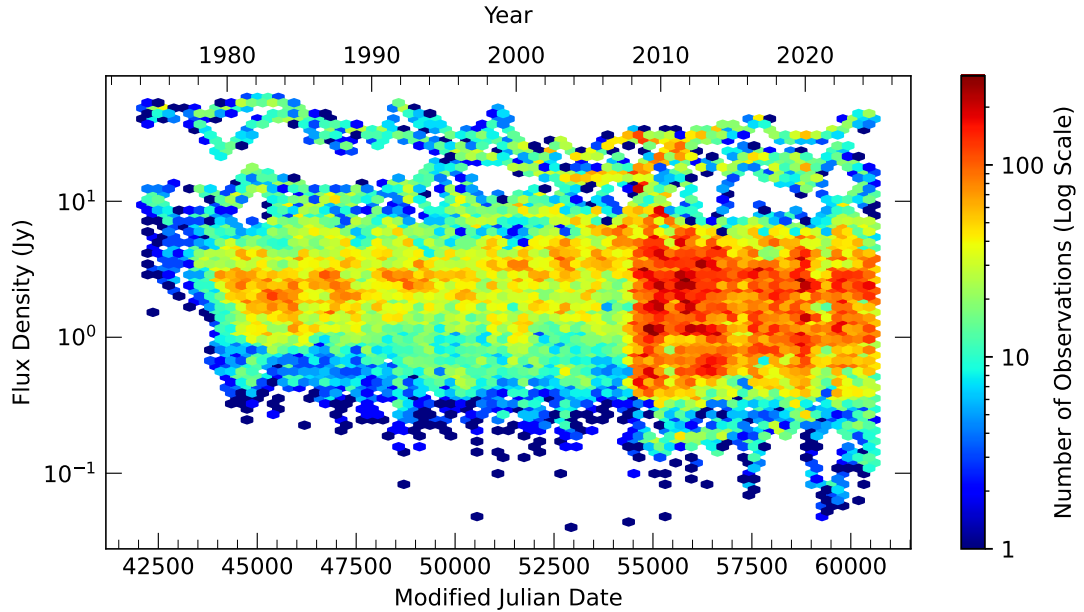


Figure 4.2: It corresponds to the full OVRO+UMRAO sample and allows us to visualize the number of observations over time and the range of flux densities in which they are distributed. Each hexagon represents a two-dimensional bin in the (MJD, flux) plane and indicates how many observations fall within it; the color of each hexagon is determined by that number of observations. The figure illustrates the long historical baseline provided by UMRAO and the denser sampling contributed by OVRO after 2008.

As an illustrative example, Figure 4.3 shows the multifrequency light curve of the source OJ 287, including UMRAO data at 4.8, 8.0, and 14.5 GHz, together with OVRO data at 15 GHz. However, for the analysis developed in this thesis, only the UMRAO measurements at 14.5 GHz and the OVRO measurements at

15 GHz are used. Before constructing the combined light curves, the data were inspected and filtered to remove problematic measurements, such as negative flux densities, missing or anomalously large uncertainties, and records containing invalid characters. In addition, the source identification was homogenized by adopting the J2000 names used by OVRO, since the UMRAO records were originally associated with B1950 names. Finally, the measurements were separated by frequency to select the relevant 14.5 GHz subset.

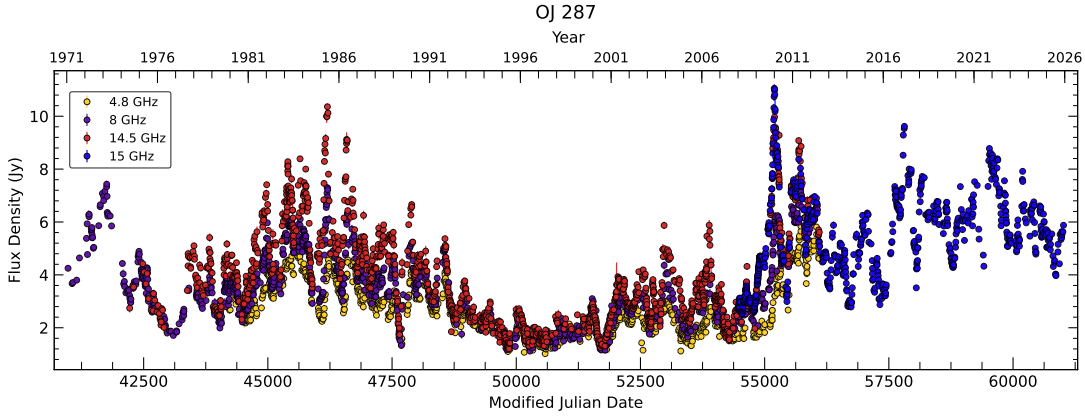


Figure 4.3: Example of a multifrequency radio light curve constructed from OVRO and UMRAO observations.

4.2. OVRO and UMRAO Data Combination

To combine the light curves obtained by the OVRO 40 m telescope at 15 GHz and the UMRAO 26 m telescope at 14.5 GHz, it is necessary to evaluate whether the small difference in observing frequency may introduce a significant systematic offset in the measured flux density.

Assuming that the synchrotron emission of the source can be locally described by a simple power law, the flux density S_ν as a function of frequency ν is expressed as

$$S_\nu = S_0 \nu^\alpha, \quad (4.2)$$

where α is the spectral index. Under this assumption, the expected ratio between the flux densities measured at the two observing frequencies is

$$R = \frac{S_{\text{OVRO}}}{S_{\text{UMRAO}}} = \left(\frac{15.0 \text{ GHz}}{14.5 \text{ GHz}} \right)^\alpha \approx (1.0345)^\alpha. \quad (4.3)$$

The corresponding fractional deviation between the two expected flux densities can be defined as

$$\frac{\Delta S}{S} \equiv \left| \frac{S_{\text{OVRO}} - S_{\text{UMRAO}}}{S_{\text{UMRAO}}} \right|. \quad (4.4)$$

Since

$$R = \frac{S_{\text{OVRO}}}{S_{\text{UMRAO}}}, \quad (4.5)$$

the fractional deviation can be written equivalently as

$$\frac{\Delta S}{S} = |R - 1|. \quad (4.6)$$

In percentage units, this becomes

$$\Delta S_{\%} = 100 |R - 1|. \quad (4.7)$$

For the type of sources commonly monitored in these programs, such as blazars and flat-spectrum AGNs, the spectral index in this radio-frequency band can be taken, as a practical approximation, to lie within the range $-0.5 \leq \alpha \leq +0.5$. Evaluating the ratio for the two limiting cases of this range gives

$$\alpha = -0.5 : \quad R = (1.0345)^{-0.5} \approx 0.983, \quad \Delta S_{\%} = 100 |0.983 - 1| \approx 1.7\%, \quad (4.8)$$

$$\alpha = +0.5 : \quad R = (1.0345)^{+0.5} \approx 1.017, \quad \Delta S_{\%} = 100 |1.017 - 1| \approx 1.7\%. \quad (4.9)$$

Therefore, for this representative range of spectral indices, the expected flux-density difference due purely to the separation between 14.5 GHz and 15.0 GHz is below approximately 2%. This value is comparable to, or smaller than, the typical absolute calibration uncertainties associated with single-dish radio monitoring, including flux-scale and atmospheric contributions, which are commonly at the level of a few percent. This can be summarized as

$$|1 - R| \lesssim \sigma_{\text{calibration}}. \quad (4.10)$$

The small fractional frequency separation therefore suggests that the expected spectral offset between the two observing bands is minor for the purposes of long-term variability analysis. This supports the use of a directly combined OVRO+UMRAO light curve as a practical long-baseline radio dataset at

approximately 14.5–15 GHz. However, this combination should be understood as an approximation supported by the small frequency difference and by the calibrated nature of both datasets, rather than as a proof that the two measurements are instrumentally identical.

The OVRO-only, UMRAO-only, and combined OVRO+UMRAO light curves are examined separately using GLS and WWZ analyses. These independent checks allow us to verify whether the dominant periodic features are present in both monitoring programs, whether they remain coherent in time.

4.3. Time systems, units, and flux-density scales

All observing epochs used in this work are expressed in Modified Julian Date (MJD). When calendar dates are reported for reference, they are given in UTC. Time intervals, temporal baselines, cadences, and observing-window widths are reported in days unless otherwise stated. Flux densities are expressed in janskys (Jy), and their associated uncertainties are denoted by σ_S .

For each monitoring program, the cadence was estimated as follows.

The UO sample is defined as:

$$\mathcal{S}_{\text{UO}} = \{s_1, s_2, \dots, s_{83}\}, \quad |\mathcal{S}_{\text{UO}}| = 83 \quad (4.11)$$

For each source $s_j \in \mathcal{S}_{\text{UO}}$, with $j = 1, \dots, 83$, we consider three light curve sets,

$$M \in \{\text{UMRAO}, \text{OVRO}, \text{UO}\}, \quad \text{UO} \equiv \text{OVRO} + \text{UMRAO} \quad (4.12)$$

For a given source s_j and monitoring set M , the ordered observing epochs are

$$t_{j,M,1} < t_{j,M,2} < \dots < t_{j,M,N_{j,M}}, \quad (4.13)$$

where $N_{j,M}$ is the number of observations. The consecutive time separations are

$$\Delta t_{j,M,i} = t_{j,M,i+1} - t_{j,M,i}, \quad i = 1, \dots, N_{j,M} - 1 \quad (4.14)$$

The per-source median cadence is defined as

$$\widetilde{\Delta t}_{j,M} = \text{median}_i (\Delta t_{j,M,i}), \quad (4.15)$$

and the per-source mean cadence as

$$\langle \Delta t_{j,M} \rangle = \frac{1}{N_{j,M} - 1} \sum_{i=1}^{N_{j,M}-1} \Delta t_{j,M,i} \quad (4.16)$$

For each monitoring set M , the histograms (4.4) show the distributions over the 83 sources of either

$$x_{j,M} = \widetilde{\Delta t}_{j,M}, \quad (4.17)$$

and

$$x_{j,M} = \langle \Delta t_{j,M} \rangle \quad (4.18)$$

The sample mean and sample median reported in each panel are therefore

$$\bar{x}_M = \frac{1}{83} \sum_{j=1}^{83} x_{j,M}, \quad (4.19)$$

with,

$$\tilde{x}_M = \text{median}(x_{1,M}, x_{2,M}, \dots, x_{83,M}) \quad (4.20)$$

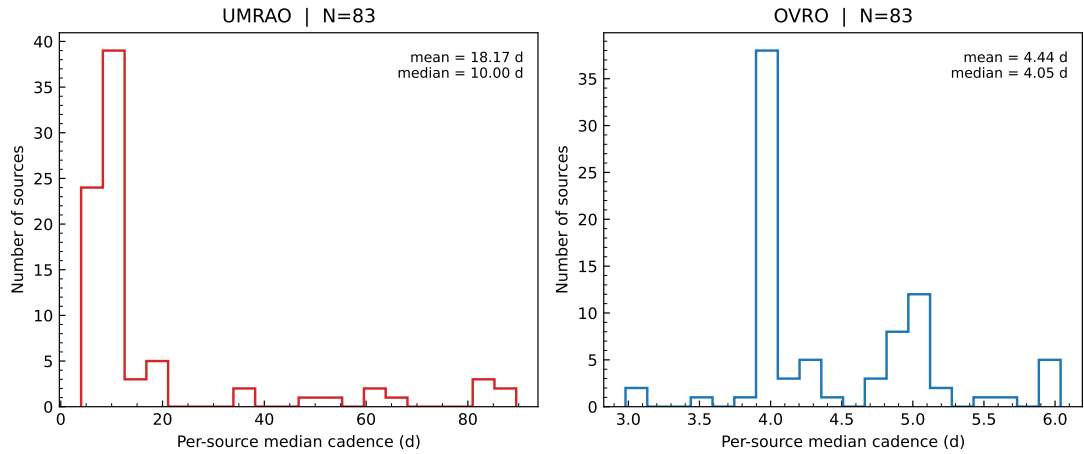


Figure 4.4: Histogram of the per-source median cadence. The red and blue, curves correspond to UMRAO, OVRO, respectively.

The following histograms summarize key properties of the OVRO+UMRAO sample:

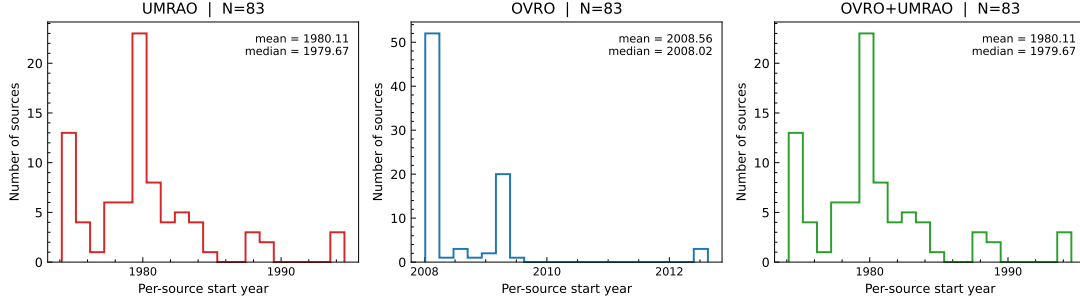


Figure 4.5: Start OVRO+UMRAO sample

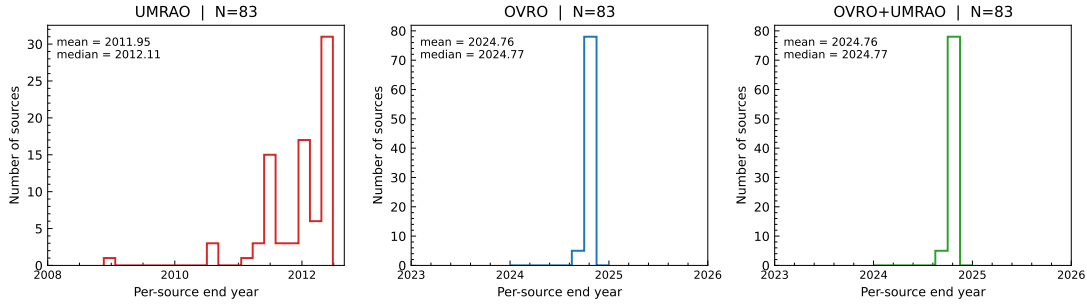


Figure 4.6: End OVRO+UMRAO sample

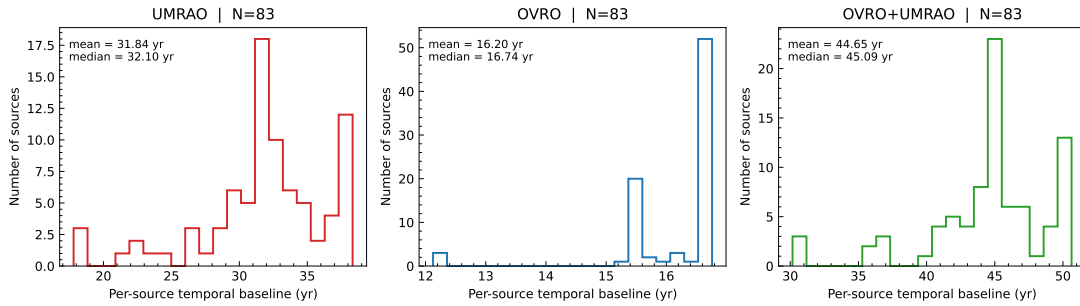


Figure 4.7: Temporal baseline in Years

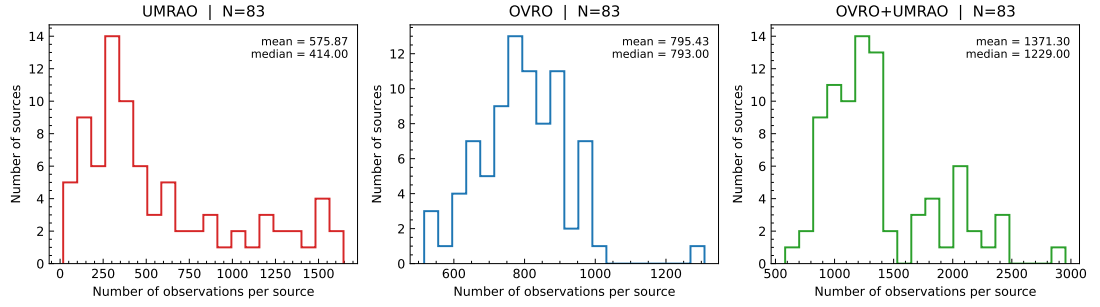


Figure 4.8: Number of observations

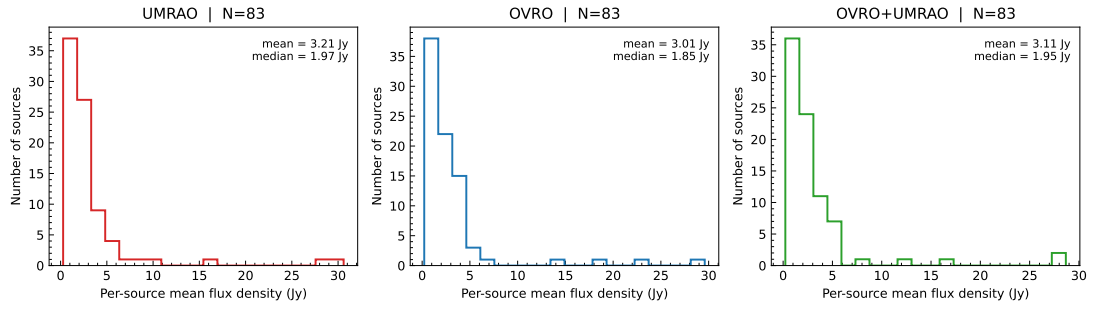
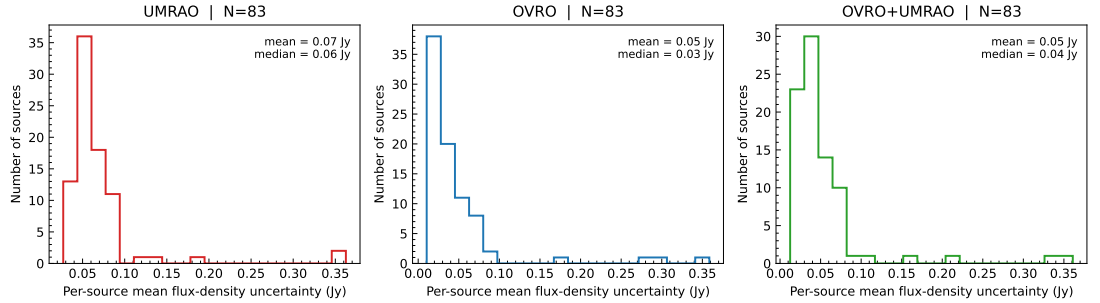
Figure 4.9: Mean flux density [Jy]Figure 4.10: Mean flux uncertainties (σ_s) [Jy]

Table 4.1: Summary of temporal coverage, sampling, and flux-density statistics for the UMRAO, OVRO, and combined OVRO+UMRAO light curves.

Program	ν (GHz)	N_{src}	$N_{\text{obs}}^{\text{tot}}$	Year range	T_{span} (yr)	$\langle T_{\text{src}} \rangle$ (yr)	$\langle N_{\text{obs}} \rangle_{\text{src}}$	$\langle S \rangle$ (Jy)	$\langle \sigma_S \rangle$ (Jy)	$\widetilde{\Delta t}$ (d)
UMRAO	14.5	83	47797	1974.14–2012.49	38.35	31.84	575.9	3.211	0.068	10.00
OVRO	15.0	83	66021	2008.01–2024.78	16.76	16.20	795.4	3.007	0.046	4.05
OVRO+UMRAO	14.5/15.0	83	113818	1974.14–2024.78	50.63	44.65	1371.3	3.112	0.053	5.21

Note. N_{src} is the number of sources. $N_{\text{obs}}^{\text{tot}}$ is the total number of observations in each dataset, while $\langle N_{\text{obs}} \rangle_{\text{src}}$ is the mean number of observations per source. The year range is computed from the earliest and latest epochs among all sources, and T_{span} is the corresponding global temporal baseline. $\langle T_{\text{src}} \rangle$ is the unweighted mean of the per-source temporal baselines. $\langle S \rangle$ and $\langle \sigma_S \rangle$ are unweighted means of the per-source mean flux density and mean flux-density uncertainty. $\widetilde{\Delta t}$ is the sample median of the per-source median cadences.

4.4. Classification and redshift distribution of the OVRO+UMRAO sample

Table 4.2 presents the 83 sources in the OVRO+UMRAO sample, together with their main identifiers, coordinates, AGN classifications, and adopted redshifts. Figure 4.11 shows the redshift distribution for the sources with an available value. The distribution has a mean of $\bar{z} = 0.76$ and a median of $\tilde{z} = 0.65$. The latter provides a more robust measure of the characteristic redshift of the sample, as it is less sensitive to sources in the high-redshift tail.

Table 4.2: Source Properties and Redshifts for the OVRO+UMRAO Sample.

J2000 Name	B1950 Name	Common Name	RA	DEC	AGN Class	z
J0010+1058	0007+106	IIIZW2	00:10:31.008	10:58:28.920	LSP G	0.089
J0019+7327	0016+731	S5 0016+73	00:19:45.792	73:27:29.880	LSP LPQ	1.781
J0050-0929	0048-097	PKS 0048-09	00:50:41.328	-9:29:04.920	ISP BL Lac	0.635
J0102+5824	0059+581	TXS 0059+581	01:02:45.768	58:24:11.160	LSP Quasar	0.644
J0108+0135	0106+013	4C +01.02	01:08:38.760	01:34:59.880	LSP HPQ	2.109
J0111+3906 †	0108+388	S4 0108+38	01:11:37.320	39:06:28.080	G	0.668
J0112+2244	0109+224	S2 0109+22	01:12:05.832	22:44:38.040	LSP BL Lac	U
J0136+4751	0133+476	DA 55	01:36:58.584	47:51:29.160	LSP HPQ	0.859
J0204+1514	0202+149	4C +15.05	02:04:50.400	15:14:11.040	LSP HPQ	0.405
J0217+7349	0212+735	S5 0212+73	02:17:30.816	73:49:32.160	LSP HPQ	2.346
J0217+0144	0215+015	OD 026	02:17:48.960	01:44:48.840	LSP HPQ	1.715
J0228+6721	0224+671	4C +67.05	02:28:50.040	67:21:02.880	LSP Quasar	0.523
J0237+2848	0234+285	4C +28.07	02:37:52.416	28:48:09.000	LSP HPQ	1.206
J0238+1636	0235+164	PKS 0235+164	02:38:38.928	16:36:59.040	LSP HPQ	0.940
J0259+0747	0256+075	OD 94.7	02:59:27.072	07:47:39.480	LSP HPQ	0.893
J0303+4716	0300+470	4C +47.08	03:03:35.232	47:16:16.320	LSP BL Lac	U
J0309+1029	0306+102	TXS 0306+102	03:09:03.624	10:29:16.080	LSP Quasar	0.862
J0319+4130	0316+413	3C 84	03:19:48.168	41:30:42.120	LSP G	0.018
J0336+3218	0333+321	NRAO 140	03:36:30.096	32:18:29.520	LSP LPQ	1.259
J0339-0146	0336-019	CTA 26	03:39:30.936	-1:46:35.040	LSP HPQ	0.847
J0418+3801	0415+379	3C 111	04:18:21.288	38:01:35.760	LSP G	0.049
J0423-0120	0420-014	PKS 0420-01	04:23:15.792	-1:20:33.000	LSP HPQ	0.913
J0424+0036	0422+004	PKS 0422+00	04:24:46.848	00:36:06.480	ISP BL Lac	0.268
J0433+0521	0430+052	3C 120	04:33:11.088	05:21:15.120	LSP G	0.033
J0501-0159	0458-020	S3 0458-02	05:01:12.816	-1:59:13.920	LSP HPQ	2.286
J0530+1331	0528+134	PKS 0528+134	05:30:56.424	13:31:54.840	LSP HPQ	2.070
J0607-0834	0605-085	OH -010	06:07:59.712	-8:34:50.160	LSP Quasar	0.872
J0609-1542	0607-157	PKS 0607-15	06:09:40.944	-15:42:39.960	LSP Quasar	0.322
J0721+7120	0716+714	TXS 0716+714	07:21:53.520	71:20:35.880	ISP BL Lac	U
J0730-1141	0727-115	PKS 0727-11	07:30:19.200	-11:41:12.480	LSP Quasar	1.591

Continued on next page

J2000 Name	B1950 Name	Common Name	RA	DEC	AGN Class	z
J0738+1742	0735+178	OI 158	07:38:07.440	17:42:19.080	LSP BL Lac	U
J0739+0137	0736+017	OI 061	07:39:18.000	01:37:04.080	LSP HPQ	0.189
J0757+0956	0754+100	PKS 0754+100	07:57:06.720	09:56:34.080	LSP BL Lac	0.266
J0808+4950	0804+499	OJ 508	08:08:39.600	49:50:35.880	LSP HPQ	1.435
J0818+4222	0814+425	OJ 425	08:18:16.080	42:22:45.120	LSP BL Lac	U
J0831+0429	0829+046	OJ 049	08:31:48.960	04:29:39.120	LSP BL Lac	0.174
J0854+2006	0851+202	OJ 287	08:54:48.960	20:06:29.880	LSP BL Lac	0.306
J0958+6533	0954+658	S4 0954+65	09:58:47.280	65:33:54.000	LSP BL Lac	0.369
J1058+0133	1055+018	4C +01.28	10:58:29.520	01:33:57.960	LSP HPQ	0.891
J1104+3812*	1101+384	Mrk 421	11:04:27.314	38:12:31.799	HSP BL Lac	0.029
J1150+2417	1147+245	IVS B1147+245	11:50:19.200	24:17:53.160	ISP BL Lac	0.209
J1159+2914	1156+295	4C +29.45	11:59:31.920	29:14:43.080	LSP HPQ	0.725
J1217+3007*	1215+303	ON 325	12:17:52.082	30:07:00.636	HSP BL Lac	0.130
J1221+2813*	1219+285	W Comae	12:21:31.691	28:13:58.500	ISP BL Lac	0.102
J1224+2122	1222+216	4C +21.35	12:24:54.480	21:22:46.560	LSP HPQ	0.433
J1229+0203	1226+023	3C 273	12:29:06.720	02:03:07.920	LSP LPQ	0.158
J1256-0547	1253-055	3C 279	12:56:11.280	-5:47:21.120	LSP HPQ	0.536
J1305-1033	1302-102	PG 1302-102	13:05:33.120	-10:33:19.080	LSP LPQ	0.278
J1309+1154*	1307+121	OP 112	13:09:33.932	11:54:24.553	LSP BL Lac	U
J1310+3220	1308+326	OP 313	13:10:28.560	32:20:43.080	LSP HPQ	0.995
J1337-1257	1334-127	PKS 1335-127	13:37:39.840	-12:57:24.120	LSP HPQ	0.539
J1415+1320	1413+135	PKS B1413+135	14:15:58.800	13:20:22.920	LSP BL Lac	0.247
J1419+5423	1418+546	OQ 530	14:19:46.560	54:23:13.920	LSP BL Lac	0.152
J1555+1111*	1553+113	PG 1553+113	15 55 43.044	11:11:24.366	HSP BL Lac	0.433
J1512-0905	1510-089	PKS 1510-089	15:12:50.640	-9:05:59.640	LSP BL Lac	0.605
J1540+1447	1538+149	4C +14.60	15:40:49.440	14:47:44.880	LSP BL Lac	0.433
J1613+3412	1611+343	DA 406	16:13:41.040	34:12:47.160	LSP LPQ	1.4
J1635+3808	1633+382	4C +38.41	16:35:15.600	38:08:03.840	LSP HPQ	1.815
J1642+6856	1642+690	4C +69.21	16:42:07.920	68:56:39.120	LSP HPQ	0.751
J1642+3948	1641+399	3C 345	16:42:58.800	39:48:37.080	LSP HPQ	0.593
J1653+3945*	1652+398	Mrk 501	16:53:52.217	39:45:36.609	HSP BL Lac	0.034
J1719+1745*	1717+178	OT 129	17:19:13.048	17:45:06.437	LSP BL Lac	U
J1733-1304	1730-130	NRAO 530	17:33:02.640	-13:04:49.080	LSP HPQ	0.902
J1740+5211	1739+522	TXS 1739+522	17:40:36.960	52:11:43.080	LSP HPQ	1.379
J1743-0350	1741-038	PKS 1741-03	17:43:58.800	-3:50:03.840	LSP HPQ	1.054
J1748+7005*	1749+701	S4 1749+70	17:48:32.840	70:05:50.769	LSP BL Lac	0.770
J1751+0939	1749+096	OT 081	17:51:32.880	09:39:00.000	LSP BL Lac	0.322
J1800+7828	1803+784	S5 1803+784	18:00:45.600	78:28:04.080	LSP BL Lac	0.683
J1806+6949	1807+698	3C 371	18:06:50.640	69:49:27.840	ISP BL Lac	0.051
J1824+5651	1823+568	4C +56.27	18:24:06.960	56:51:01.080	LSP BL Lac	0.663
J1927+7358	1928+738	4C +73.18	19:27:48.480	73:58:00.840	LSP LPQ	0.302
J2005+7752	2007+777	S5 2007+77	20:05:30.960	77:52:42.960	LSP BL Lac	0.342
J2007+4029	2005+403	TXS 2005+403	20:07:44.880	40:29:48.480	LSP Quasar	1.736
J2022+6136 †	2021+614	TXS 2021+614	20:22:06.720	61:36:57.960	LSP G	0.227
J2123+0535	2121+053	OX 036	21:23:44.400	05:35:21.840	LSP HPQ	1.941
J2134-0153	2131-021	IVS B2131-021	21:34:10.320	-1:53:17.160	LSP HPQ	1.283

Continued on next page

J2000 Name	B1950 Name	Common Name	RA	DEC	AGN Class	z
J2158-1501	2155-152	PKS 2155-152	21:58:06.240	-15:01:09.120	LSP HPQ	0.672
J2225-0457	2223-052	3C 446	22:25:47.280	-4:57:01.440	LSP HPQ	1.404
J2232+1143	2230+114	CTA 102	22:32:36.480	11:43:50.880	LSP HPQ	1.037
J2236+2828	2234+282	CTD 135	22:36:22.560	28:28:57.000	LSP BL Lac	0.79
J2253+1608	2251+158	3C 454.3	22:53:57.840	16:08:53.160	LSP HPQ	0.859
J2257+0743	2254+074	PKS 2254+074	22:57:17.280	07:43:12.360	LSP BL Lac	0.190
J2348-1631	2345-167	PKS 2345-16	23:48:02.640	-16:31:12.000	LSP HPQ	0.576

Note. AGN classifications follow the MOJAVE program. Most sources belong to the *MOJAVE 1.5 Jy Quarter Century Flux-Limited AGN Sample*; sources marked with (*) belong instead to the *MOJAVE Hard Spectrum Fermi AGN Sample*. All reported redshifts are spectroscopic, except for PG 1553+113; “U” denotes an unknown redshift. For PG 1553+113, we adopt $z = 0.433$, inferred indirectly from independent constraints rather than from a direct spectroscopic measurement of the source, following Johnson et al. (2019) and Dorigo Jones et al. (2022). According to the MOJAVE classification, all sources in this sample are classified as *Flat* ($\alpha > -0.7$), except those marked with †, which have optical class *G* (radio galaxy) and radio-spectrum class *GPS2*. Classification definitions follow the *MOJAVE Field Descriptions*.

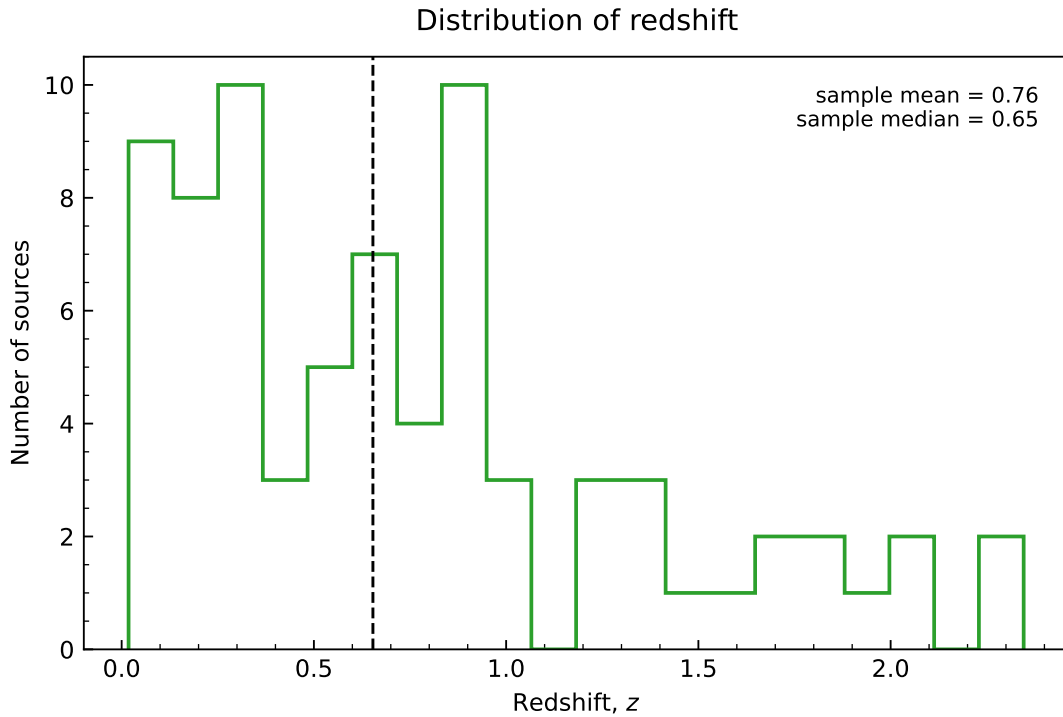


Figure 4.11: Distribution of redshifts for the OVRO+UMRAO sample. The green histogram shows the number of sources per redshift bin, and the black dashed line marks the sample median.

4.5. Periodicity Analysis Methods Applied to the OVRO and UMRAO Data

The preliminary analysis was applied to the light curves of the 83 sources, considering both the combined data and the data obtained with each radio telescope separately. First, the Generalized Lomb–Scargle Periodogram (GLSP) was computed, and the three highest-power peaks were identified, as illustrated in Fig. 4.12 and in the left panel of Fig. 4.13. The corresponding periods were then used for least-squares sinusoidal fitting, and the resulting fits were overlaid on the light curves, as shown in the right panel of Fig. 4.13.

To cross-check the periodicities identified by the GLSP and examine the temporal persistence of periodic features, a complementary search was conducted using the weighted wavelet Z-transform (WWZ). The period range adopted for this study was 100–11,000 days (approximately 0.27–30.12 years), as used for the results presented in Fig. 4.15 and in the corresponding appendix panels. The right panel of Fig. 4.14 provides an illustrative example of the WWZ method. Joint inspection of the GLSP and WWZ results enabled an assessment of the relative prominence of the detected signals and their consistency with the observed light curves. As an additional validation step, the results were compared with those obtained using the sine wave fitting (SWF) method (Appendix E).

This section and the following sections present representative examples of the application of these methods, together with a summary of the results obtained for the full sample. The individual results of this analysis are presented in Appendix A, whereas the analytical and theoretical foundations of the methods employed are described in Appendix E and G.

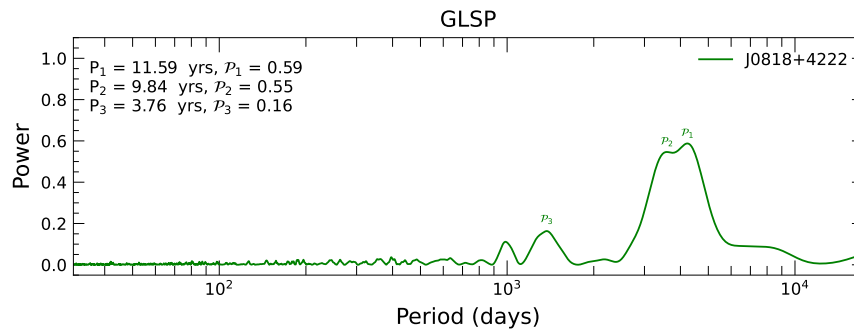


Figure 4.12: GLSP example. The detected main period, corresponding to the highest peak, is 11.59 years, with a power of 0.59.

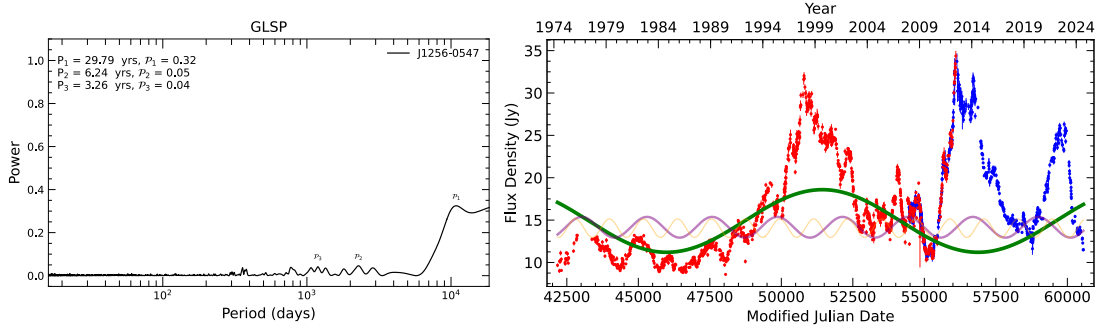


Figure 4.13: Preliminary sinusoidal least-squares visualization of the GLSP periods for the source J1256–0547, also known as 3C 279. Left: GLSP. Right: light curve with fits for P_1 , P_2 , and P_3 shown in green, purple, and orange, respectively; red and blue points denote UMRAO and OVRO data.

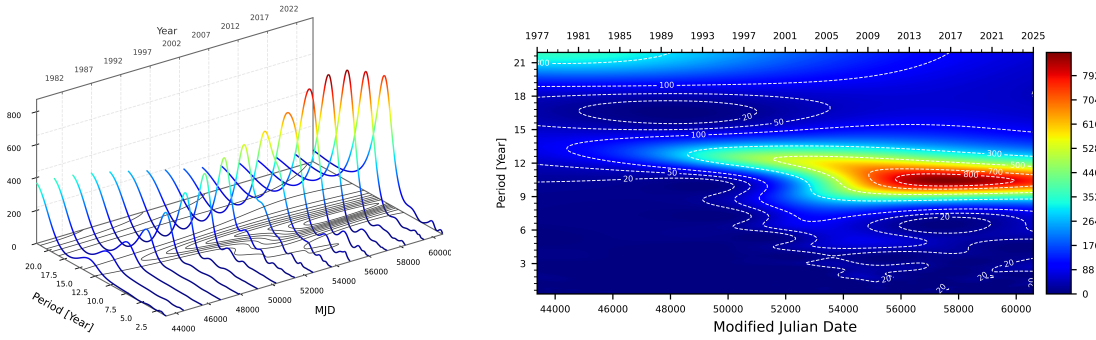


Figure 4.14: Example of the Weighted Wavelet Z-transform (WWZ) analysis for the source J0818+4222. Left: three-dimensional representation of the WWZ spectrum. Right: conventional time and period representation with superimposed contours. The color scale indicates the WWZ amplitude. A persistent periodicity of approximately 12 years is observed, with increased power from ~ 2009 onward. This feature is fully consistent with the Generalized Lomb–Scargle (GLS) result shown in Figure 4.12. The WWZ was computed using $c = 0.0125$, closely approximating the value $c = 1/(8\pi^2)$ suggested by Foster (1996); this dimensionless decay parameter controls the Gaussian weighting around each central time τ and thus the trade-off between time and frequency resolution. A detailed description of the GLSP and WWZ methods is provided in Appendix G.

4.6. Preliminary Periodicity Diagnostics and Results

As a preliminary but essential diagnostic stage, this section applies the three tools defined above to the OVRO, UMRAO, and combined OVRO+UMRAO light curves. The GLSP is used to identify the dominant periodic components, sinusoidal least squares fits are applied to the three strongest GLSP peaks to assess their consistency with the observed variability, and the WWZ is used to examine their temporal persistence. This approach provides a first selection of periodicity candidates for subsequent statistical validation (see Figure 4.15).

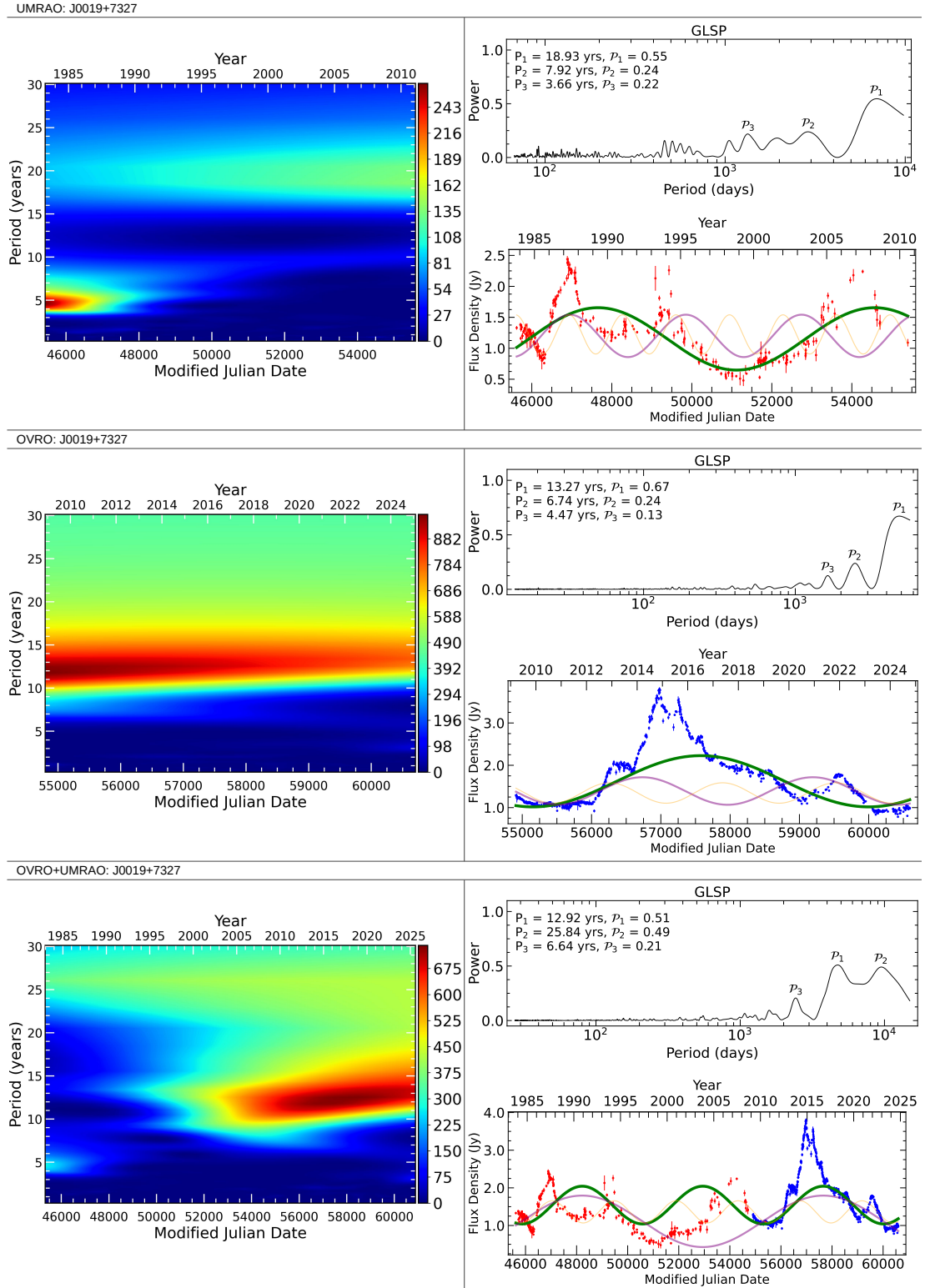


Figure 4.15: Example diagnostic panels for J0019+7327. The green, purple, and orange curves show the sinusoidal fits for the GLSP periods P_1 , P_2 , and P_3 , respectively. The complete set of panels for all sources is provided in Appendix A.

The following plots (figures 4.16, 4.17, 4.18 and 4.19), summarize global properties of the GLSP analyses applied to the UMRAO, OVRO, and OVRO+UMRAO light curves. In particular, we examine the distribution of the strongest period, P_1 , together with its associated power, $\mathcal{P}_1$. These diagnostics allow us to compare the stability of the dominant periodic structure across configurations, identify period clustering and differences associated with temporal coverage or observing cadence. This stage does not constitute a final significance test, but rather an initial inspection aimed at selecting candidates for subsequent statistical analyses. The evaluation will later be complemented by a detailed visual review of the individual diagnostic panels.

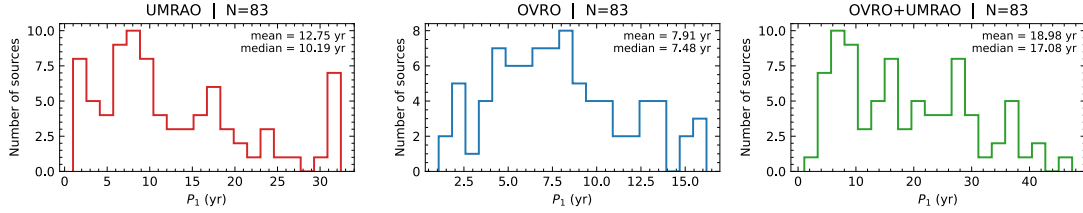


Figure 4.16: Distribution of the dominant period P_1 obtained from the GLSP analysis for UMRAO, OVRO, and OVRO+UMRAO, expressed in years. The columns show the three data configurations. The mean and median of each distribution are indicated in each panel. It can be observed that the UMRAO and OVRO light-curve distributions differ markedly, indicating that most sources do not show stable periodicity.

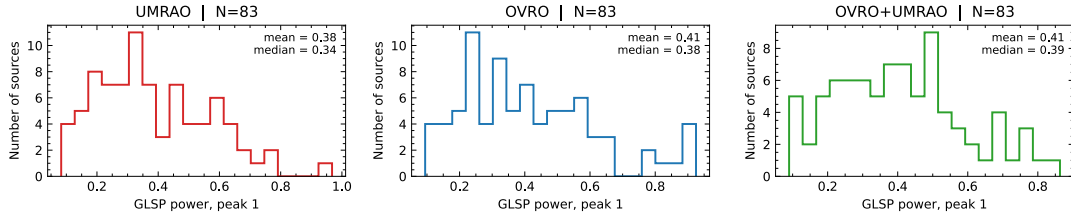


Figure 4.17: Distribution of the GLSP power $\mathcal{P}_1$ associated with the dominant period P_1 for UMRAO, OVRO, and OVRO+UMRAO. The columns show the three data configurations. The text boxes indicate the mean and median of each distribution. The similar power of the dominant peak across the individual and combined datasets indicates consistency, suggesting that differences in cadence and temporal baseline do not systematically affect the detected GLSP power.

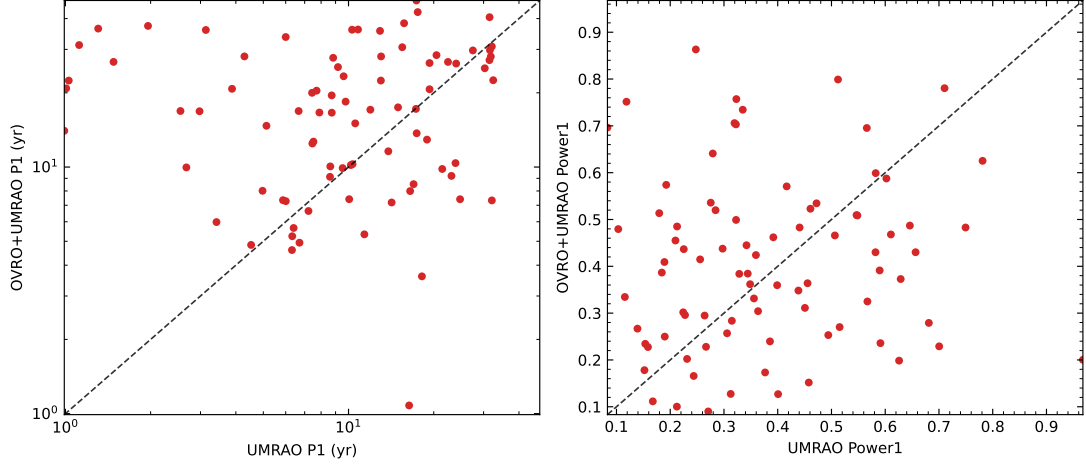


Figure 4.18: Source-by-source comparison between UMRAO and OVRO+UMRAO for the dominant period P_1 and its associated GLSP power $\mathcal{P}_1$. The dashed line indicates the one-to-one relation. The complete absence of clustering of data along the dotted line shows very clearly that the periodicities identified by the GLSP method in these 83 blazars are not reliable and should be treated with great caution and skepticism.

Within this preliminary diagnostic framework, the dominant period identified by the GLSP does not exhibit clear stability across the UMRAO, OVRO, and combined OVRO+UMRAO configurations (see Table 4.3). The mean (median) dominant period is 18.98 (17.08) yr for OVRO+UMRAO, compared with 12.75 (10.19) yr for UMRAO and 7.91 (7.48) yr for OVRO.

For the source-by-source comparisons, the fractional period difference is defined as $\delta P_1 = (P_1 - P_{1,\text{UO}})/P_{1,\text{UO}}$. The median absolute differences are $\text{med}(|\delta P_1|) = 0.494$ for UMRAO and 0.578 for OVRO. Only $f_{10}(P_1) = 14.5\%$ and 27.7% of the respective sources have dominant periods that differ by no more than 10% from the combined results, where $f_{10}(P_1)$ denotes the fraction satisfying $|\delta P_1| \leq 0.10$. The associated powers show similarly limited agreement, with $\text{med}(|\delta \mathcal{P}_1|) = 0.448$ and 0.318, and $f_{10}(\mathcal{P}_1) = 10.8\%$ and 16.9% , for UMRAO and OVRO, respectively.

Overall, neither the period nor the power of the dominant GLSP peak is generally preserved between the individual and combined configurations. These results should nevertheless be interpreted only as a preliminary statistical diagnostic, since they may be influenced by temporal coverage, cadence, the number of observations, and noise properties, and do not by themselves establish the presence or absence of a stable physical periodicity.

In addition, this possible sinusoidal dominance must be supported by the WWZ analysis, verifying that the signal remains temporally coherent and does not

Table 4.3: Summary of the dominant GLSP period, its associated power, and their comparison with the combined light curves.

Quantity	UMRAO	OVRO	OVRO+UMRAO
Strongest Peak			
Mean P_1 (yr)	12.75	7.91	18.98
Median P_1 (yr)	10.19	7.48	17.08
Mean $\mathcal{P}_1$	0.384	0.412	0.408
Median $\mathcal{P}_1$	0.344	0.379	0.391
Comparison			
N_{common}	83	83	ref.
$\text{med}(\delta P_1)$	0.494	0.578	ref.
$f_{10}(P_1)$	14.5%	27.7%	ref.
$\text{med}(\delta \mathcal{P}_1)$	0.448	0.318	ref.
$f_{10}(\mathcal{P}_1)$	10.8%	16.9%	ref.

Note. P_1 is the dominant period identified by the GLSP, and $\mathcal{P}_1$ is its associated power. Periods are given in years. The fractional differences are defined as $\delta P_1 = (P_1 - P_{1,\text{UO}})/P_{1,\text{UO}}$ and $\delta \mathcal{P}_1 = (\mathcal{P}_1 - \mathcal{P}_{1,\text{UO}})/\mathcal{P}_{1,\text{UO}}$. The quantity f_{10} is the fraction of sources with an absolute fractional difference less than or equal to 0.10. All summary statistics are based on the same sample of 83 sources.

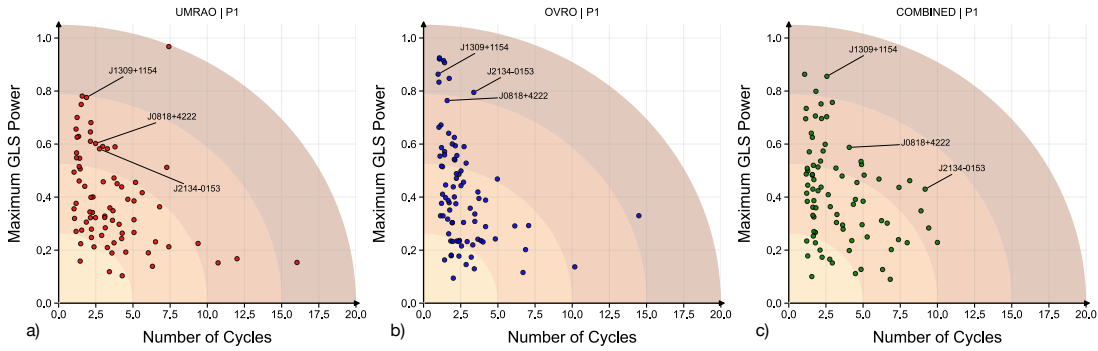


Figure 4.19: GLSP power as a function of the number of cycles for the main peak, P_1 , for each sample: UMRAO-only in red, OVRO-only in blue, and the combined sample in green. The outline colors are included only as visual references. Three sources are shown as examples to illustrate how the GLSP power can vary depending on the observing epoch and/or the radio telescope used.

correspond to a localized, transient, or sampling related structure. Therefore, this condition should be interpreted as a rare diagnostic pattern, not as a final statistical confirmation of periodicity.

Figure 4.20 provides a global WWZ view of the combined OVRO+UMRAO light curves, with the sources ordered by right ascension. The WWZ power is scaled by the maximum value detected over the full sample, so the color scale preserves the

relative amplitude between sources without normalizing each object independently. The marginal profile on the left shows, for each source, its maximum normalized WWZ value, regardless of the time or period at which this maximum occurs. In this context, J1613+3412 shows the highest WWZ power in the sample; however, its dominant structure appears mainly from 2008 onward, and the fundamental ridge is relatively broad, suggesting, in a preliminary sense, a less well defined periodicity. This contrasts with J2134–0153 (O’Neill et al., 2022), which displays a much narrower structure together with high WWZ power. Other visually interesting cases include J0818+4222, J1150+2417, J1309+1154, and C1224+2122. In particular, J1309+1154 stands out because it combines a narrow ridge, temporal persistence, and a relevant WWZ power within the sample. This figure should therefore be understood as a global and orientative view; a detailed assessment of the temporal periodic behavior of each source requires a careful inspection of the individual objects.

A complementary view is presented in Fig. 4.21, where the left-hand profile shows a proposed diagnostic metric combining WWZ power, period resolution, and temporal persistence. This metric is intended solely as a visual guide for comparing sources; further details are provided in Appendix F.

To summarize the global WWZ results for the 83 sources in the combined OVRO+UMRAO sample, Fig. 4.22 presents four distributions derived from the maximum WWZ power of each source. Here, W denotes the WWZ power and the subscript j identifies an individual source. Thus, $W_j^{\max}$ is the maximum power in the WWZ map of source j within the analysed period and time ranges, while $W_{\text{global}}^{\max} = \max_j W_j^{\max}$ is the largest of these values across the sample. The upper panels show the normalized maxima, $W_j^{\max}/W_{\text{global}}^{\max}$, and the absolute maxima, $W_j^{\max}$, respectively, with median values of 0.159 and 382. Most sources have relatively low peak powers compared with the sample maximum, while a small number form an extended high-power tail. The lower-left panel shows the periods associated with these maxima, with a mean of 16.5 yr and a median of 16 yr. The lower-right panel shows the corresponding calendar years, indicating when the maximum WWZ power occurs for each source. Each source contributes one entry to each histogram, so these distributions summarize the strength and location of the maxima without measuring their temporal persistence. The shaded subset in the upper-right panel highlights sources with relatively high peak powers and

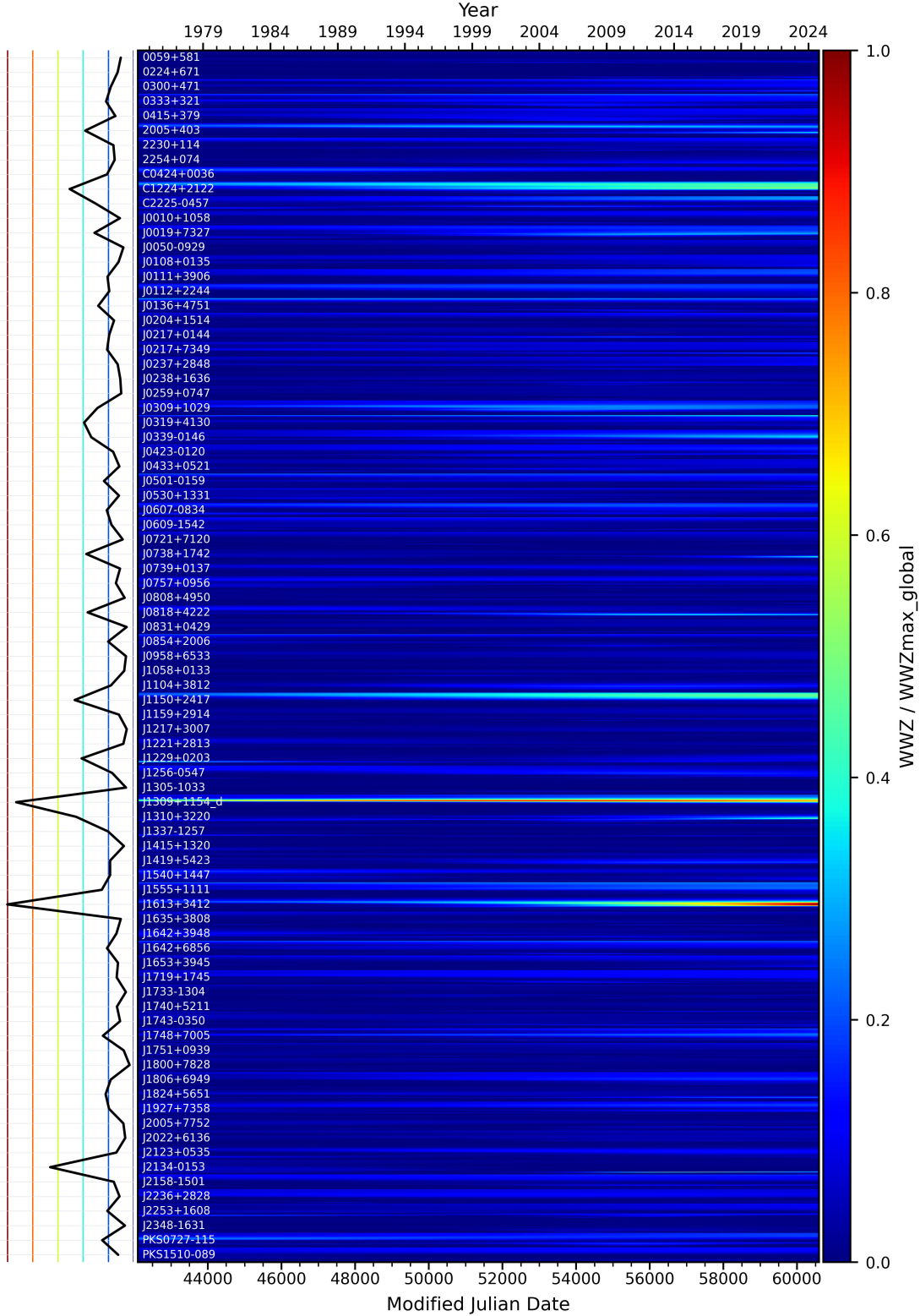


Figure 4.20: WWZ mosaic of the OVRO+UMRAO sample, ordered by right ascension. Peak values in the left-hand curves and the color scale are normalized to the sample’s global maximum WWZ power. Although J1613+3412 exhibits the highest WWZ power, its lack of temporal persistence does not support a plausible periodic interpretation; therefore, the mosaic is intended primarily for comparative reference. Superscript “d” denotes the detrended light curve of J1309+1154. The complete set of panels is provided in Appendix A.

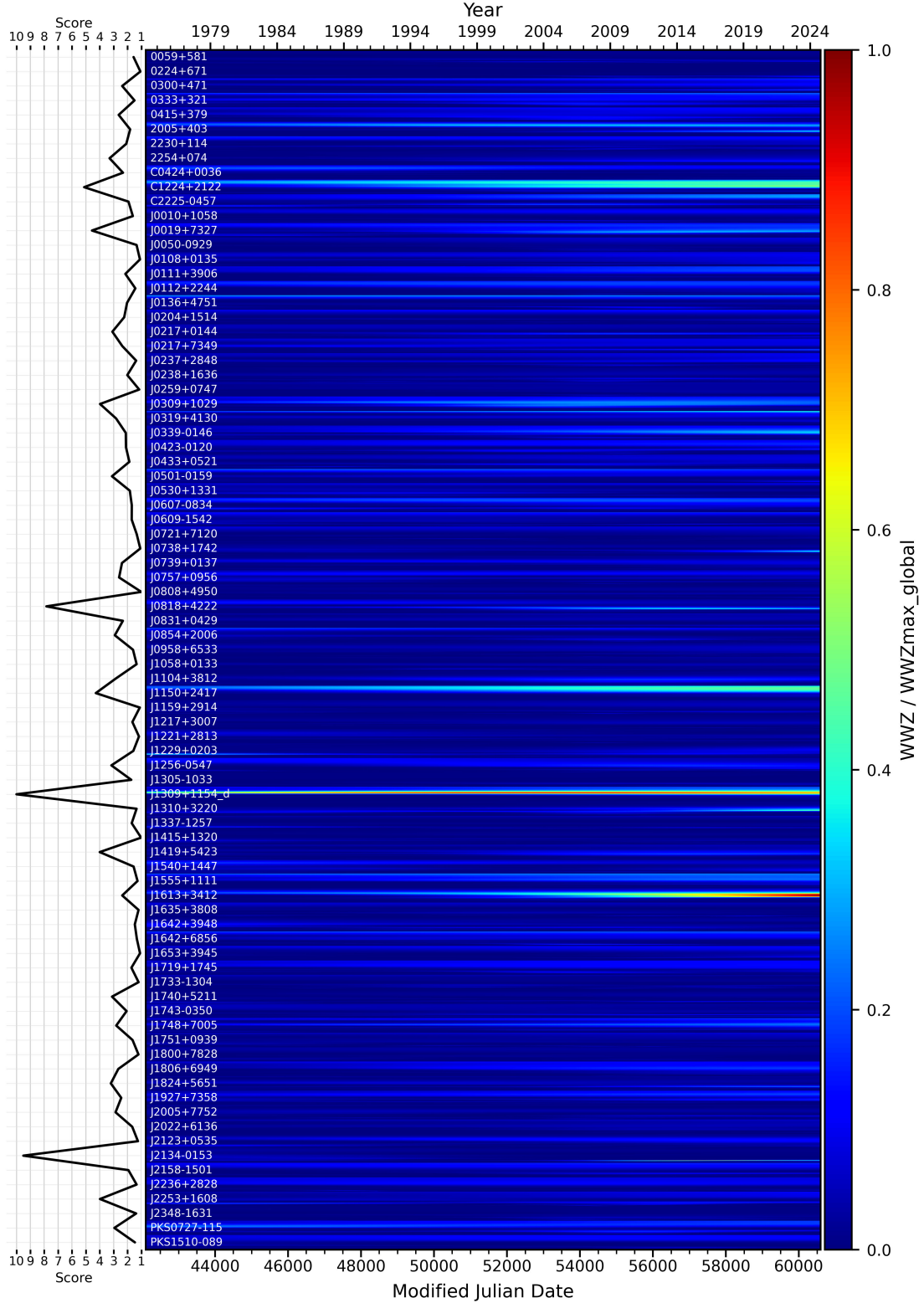


Figure 4.21: Global WWZ mosaic for OVRO+UMRAO. The curve on the left uses a proposed diagnostic metric conceived solely as a global visual guide. This metric considers WWZ power, period resolution, and temporal persistence to illustrate that these parameters are not independent when assessing a plausible periodicity. In this example, the metric was calibrated using the detrended light curve of J1309+1154, and the highest peaks are observed for J2134–0153 (PKS 2131–021) and J0818+4222. Further details are provided in Appendix F.

does not constitute a statistical significance selection.

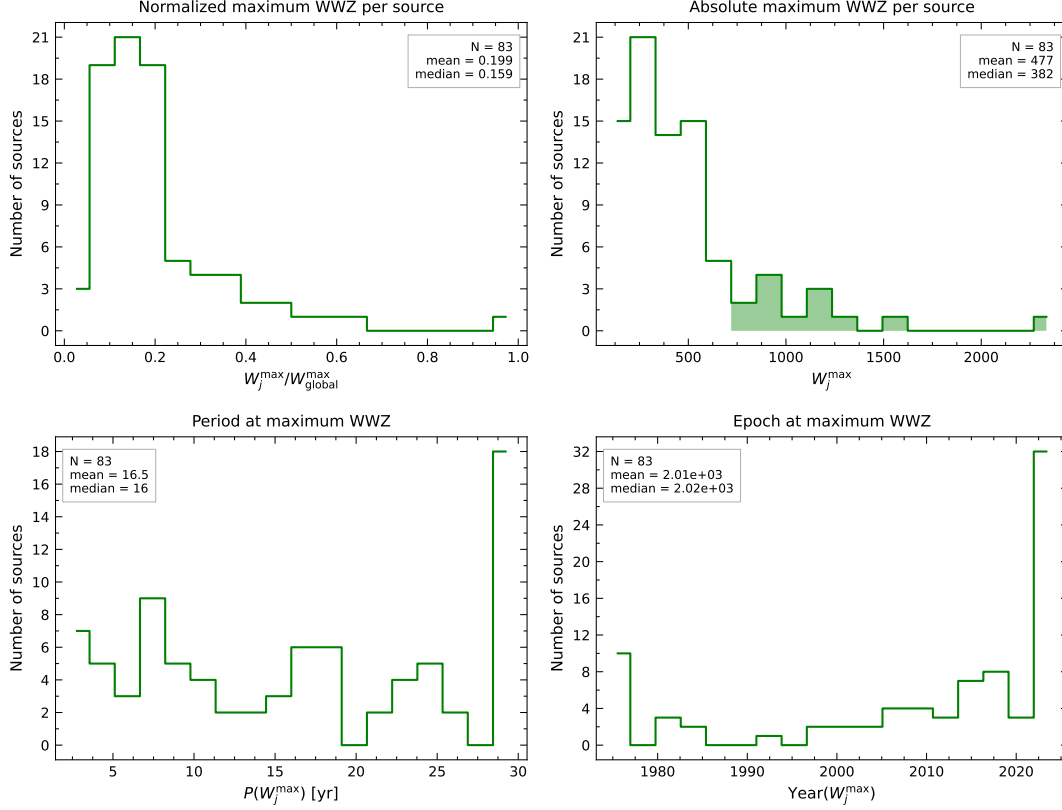


Figure 4.22: Global distributions derived from the WWZ analysis of the combined OVRO+UMRAO light curves. The panels show the maximum WWZ power normalized by the global maximum of the sample (top left), the absolute maximum WWZ power (top right), the period associated with this maximum (bottom left), and the year in which it occurs (bottom right). In the top-right panel, the shaded area identifies sources with $W_j^{\max}/W_{\text{global}}^{\max} \geq 0.30$.

4.7. Global Preliminary Results

From the global analysis of the sample, the source J1309+1154 emerges as the most interesting case. This source, classified as a blazar, is located at RA = 13:09:33.932 and Dec = +11:54:24.553, and its light curve shows features compatible with a persistent periodic modulation, making it a candidate supermassive black hole binary system. The GLSP results show notable consistency among the UMRAO, OVRO, and combined OVRO+UMRAO light curves. In particular, the main peak reaches a power close to $\mathcal{P}_1 \simeq 0.78$, standing out clearly above the secondary peaks. Moreover, this maximum appears as a narrow and well defined structure in the periodogram, reinforcing its relevance within the set of inspected sources.

Based on this dominant periodicity, a least squares sinusoidal fit was performed using the main period identified by the GLSP. The fit shows clear visual agreement with the combined light curve, reproducing a sinusoidal variation that spans approximately 2.5 cycles over the available temporal baseline (Fig. 4.19), with a period close to 17.2 years. This consistency between the periodogram and the global shape of the light curve provides an initial diagnostic indication of interest, although it does not by itself constitute a statistical confirmation of periodicity.

The WWZ analysis also shows power concentrated around the same period interval, suggesting that the signal maintains some degree of temporal coherence. However, the periodicity appears to be more prominent in the portion dominated by the OVRO data, so it will be necessary to carefully evaluate whether the detection is influenced by differences in cadence, temporal coverage, or the transition between the UMRAO and OVRO data sets. In addition, the light curve presents a secular decreasing trend, which must be considered in subsequent analyses, since it could produce slight changes in the recovered periods and powers. Consequently, J1309+1154 (Fig. 4.23) will be treated as a priority candidate and subjected to more rigorous statistical tests, including evaluations against red noise, temporal stability analyses, and robustness tests against the treatment of long term trends.

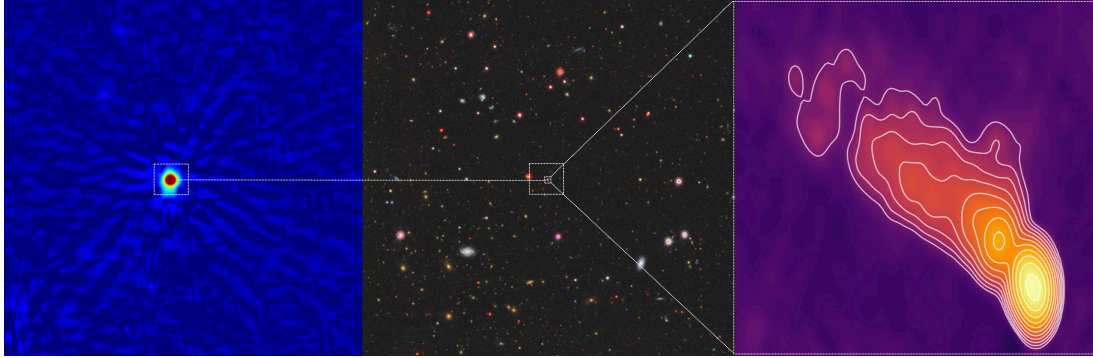


Figure 4.23: Illustrative multiwavelength view of J1309+1154, combining LoTSS/LOFAR emission at 144 MHz, the optical Pan-STARRS image, and the high-resolution VLBA/MOJAVE structure at 15 GHz. This comparison is intended as a visual guide and does not represent a strict angular-scale matching between the panels.

4.8. Introduction of the Source J1309+1154

J1309+1154 was identified as one of the most relevant sources in the sample after applying the preliminary inspection scheme defined in this section. This diagnostic combines the GLSP, sinusoidal fits associated with the main periodogram peaks, and the WWZ analysis, together with descriptive statistics and visual inspection of the diagnostic panels, (Appendix A, figure 4.20 and figure 4.22). In this context, the combined OVRO+UMRAO light curve shows a long-term modulation compatible with a dominant sinusoidal component, while the WWZ analysis suggests a relatively narrow and temporally persistent structure. This combination of temporal regularity, visual consistency among the preliminary methods, and significant WWZ power makes J1309+1154 a particularly relevant source and a promising SMBHB candidate to be tested in detail.

The source appears in the literature and astronomical databases under several identifiers, including 1307+121, OP 112, 4C 12.46, and MC2 1307+12. Its presence in early radio catalogues indicates that it has been known since the early stages of extragalactic radio astronomy. Among the most relevant historical detections are those associated with the Fourth Cambridge Survey at 178 MHz, the Ohio Sky Survey at 1415 MHz, and the Molonglo catalogues at 408 MHz. These early radio detections provide the historical basis for its identification before the development of modern variability monitoring programs. Subsequently, the source was included in the sample of 41 radio-loud BL Lac objects analyzed by [Aller et al. \(1999\)](#), using observations obtained with the UMRAO 26 m telescope at 4.8, 8.0, and 14.5 GHz. The study characterized total-flux variability and linear polarization and

found that, as a population, BL Lac objects exhibit greater total-flux variability than the QSOs in the Pearson–Readhead sample, supporting the existence of intrinsic differences between their parsec-scale relativistic jets.

This historical radio context is complemented by the modern classification reported by MOJAVE. According to this information, J1309+1154 is classified as a flat radio spectrum source, belongs to the LSP SED class, and has an optical identification of type B. However, no spectroscopic redshift is currently defined for this object. Therefore, any physical interpretation that depends directly on distance, intrinsic luminosity, or projected linear scales must be treated with caution.

Given these properties, J1309+1154 is considered here as a priority source for subsequent analysis. In particular, it will be subjected to more rigorous statistical tests aimed at evaluating the significance of the periodic signal, its temporal stability, and its possible physical origin. These tests must consider both the combined OVRO+UMRAO light curve and the UMRAO and OVRO light curves separately, in order to verify whether the signal persists under different observational configurations and is not dominated by sampling effects, cadence differences, or instrumental combination. Under the criteria described above, J2134–0153, also known as PKS 2131–021, also emerges as a remarkably clear and strong candidate. This source has already been studied in detail by O’Neill *et al.* (2022). Therefore, it will not be considered as the main target of this investigation. Instead, it will be used, when appropriate, as a control source for comparison with the candidates identified in the present analysis.

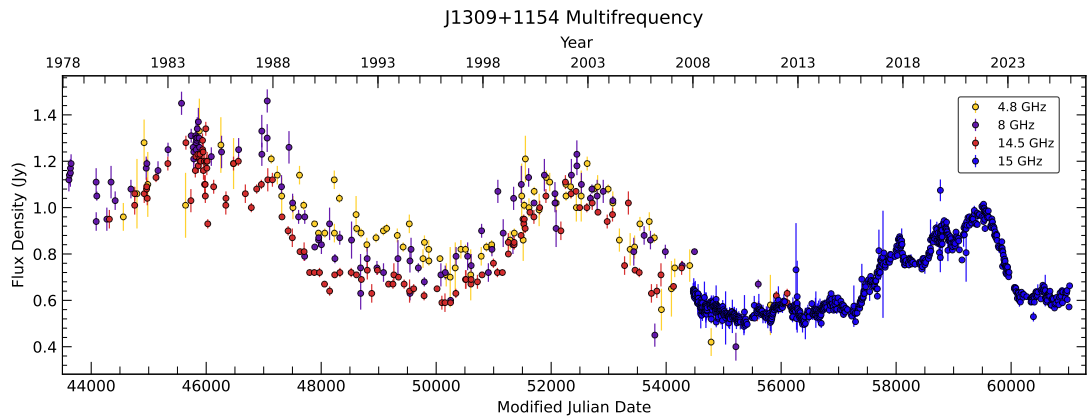


Figure 4.24: Multifrequency OVRO+UMRAO radio light curves of J1309+1154. The present analysis uses the high-frequency light curve, combining OVRO 15 GHz and UMRAO 14.5 GHz data. The curve also shows a noticeable long-term declining linear trend that must be considered in the periodicity analysis.

Chapter 5

Research Programme: Searching for SMBHB Candidates

Over the last few years, two blazars from the Owens Valley Radio Observatory (OVRO) 40 m monitoring program, PKS J0805–0111 and PKS 2131–021, have emerged as strong candidates for supermassive black hole binaries (SMBHBs). In both cases, the light curves exhibit remarkably persistent sinusoidal variations, and periodicity analyses have shown that these are unlikely to be explained by the random variability expected from the steep power spectral density of blazar light curves.

These two sources naturally raised a broader question: if such behavior can be found within the OVRO monitoring program, what might be learned from revisiting a smaller subset of sources for which substantially longer radio light curves are available?. In this context, the 83 blazars common to the OVRO and UMRAO monitoring programs constitute a particularly valuable sample, with a 46-year time baseline and an excellent combined cadence that is approximately weekly on average.

Moreover, because the combined light curves are constructed from two independent monitoring programs, they allow a more stringent assessment of whether any candidate periodic behavior remains coherent, both in the combined dataset and when the OVRO and UMRAO data are examined separately.

The first step was to perform standard analyses for each source: GLSP (Generalized Lomb–Scargle Periodogram), WWZ (Weighted Wavelet Z-transform), and a time-

domain corroboration based on simple sinusoidal least-squares fits, both for each telescope individually and for the combined light curve. The idea was not to rely on a single method, but rather to examine the problem from several angles and assess its reliability.

At this stage of the investigation, and in agreement with the conclusions presented at the end of the previous chapter, J1309+1154 has emerged as the most robust candidate within the analyzed sample. Nevertheless, interpreting a periodic signal as possible evidence of a supermassive black hole binary system requires a progressive validation process. In particular, before confronting the observed signal with physical models of periodicity, such as the K.O., M.K.O., and Wind-Jet models discussed by O’Neill et al. (2022); Kiehlmann et al. (2025); Sullivan et al. (2026), it is necessary to assess whether the detected periodicity can be regarded as intrinsic to the source or whether it could instead be apparently reproduced by the stochastic red-noise variability characteristic of blazars.

In this context, the following section presents a series of tests, observational evidence, and complementary inspections aimed at providing preliminary support for the periodicity identified in J1309+1154. These evaluations do not constitute a definitive confirmation by themselves, but they allow the coherence of the signal to be examined from different perspectives before applying the statistical significance analysis commonly used in this type of study.

5.1. Evidence of periodicity in J1309+1154

As shown in Figure 5.1, the GLSP periodograms obtained for the different datasets reveal a dominant signal that is consistent across telescopes. In particular, the main peak associated with J1309+1154 reaches a normalized power above 80%, indicating a prominent periodicity in the analyzed light curve. This feature is especially relevant because blazar light curves are commonly dominated by stochastic red-noise variability. Therefore, a modulation that is visually close to a coherent sinusoidal component represents an uncommon behavior among this type of source. For further context, the reader is referred to Appendix A, where J1309+1154 clearly stands out with respect to the rest of the analyzed sample.

The results obtained for the detrended light curve are presented in Figure 5.1

and 5.2. In this analysis, an oversampling factor of $\zeta = 29$ was adopted, and the search was restricted to periods longer than 100 days in order to avoid timescales dominated by short-term variability or by sampling-related effects. This periodicity was also independently examined by Molina et al. (2026) using a sine-wave fitting (SWF) method based on Markov Chain Monte Carlo (MCMC), whose results are summarized in Table E.2 of Appendix E.

As a complementary test, the Weighted Wavelet Z-transform (WWZ) method was applied to evaluate the temporal persistence of the periodic signal. For this analysis, a Gaussian weighting parameter of $c = 0.0125$ was adopted, following the value suggested by Foster (1996), and a period range from 100 to 11,000 days (approximately 0.27–30.12 years) was explored. The results, shown in Figure 5.2, allow us to examine whether the periodicity identified by the GLSP remains coherent across the temporal baseline of the light curve.

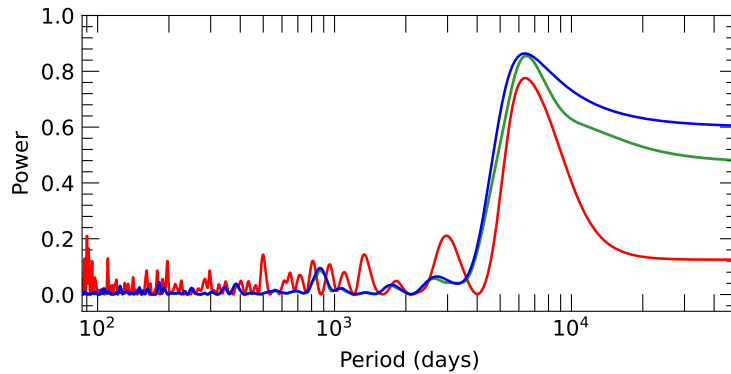


Figure 5.1: Unified GLSP, with red, blue, and green curves corresponding to UMRAO, OVRO, and the combined dataset, respectively.

Table 5.1: Period estimates for J1309+1154 from SWF, GLSP, and WWZ analyses.

Method	P (days)	P (days)	P (days)
	UMRAO	OVRO	OVRO+UMRAO
SWF	6391^{+115}_{-107}	6201 ± 48	6551 ± 26 ^(a)
GLSP	6395.53	6256.73	6286.44
WWZ ^(b)	≈ 6757.13	≈ 6537.98	≈ 6611.03

^(a) Value obtained using SWF for the detrended combined light curve. ^(b) The WWZ values are based on visual inspection and should therefore be interpreted as approximate. Due to the nature of a WWZ map, defining a unique exact period is non-trivial, since the signal may appear as a broad or time-dependent ridge in the time–period plane. The periods estimated using SWF for the remaining sources are listed in Table E.2 of Appendix E.

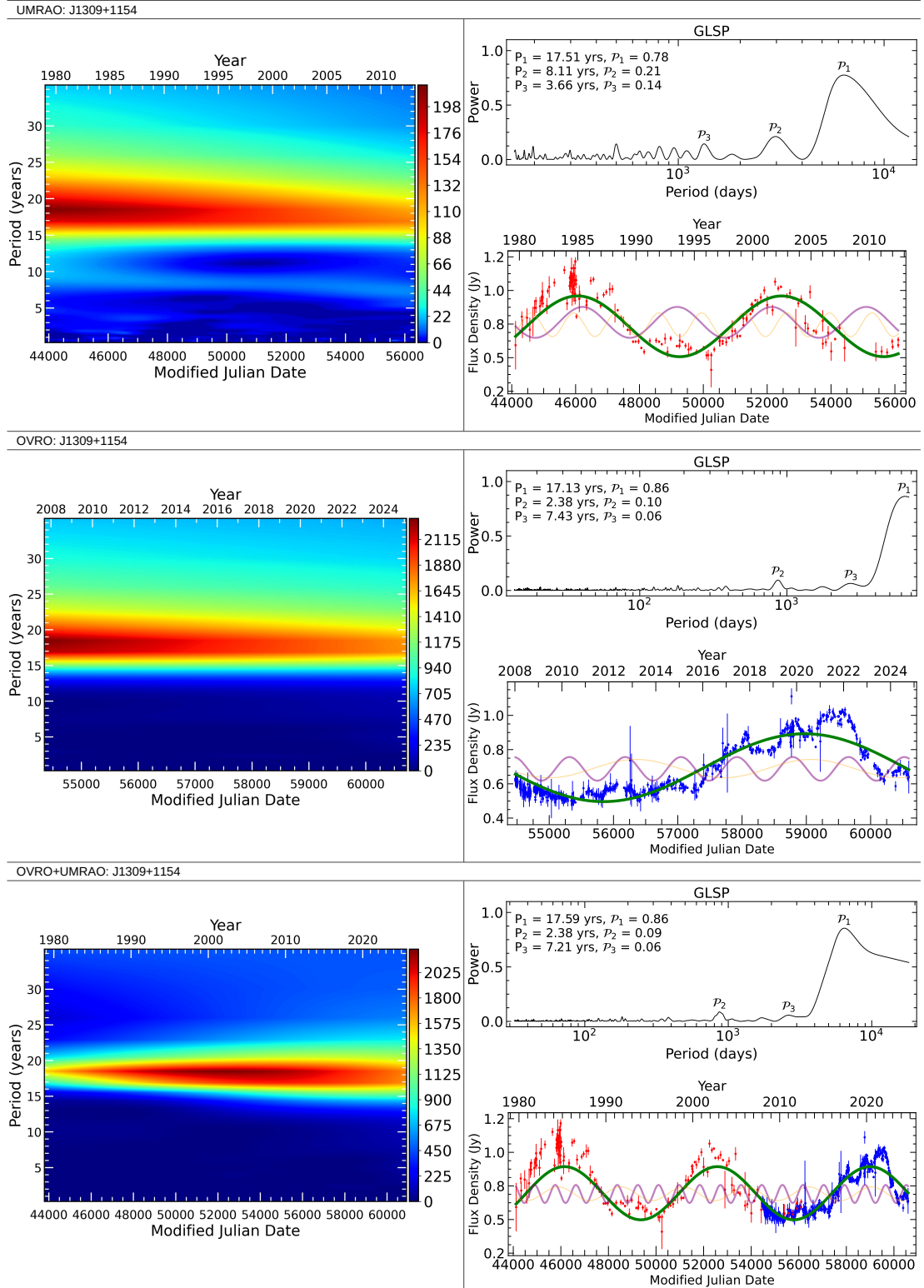


Figure 5.2: Preliminary detrended analyses of J1309+1154 for UMRAO, OVRO, and the combined OVRO+UMRAO data. The sinusoidal fits use the three strongest GLSP peaks. The harmonic near ~ 9 yr is clearly visible in the UMRAO WWZ. The UMRAO WWZ is displayed using a square-root power scale, whereas the OVRO and combined OVRO+UMRAO WWZ plots are displayed using a linear power scale. See Appendix A for the full sample.

5.1.1 Harmonic in PKS J1309+1154

Based on the preliminary results presented in Sections 4.6 and 4.7, persistent sinusoidal modulation is expected to be uncommon in AGN light curves because these objects are generally dominated by stochastic red-noise variability. Therefore, PKS J1309+1154 stands out from the rest of the sample.

The periodicity of PKS J1309+1154 was detected in 2024 and reported by (Molina et al., 2026). It has a period of 17.9 years. The observations cover approximately 46 years, corresponding to about 2.5 cycles.

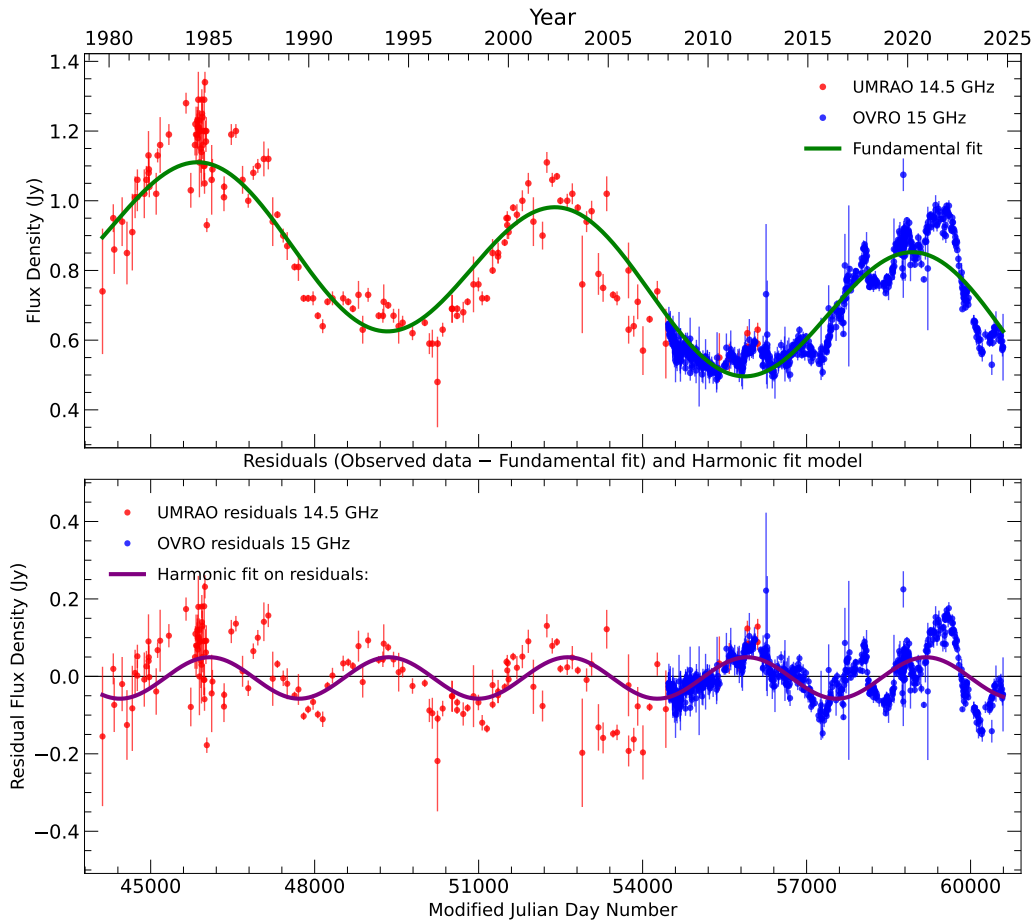


Figure 5.3: Top: Least-squares sinusoidal fit (green line) obtained using the SWF model described by Molina et al. (2026) (Equation E.3; Table E.1). Bottom: Sinusoidal fit to the residuals (purple line), with a period of approximately 9 years.

The upper panel of Figure 5.3 shows the UMRAO and OVRO light curves in red and blue respectively. The green line represents a least-squares sinusoidal fit. The model includes a linear trend with a slope of $-0.00755 \text{ Jy yr}^{-1}$ and a sinusoidal amplitude of 0.22 Jy. The light curve shows maxima around 1984, 2002 and 2020.

The lower panel shows the residual light curve and its sinusoidal fit in purple. This component has a period of approximately 9 years and a phase relationship consistent with the fundamental periodicity. A subsequent paper examined this signal and analyzed 11 regions (Readhead et al., 2026). The authors showed that the signal is not an artifact caused by subtracting the fundamental component. It is also detected in the original light curve using the WWZ method, as shown in Figure 5.4, and remains coherent throughout the 46-year observing period. Also it was found that the deviations from the fundamental periodicity alternate as predicted by the harmonic in all 11 regions. The statistical significance of this result is $p = 4.9 \times 10^{-4}$, which is equivalent to 3.3σ (see Appendix H).

The period ratio is approximately $P/2$ and the two signals maintain a coherent phase relationship. These properties are consistent with orbital Doppler modulation or with two relativistic jets associated with a binary system. The results provide strong evidence for a supermassive black hole binary (SMBHB), although they are not conclusive and are not unique to this interpretation.

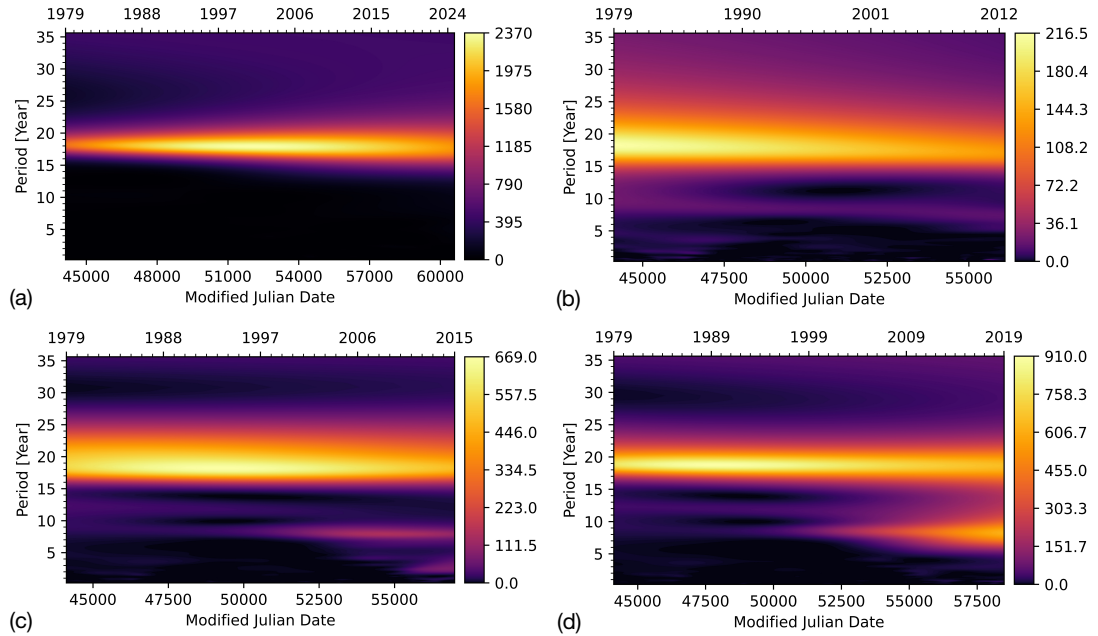


Figure 5.4: WWZ maps of PKS J1309+1154. Panel (a) shows the WWZ spectrum computed from the full UMRao+OVRO dataset on a linear power scale, where the fundamental periodicity dominates and the harmonic is not clearly resolved. Panel (b) shows the UMRao data alone from 1979 to 2012. Panels (c) and (d) show the UMRao data with OVRO observations added up to 2015 and 2019, respectively. Panels (b), (c), and (d) are displayed using a square-root power scale, which enhances the weaker feature near ~ 9 yr, consistent with a first harmonic in a 2:1 relation with the fundamental period.

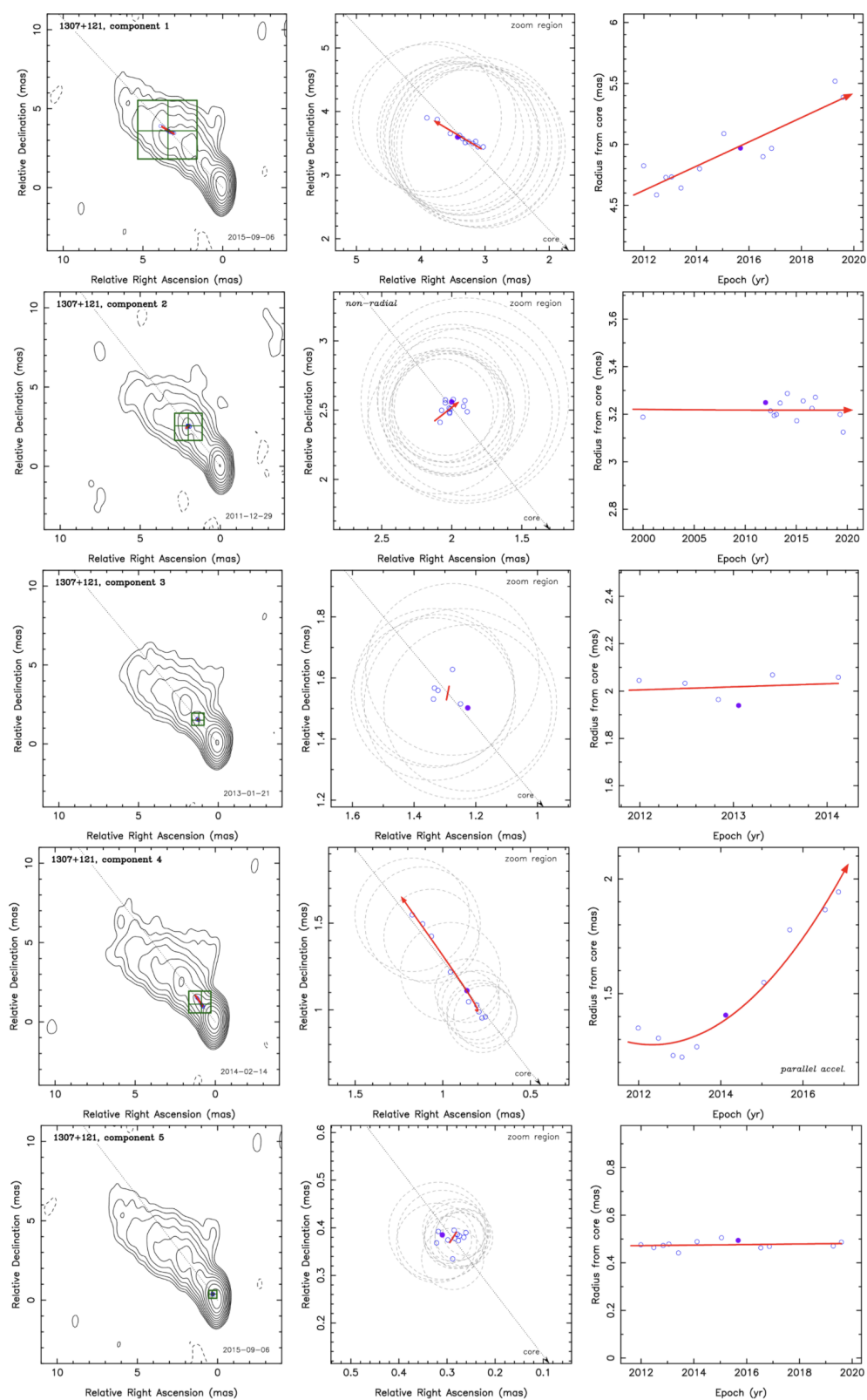
5.1.2 VLBA observations of PKS J1309+1154

The 15 GHz VLBA observations from the Monitoring of Jets in Active Galactic Nuclei with VLBA Experiments program MOJAVE, (Lister et al., 2021) allow the parsec scale structure of PKS J1309+1154 to be studied with an angular resolution that is not accessible from single dish observation. This is essential to assess whether the periodic modulation observed in UMRAO+OVRO is associated with the compact central region or with more extended jet components. The MOJAVE data set for PKS J1309+1154 comprises 13 observing epochs: one in 1999 and the remaining 12 between 2011 and 2019, making it possible to trace the temporal evolution of its VLBA components.

The flux densities of the core and jet, together with the modeled components, were taken from the MOJAVE catalog available through: [VizieR, J/ApJ/923/30/table4](https://vizier.cfa.harvard.edu/vizier/VizieR, J/ApJ/923/30/table4), based on the supplementary material of (Lister et al., 2021). The component models were obtained through Gaussian fits to the VLBA interferometric visibilities using DIFMAP (Shepherd et al., 1994; Shepherd, 1997). From these fits, the radial separation from the core, temporal evolution, and relative contribution of each component to the total emission were estimated.

Figures 5.5 and 5.6 (upper) summarize the kinematics of the identified components. Some external components show appreciable radial displacements, whereas component 5, the closest one to the core, remains at a small and nearly constant separation over the available epochs. This stability suggests that component 5 is not a moving feature propagating along the jet, but rather a compact and stationary structure associated with the core or the inner jet.

The connection between the VLBA structure and the integrated variability is shown in Figure 5.6 (lower). The comparison between the UMRAO+OVRO light curve, the total MOJAVE flux, the core, the core+5 contribution, and the individual components indicates that the long-term variability is dominated by the compact central region. In particular, the evolution of the core and of the core+5 combination follows the modulation observed in the single dish light curve more closely than the external components. Therefore, the periodicity detected in PKS J1309+1154 appears to be mainly associated with the nuclear region or the unresolved inner jet, rather than with moving structures located farther downstream in the jet.



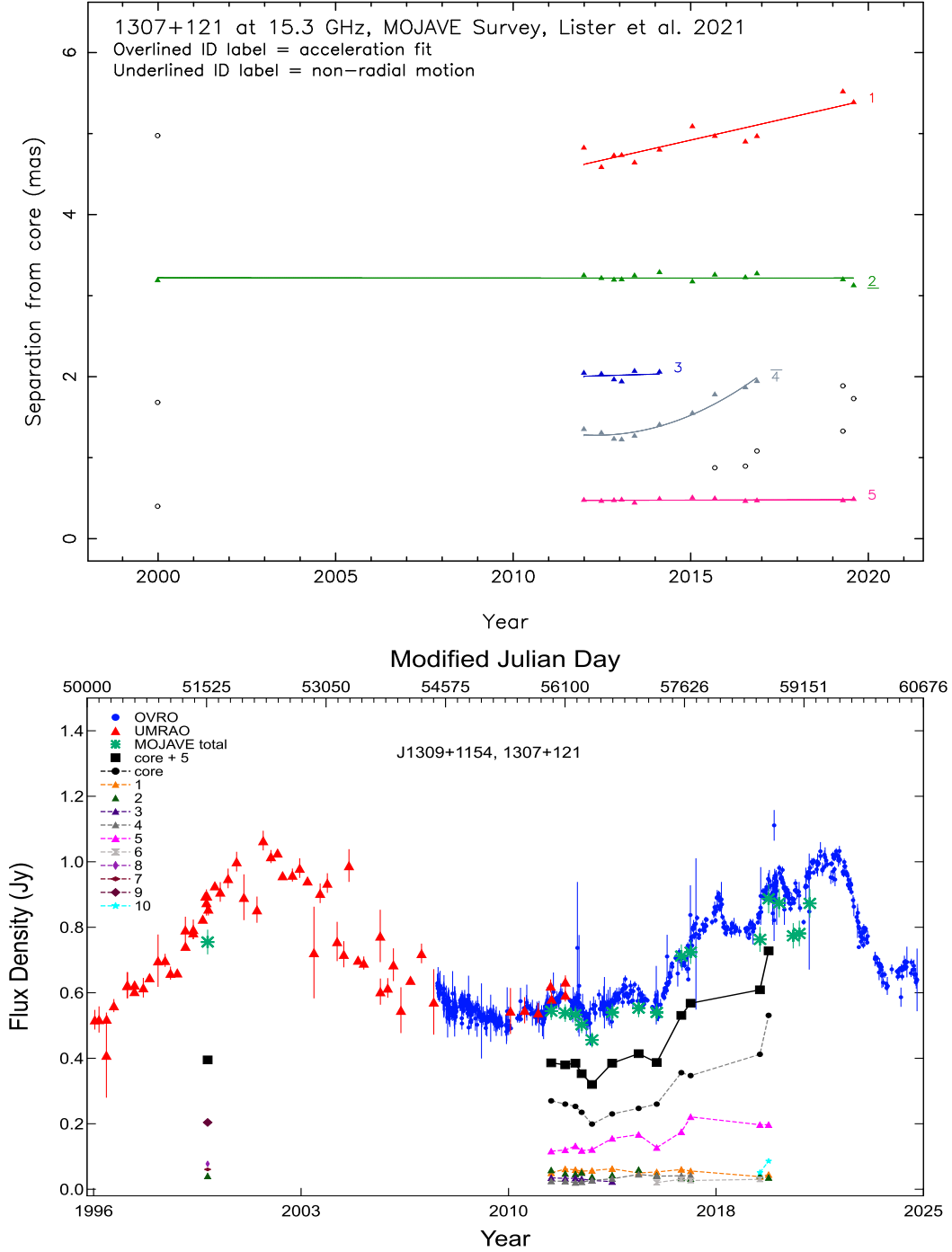


Figure 5.6: Top: component separation from the core as a function of time (Lister et al., 2021). Bottom: UMRAO/OVRO light curve of J1309+1154 compared with MOJAVE total flux and VLBA component flux densities from DIFMAP model fitting. The core and nearby component 5 broadly follow the single-dish modulation, unlike the more distant jet components. (Molina et al., 2026)

5.1.3 ALMA Observations of PKS J1309+1154

The ALMA observations presented by [Molina et al. \(2026\)](#) provide a relevant millimeter-frequency comparison for the periodic modulation detected in PKS J1309+1154. In particular, the Band 3 data, obtained at 91.5 and 104 GHz, allow the temporal evolution of the source to be compared with the 15 GHz OVRO light curve over the common observing window. Because the number of available ALMA epochs is limited and the measurements at both frequencies are closely matched in time, these observations are treated jointly as Band 3 data.

As shown in Figure 5.7, the ALMA points follow the same broad sinusoidal trend observed by OVRO, providing a multi-frequency consistency check of the long-term modulation. For this comparison, the period derived from the full UMRAO+OVRO temporal baseline is not adopted. Instead, a least-squares sinusoidal fit is performed on the OVRO light curve over its own observing window, since correlated non-sinusoidal variability can introduce apparent period differences when different temporal intervals are fitted. This fit yields a period of 6201 days, equivalent to 16.98 yr, which is subsequently held fixed when fitting a sinusoid to the ALMA data.

Although the ALMA fit does not provide an independent determination of the period, which is fixed to the OVRO value, it shows temporal evolution consistent with the same modulation. Taking the OVRO fit as the temporal reference, the ALMA fit has an offset of -300^{+260}_{-200} days; the negative sign indicates that the ALMA modulation leads the OVRO modulation, although with large uncertainties. This possible lead is compatible with higher-frequency emission originating farther upstream in the jet, as discussed in the interpretation of the frequency-dependent phase shifts in PKS 2131–021 by [Kiehlmann et al. \(2025\)](#).

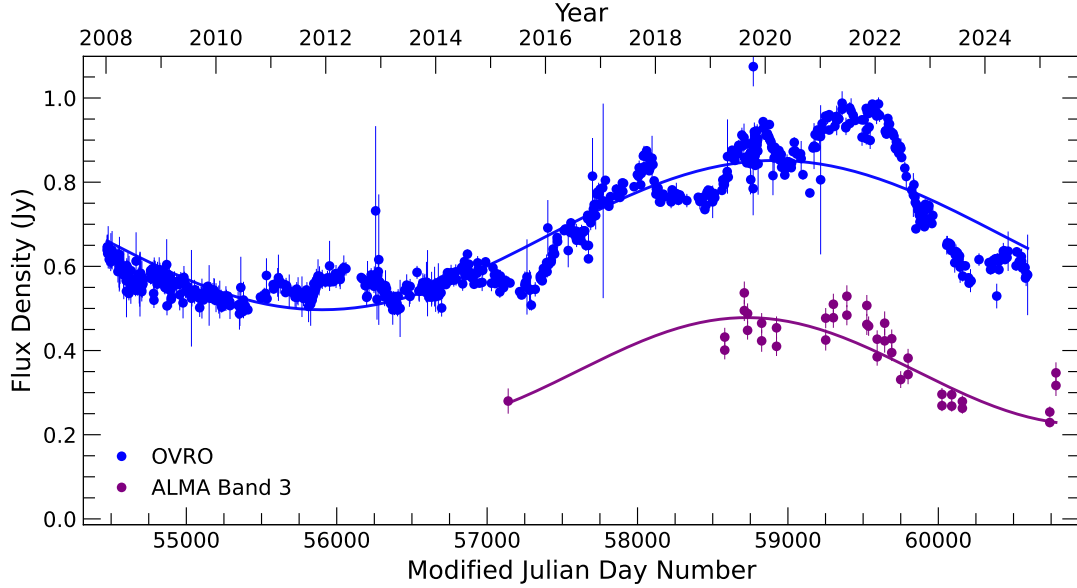


Figure 5.7: OVRO 15 GHz (blue) and ALMA Band 3 (purple) light curves of PKS J1309+1154. Solid curves show the least-squares sinusoidal fit to OVRO ($P = 6201$ days) and the fit to ALMA with this period held fixed. The ALMA fit has a temporal offset of -300^{+260}_{-200} days relative to OVRO, indicating that ALMA leads OVRO (Molina et al., 2026).

5.1.4 Metsähovi observations of PKS J1309+1154

The Metsähovi observations at 37 GHz provide additional verification of the long-term modulation detected in the UMRAO+OVRO light curves. Although the sampling is sparser than that of OVRO and the uncertainties are larger, the 37 GHz light curve shows an evolution that is qualitatively consistent with the main periodicity of PKS J1309+1154. In particular, an apparent maximum is observed near the beginning of the series, around 2002, followed by a progressive decrease to a relative minimum between approximately 2008 and 2011. Subsequently, the source flux increases again and reaches a new maximum around 2019.8–2020.0.

The temporal separation between the apparent maxima at ~ 2002 and ~ 2020 is approximately 18 years, in good agreement with the period of 17.9 years inferred from the UMRAO+OVRO light curve. This agreement is relevant because Metsähovi observes at a considerably higher frequency, 37 GHz, so the presence of a compatible modulation suggests that the periodicity is not restricted to the 14.5/15 GHz observations. Nevertheless, because of the lower cadence and the more sparsely distributed nature of the data, these observations should be interpreted as qualitative, multifrequency confirmation rather than as an independent high-

precision measurement of the period. The 37 GHz single-dish total flux-density observations were obtained with the 14-metre telescope at the Aalto University Metsähovi Radio Observatory in Finland and were provided by M. Tornikoski (private communication).

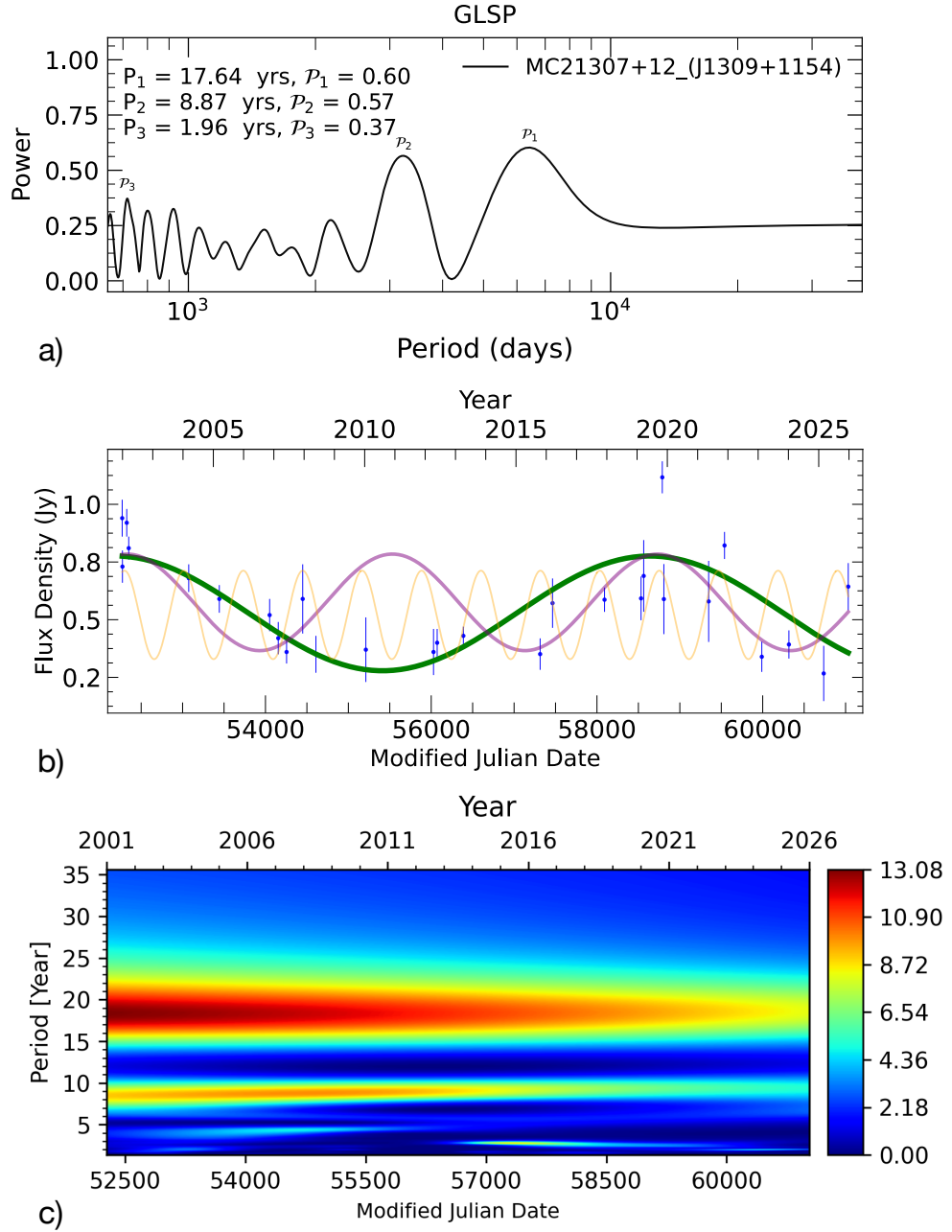


Figure 5.8: Preliminary periodicity analysis of J1309+1154 using the Metsähovi dataset: (a) GLSP; (b) light curve with sinusoidal fits corresponding to the three highest-power GLSP peaks (17.64, 8.87, and 1.96 yr, shown in green, purple, and orange, respectively); and (c) WWZ map. A secondary WWZ feature near 9 yr is consistent with a possible harmonic of the approximately 18 yr modulation.

5.2. Multifrequency radio light curves of J1309+1154

Figure 5.9 brings together the radio light curves of J1309+1154 from UMRAO, OVRO, Metsähovi, MOJAVE/VLBA, and ALMA to compare the long-term modulation across frequencies. Within the overlapping observing intervals, the datasets display broadly similar rises and declines in flux density, despite differences in flux-density levels, sampling, and temporal coverage. This visual correspondence suggests correlated long-term variability across the sampled radio bands.

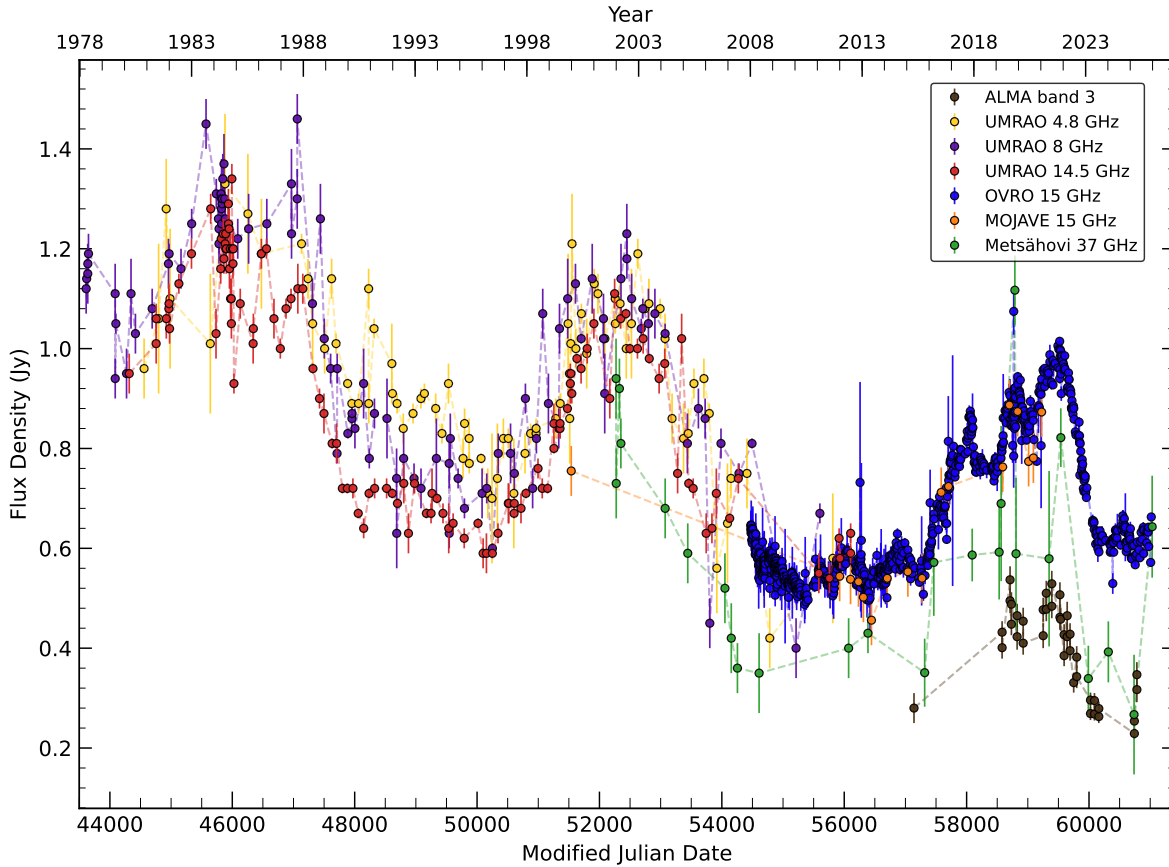


Figure 5.9: Multifrequency radio light curves of J1309+1154 combining single-dish total-flux-density measurements from UMRAO (26-m telescope; 4.8, 8.0, and 14.5 GHz), OVRO (40-m telescope; 15 GHz), and Metsähovi (14-m telescope; 37 GHz), together with MOJAVE/VLBA measurements near 15 GHz and ALMA Band 3 observations at 91.5 and 103.5 GHz.

5.3. Stochastic Modeling of Blazar Light Curves

The search for periodicities in blazar light curves requires a careful statistical assessment, since the intrinsic variability of these sources is often dominated by irregular flares, red noise, and non Gaussian flux density distributions. In this context, a prominent peak in a periodogram does not by itself constitute evidence for a physical periodicity, since apparently periodic structures can arise naturally from stochastic processes with a steep power spectral density. For this reason, the significance of a candidate periodicity is evaluated through a procedure based on light curve simulations that reproduce the relevant properties of the observed source, including the slope of the power spectral density, the probability distribution of the flux densities, the temporal sampling, and the measurement uncertainties.

The purpose of these simulations is not to demonstrate the existence of a periodic signal, but to construct a reference distribution to estimate how often stochastic variability, compatible with the observed properties of the source, can produce power maxima in a periodogram, such as the GLSP, equal to or larger than those measured in the real data.

The comparison between the observed statistic and the distribution obtained from the simulations allows an empirical probability to be assigned to the detected signal, which can be expressed as an equivalent significance in units of σ .

Under this approach, a periodicity is considered consistent with stochastic variability when signals of comparable power appear with non-negligible frequency in the simulated light curves. Conversely, a periodicity that exceeds the adopted significance threshold can be considered a candidate physical periodicity. This classification must be interpreted conservatively: a high statistical significance does not by itself prove the presence of a supermassive black hole binary system, but rather indicates that the purely stochastic explanation is insufficient within the variability model considered.

In this investigation, we adopted the same approach used by [O'Neill et al. \(2022\)](#). To describe its implementation, it is first necessary to introduce some fundamental concepts that underlie the construction of the simulations and the statistical interpretation of their results.

5.3.1 Power Spectral Density and PSD Slope

The power spectral density, or PSD, describes how the variance of a light curve is distributed over temporal frequencies. For a finite, mean-subtracted signal, the PSD can be estimated from the squared modulus of its Fourier transform, normalized by the observing duration; this estimate is known as the periodogram. For a weakly stationary stochastic signal $x(t)$ with autocovariance function $C(\tau)$, the underlying PSD is formally defined as the Fourier transform of the autocovariance,

$$P(\nu) = \int_{-\infty}^{\infty} C(\tau) e^{-2\pi i \nu \tau} d\tau, \quad (5.1)$$

where ν is the temporal frequency and τ is the time lag. Physically, larger values of $P(\nu)$ indicate a greater contribution to the variance per unit temporal frequency around the associated timescale, $T \sim 1/\nu$.

In blazar light curves, the PSD is commonly approximated by a power law over the frequency range of interest,

$$P(\nu) = A\nu^{-\beta}, \quad (5.2)$$

where A is a normalization constant and β quantifies the spectral slope. In logarithmic space, this relation becomes

$$\log P(\nu) = \log A - \beta \log \nu, \quad (5.3)$$

so that a steeper PSD, corresponding to a larger value of β , implies a larger relative contribution of variability at low temporal frequencies, or equivalently on long timescales. This property is essential for assessing the significance of candidate periodicities, since a steep PSD can produce prominent maxima in a periodogram even in the absence of a physical periodicity.

Figure 5.10 shows an illustrative approximation to the observed PSD of PKS J1309+1154, obtained using the Lomb–Scargle periodogram with Astropy’s `psd` normalization. This representation remains affected by the finite duration and irregular sampling of the observed light curve and therefore does not, by itself, recover the intrinsic PSD. In particular, the finite observing duration can bias the estimated PSD slope through red-noise leakage; for unevenly sampled

light curves, interpolation onto a regular time grid followed by the application of an appropriate window function helps mitigate this effect (Max-Moerbeck et al., 2014).

The intrinsic PSD slope is inferred through a forward Monte Carlo approach (Uttley et al., 2002), following the procedure described by O’Neill et al. (2022). Figure 5.11 illustrates this procedure for PKS J1309+1154, comparing the observed PSD with simulated PSDs using data available through 2024. For the significance analyses, the adopted slope is $\beta = 1.66$, estimated after applying a linear-trend correction to the data available through 2023. Under the convention $P(\nu) \propto \nu^{-\beta}$, this corresponds to a red-noise-dominated PSD, in which the power increases toward low temporal frequencies. Further details on the PSD-slope estimation and the inference of the intrinsic PSDs are provided in Appendix D, together with plots analogous to Figure 5.11 for all 83 sources in the OVRO+UMRAO sample. Table 5.2 lists the estimated PSD slopes, β , for the combined OVRO+UMRAO light curves of all 83 sources.

Figure 5.12 shows the distribution of the best-fitting intrinsic PSD slopes, β , for the 83 sources in the combined OVRO+UMRAO sample, estimated following the methodology described by O’Neill et al. (2022) and Max-Moerbeck et al. (2014). Most values lie between approximately 1.3 and 2.1, with a few sources extending to steeper slopes. The mean and median are 1.74 and 1.72, respectively, indicating that the sample is typically characterized by stronger variability power at low temporal frequencies under the convention $P(\nu) \propto \nu^{-\beta}$. The histogram includes only the best-fitting values and does not represent their individual uncertainties, β_{err} .

Table 5.2: Intrinsic power spectral density slopes for the OVRO+UMRAO sample, ordered by right ascension.

Source	Slope β	β_{err}	Source	Slope β	β_{err}
J0010+1058	1.46	0.09	J1217+3007	1.50	0.22
J0019+7327	2.18	0.19	J1221+2813	2.05	0.16
J0050-0929	1.62	0.10	C1224+2122	2.01	0.15
0059+581	1.70	0.10	J1229+0203	1.76	0.10
J0108+0135	2.07	0.15	J1256-0547	1.88	0.12
J0111+3906	1.73	0.34	J1305-1033	1.58	0.10

Continued on next page

Table 5.2 – continued from previous page

Source	Slope β	β_{err}	Source	Slope β	β_{err}
J0112+2244	1.94	0.12	J1309+1154	1.62	0.10
J0136+4751	1.63	0.09	J1310+3220	1.81	0.11
J0204+1514	1.80	0.11	J1337-1257	1.56	0.10
J0217+7349	1.69	0.12	J1415+1320	1.80	0.12
J0217+0144	1.69	0.10	J1419+5423	2.04	0.13
0224+671	1.81	0.14	PKS1510-089	1.54	0.08
J0237+2848	1.54	0.08	J1540+1447	1.93	0.14
J0238+1636	1.39	0.07	J1555+1111	1.94	0.34
J0259+0747	1.61	0.10	J1613+3412	2.06	0.17
0300+471	1.89	0.14	J1635+3808	1.70	0.11
J0309+1029	1.88	0.15	J1642+6856	1.49	0.09
J0319+4130	1.95	0.12	J1642+3948	1.80	0.11
0333+321	1.71	0.11	J1653+3945	1.36	0.11
J0339-0146	1.75	0.12	J1719+1745	1.72	0.11
0415+379	1.61	0.09	J1733-1304	1.41	0.07
J0423-0120	1.62	0.09	J1740+5211	1.47	0.09
C0424+0036	1.63	0.10	J1743-0350	1.81	0.12
J0433+0521	1.67	0.10	J1748+7005	1.80	0.13
J0501-0159	2.36	0.20	J1751+0939	1.39	0.07
J0530+1331	1.61	0.09	J1800+7828	1.38	0.08
J0607-0834	1.88	0.13	J1806+6949	1.81	0.13
J0609-1542	1.70	0.10	J1824+5651	1.67	0.12
J0721+7120	1.37	0.07	J1927+7358	1.67	0.11
PKS0727-115	1.83	0.11	J2005+7752	1.95	0.12
J0738+1742	1.98	0.16	2005+403	1.75	0.12
J0739+0137	1.43	0.08	J2022+6136	1.73	0.26
J0757+0956	1.68	0.11	J2123+0535	1.80	0.11
J0808+4950	1.72	0.11	J2134-0153	1.95	0.15
J0818+4222	1.93	0.15	J2158-1501	1.96	0.14
J0831+0429	1.81	0.11	C2225-0457	1.90	0.12
J0854+2006	1.69	0.09	2230+114	1.53	0.08
J0958+6533	1.68	0.11	J2236+2828	1.58	0.09
J1058+0133	1.74	0.11	J2253+1608	1.82	0.10
J1104+3812	1.34	0.06	2254+074	1.75	0.14
J1150+2417	2.71	0.50	J2348-1631	1.50	0.09
J1159+2914	1.63	0.09	...	...	...

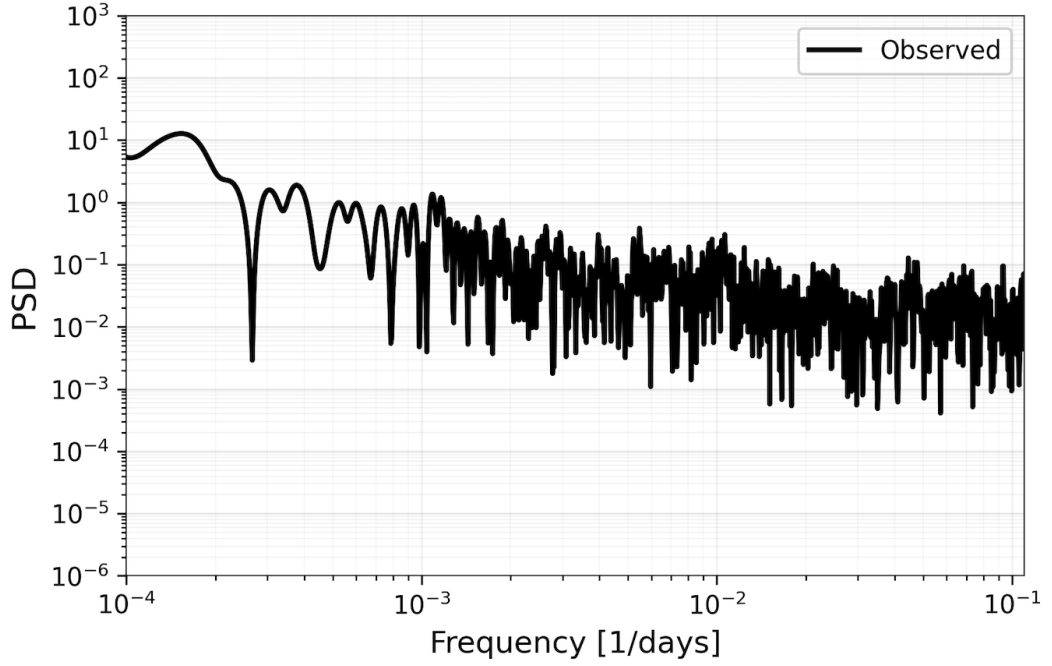


Figure 5.10: Illustrative approximation to the observed PSD of PKS J1309+1154, obtained using the Lomb–Scargle periodogram with Astropy’s `psd` normalization. This estimate remains affected by the finite duration and irregular sampling of the light curve and does not directly represent the intrinsic PSD.

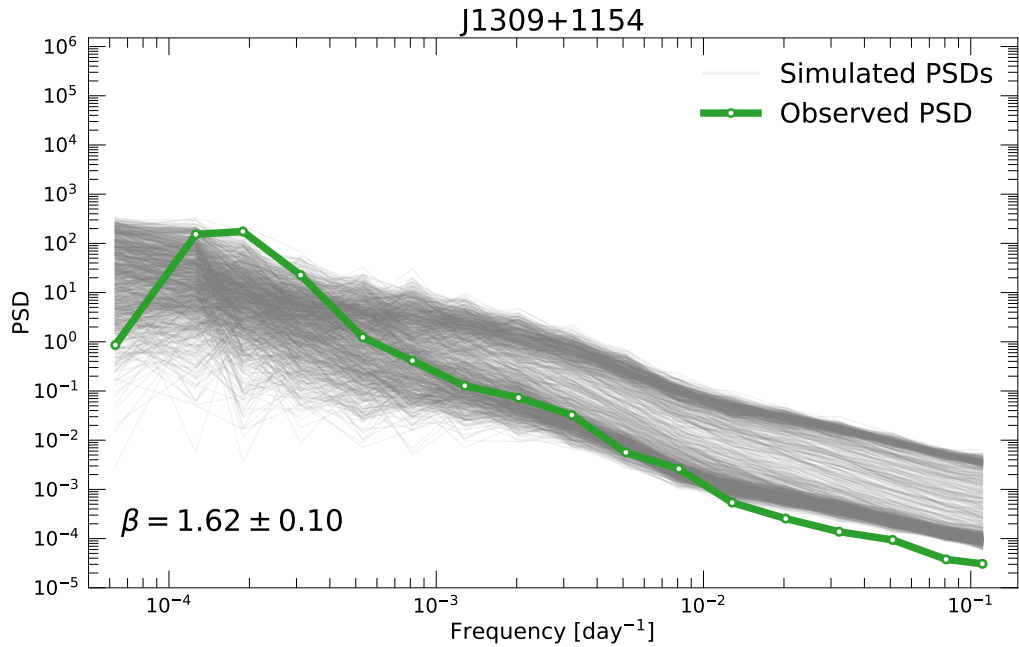


Figure 5.11: Observed PSD of PKS J1309+1154, shown in green, obtained using the procedure described by O’Neill et al. (2022). The intrinsic PSD slope, β , was estimated through a forward Monte Carlo analysis using the data available through 2024. The PSDs corresponding to the 1000 simulations are shown as gray lines. For the significance analyses, a value of $\beta = 1.66$, estimated using data available through 2023, was adopted. Details of the method and corresponding plots for all 83 sources in the OVRO+UMRAO sample are presented in Appendix D.

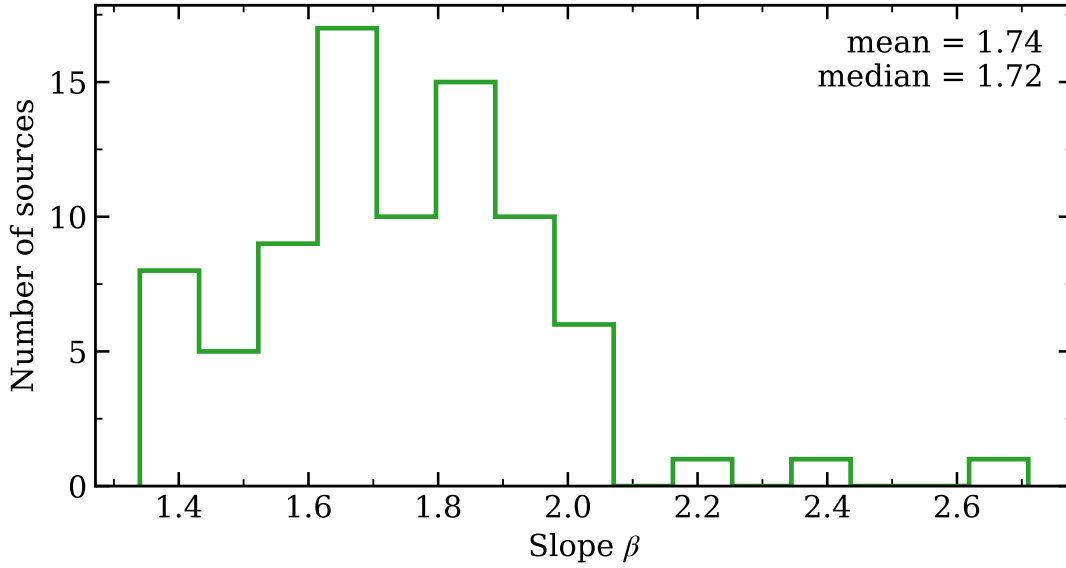


Figure 5.12: Distribution of the intrinsic PSD slopes for the OVRO+UMRAO sample. The mean and median are indicated.

5.3.2 Probability Density Function

The probability density function, or PDF, describes the distribution of the flux-density values in a light curve, without retaining the temporal order of the observations. For flux-density measurements $S_\nu(t_i)$, with $i = 1, \dots, N$, the corresponding PDF is denoted as $p(S_\nu)$, is non-negative, and satisfies the normalization condition

$$\int_{-\infty}^{\infty} p(S_\nu) dS_\nu = 1. \quad (5.4)$$

Thus, the probability of observing a flux density between S_1 and S_2 is given by

$$P(S_1 \leq S_\nu \leq S_2) = \int_{S_1}^{S_2} p(S_\nu) dS_\nu. \quad (5.5)$$

In practice, the PDF can be estimated from the empirical distribution of the flux-density measurements. A simple estimator is the normalized histogram,

$$\hat{p}_k = \frac{n_k}{N \Delta S_k}, \quad (5.6)$$

where n_k is the number of measurements in bin k , ΔS_k is the bin width, and N is the total number of observations included in the histogram. This normalization ensures that $\sum_k \hat{p}_k \Delta S_k = 1$. If the flux density is expressed in Jy, the probability

density has units of Jy^{-1} . Figure 5.13 shows the histogram of the combined OVRO+UMRAO flux-density measurements for PKS J1309+1154, illustrating their empirical distribution. This estimate reflects the available measurements and is affected by the temporal coverage, observing cadence, and measurement uncertainties.

The PDF and the PSD describe complementary properties of the variability. While the PSD quantifies how the variance is distributed over temporal frequency, the PDF describes the marginal distribution of flux-density values. Therefore, the PDF does not contain information about the temporal sequence of the observations. In this work, the empirical flux-density distribution is used together with the PSD as an observational reference for constructing simulated light curves with comparable flux-density distributions and variability power spectra.

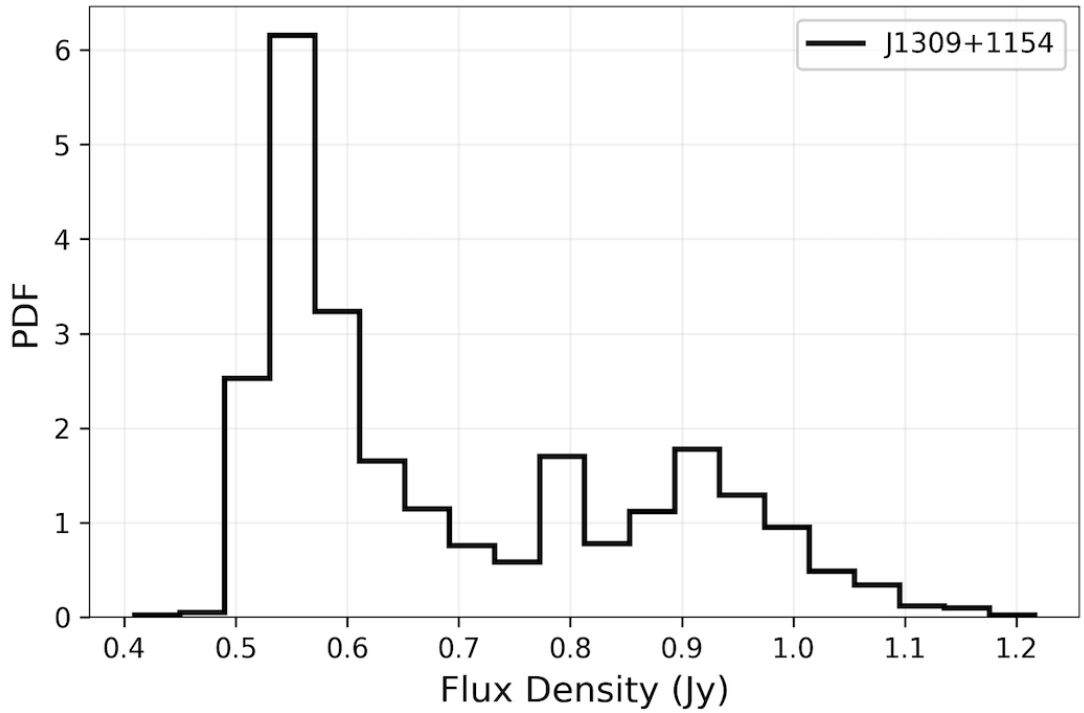


Figure 5.13: Empirical flux-density distribution of PKS J1309+1154 from the combined OVRO+UMRAO measurements.

5.3.3 Timmer and Koenig (TK)

The method proposed by [Timmer & Koenig \(1995\)](#) generates discrete stochastic simulations with a prescribed power spectrum through the direct synthesis of random Fourier coefficients. For a frequency grid $\nu_k = k/(N\Delta t)$, where N is the number of points in the series and Δt is the uniform sampling interval, the coefficients at positive frequencies below the Nyquist frequency are constructed from independent Gaussian variables. Following the unitary Fourier normalization adopted by TK, with a factor $1/\sqrt{N}$ in both the forward and inverse transforms, these coefficients are given by

$$\tilde{g}(\nu_k) = \mathcal{N}\left(0, \frac{1}{2}P(\nu_k)\right) + i\mathcal{N}\left(0, \frac{1}{2}P(\nu_k)\right), \quad (5.7)$$

where $\mathcal{N}(\mu, \sigma^2)$ denotes a normal random variable with mean μ and variance σ^2 , and $P(\nu_k)$ denotes the target discrete spectrum in this normalization. The Gaussian variables are independent between the real and imaginary components and across these positive frequencies.

The negative-frequency coefficients satisfy $\tilde{g}(-\nu_k) = \tilde{g}^*(\nu_k)$, where the asterisk denotes complex conjugation. The zero-frequency term and, when N is even, the Nyquist term are real. These conditions ensure the Hermitian symmetry needed to obtain a real-valued time series. The synthetic light curve is then obtained using the inverse discrete Fourier transform (IDFT),

$$x(t_j) = \text{IDFT} \{ \tilde{g}(\nu_k) \}. \quad (5.8)$$

For the power-law spectra considered by TK, $P(\nu) = A\nu^{-\beta}$ over the modeled positive-frequency range, the slope β is one of the main input parameters. The method randomizes both the Fourier amplitudes and phases, allowing individual periodograms to fluctuate around the prescribed spectrum. This is a central distinction from procedures that randomize only the phases while keeping the Fourier amplitudes fixed. Because the simulated series is a linear combination of Gaussian random variables, the resulting stochastic process has Gaussian marginal distributions for its light-curve values.

In applications to blazar light curves, the Gaussian nature of the TK process can be

a limitation when simulations are required to reproduce an observed non-Gaussian flux-density distribution. This motivates methods that constrain both the PSD and the PDF (Emmanoulopoulos et al., 2013). For comparison with irregularly sampled observations, synthetic light curves can be generated on a sufficiently fine uniform temporal grid and subsequently sampled at the actual observation times. Observational uncertainties can then be incorporated by adding noise consistent with the measurement errors, allowing the simulated and observed light curves to be compared under similar sampling and noise conditions (Max-Moerbeck et al., 2014).

5.3.4 Emmanoulopoulos method (EMP)

The method of Emmanoulopoulos et al. (2013) extends the previous approach by generating synthetic light curves that reproduce both a target PSD and a target PDF. It combines an initial realization with the adopted PSD, usually generated using TK, with a sample drawn from the flux density distribution to be reproduced. As in TK, the PSD can be parameterized as $P(\nu) = A\nu^{-\beta}$, so the slope β is also a fundamental input parameter of the procedure.

The algorithm applies two constraints iteratively. First, it adjusts the spectral structure of the synthetic curve by replacing its Fourier amplitudes with amplitudes consistent with the target PSD, while preserving the phases. Then, it adjusts the amplitude distribution by rank ordering the values of the curve so that its cumulative distribution matches the target PDF. These steps are repeated until convergence is reached between the PSD of the simulated curve and the adopted PSD.

The result is a stochastic realization that reproduces, in a statistically consistent way, both the temporal correlation structure described by the PSD and the marginal flux density distribution described by the PDF. For this reason, EMP is more suitable than TK when the observed light curve has a non gaussian, asymmetric, or extended tailed PDF. As in TK, the simulated curves can be resampled using the actual observational cadence and can include measurement uncertainties, allowing the construction of a reference distribution directly comparable with the observed data.

As a visual check, Figure 5.14 compares the observed light curve with three EMP realizations. This comparison provides a qualitative illustration of the algorithm’s ability to generate realistic variability patterns, which can make the observed curve difficult to distinguish from the simulations by visual inspection alone. The observed panel is not identified, in accordance with the OVRO and UMRAO collaboration’s restrictions on unpublished data.

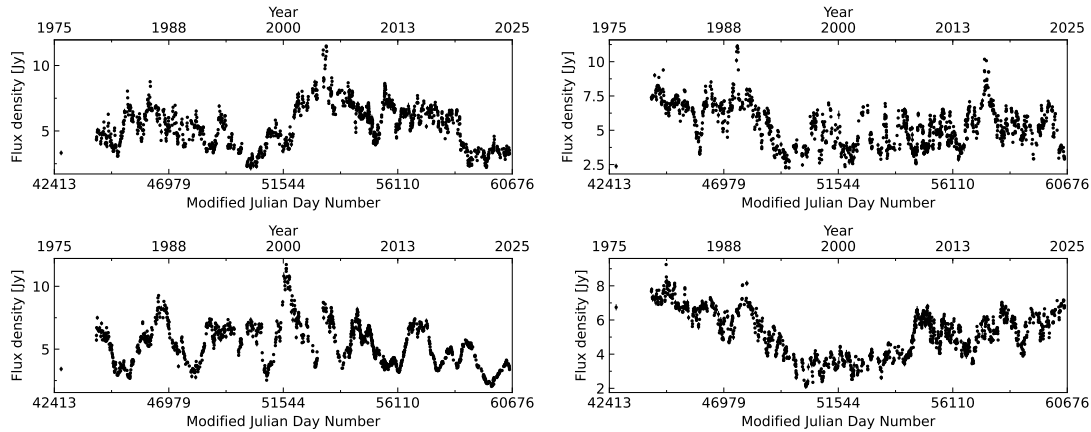


Figure 5.14: Visual comparison of one observed light curve and three simulated realizations generated with the EMP algorithm. The observed panel is not identified.

Figure 5.15 presents a more detailed comparison of TK and EMP simulations for J2134–0153 (1976–2025) and J1309+1154 (1979–2025). For this comparison, 100 realizations were generated per method, adopting estimated PSD slopes of $\beta = 1.95$ and $\beta = 1.62$, respectively, with $\text{PSD} \propto \nu^{-\beta}$. Both algorithms target the prescribed PSD, but the displayed TK realizations show approximately Gaussian flux-density distributions, most clearly for J1309+1154. For J2134–0153, none of the TK realizations in this comparison approaches the approximately 6 Jy reached by the isolated flare near the end of the observed light curve. EMP more closely reproduces the observed PDF and accommodates events with comparable flux densities, providing a more representative description of the observed flux density range.

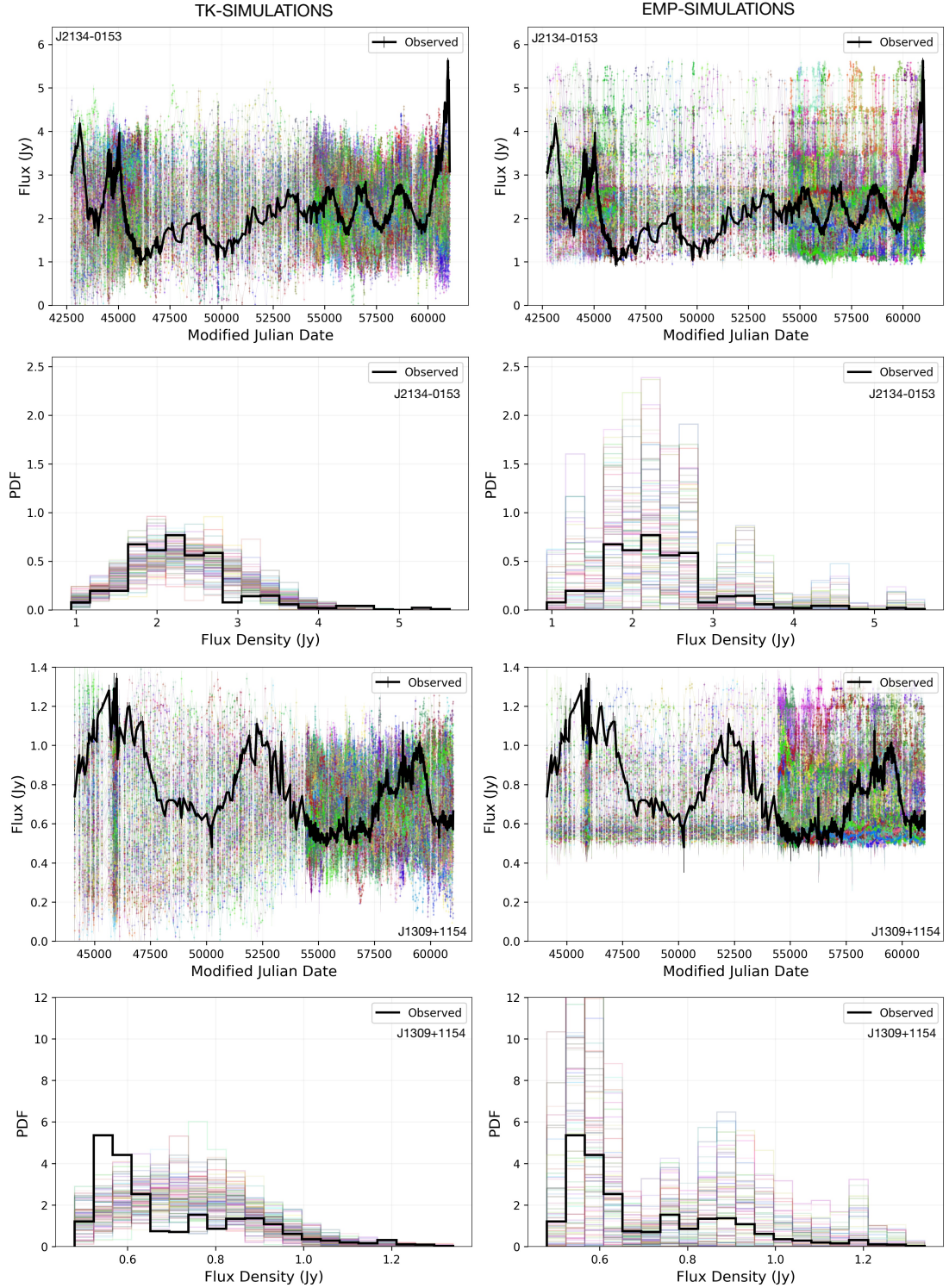


Figure 5.15: Stacked light curves and corresponding probability density functions (PDFs) for J2134–0153 (upper panels) and J1309+1154 (lower panels), using TK (left) and EMP (right) simulations. Observed light curves and PDFs are shown in black, and simulations in color. The displayed TK realizations show approximately Gaussian PDFs, most clearly for J1309+1154. For J2134–0153, the TK realizations do not reach flux densities comparable to the isolated flare near the end of the observed light curve, whereas EMP closely reproduces the observed PDF and allows events of comparable amplitude throughout the simulated time baseline.

5.3.5 Statistical significance analysis

The statistical significance of the periodicity detected in J1309+1154 was evaluated through a simulation-based procedure, following the general approach used by O’Neill et al. (2022) for PKS 2131–021. However, unlike that case, where separate epochs of sinusoidal activity were analyzed, here a single epoch was considered, covering the full available temporal baseline of the light curve.

The null hypothesis, H_0 , states that the observed GLSP peak can be produced by stochastic red noise fluctuations, without requiring an intrinsic periodic signal. To test this hypothesis, $N_{\text{sim}} = 300000$ simulated light curves were generated using the EMP algorithm. The simulations adopted a PSD slope of $\beta = 1.66$ and were analyzed with the same GLSP setup applied to the observed light curve, without linear detrending and using $\zeta = 15$. The observational uncertainties of the real light curve were preserved in the simulations, and no additional Gaussian noise was added.

The comparison between the observed GLSP and the GLSP distribution obtained from the simulations allowed both the local and global significance to be estimated. The local significance evaluates the probability of reproducing the observed power at the candidate period, whereas the global significance evaluates the probability of obtaining an equal or stronger peak at any period within the explored range.

5.3.6 Local significance

The local significance evaluates whether the observed power at the candidate period is compatible with the tail of the null distribution generated by light curve simulations at that same period. In this case, the local null hypothesis, $H_{0,\text{local}}$, states that the peak observed at T_{obs} can be produced by a stochastic red noise fluctuation at T_{obs} , without requiring an intrinsic periodicity.

Let P_{obs} be the GLSP power of the observed light curve at the candidate period T_{obs} . For each simulated light curve, the GLSP power is evaluated at that same period, or equivalently, at the corresponding frequency bin of the adopted GLSP grid. A simulation passes the local test if its power at T_{obs} is equal to or greater than P_{obs} . If N_{local} simulations satisfy this condition, the local p value is defined

as

$$p_{\text{local}} = \frac{N_{\text{local}}}{N_{\text{sim}}}. \quad (5.9)$$

Therefore, N_{local} corresponds to the number of light curve simulations that reproduce, at the candidate period, a power at least as high as the observed one. This test does not evaluate whether the main peak of each simulation occurs at T_{obs} ; it only quantifies the probability, under $H_{0,\text{local}}$, of obtaining an equal or more extreme fluctuation at that specific period. The number of simulations that do not pass the local test is given by $N_{\text{local,fail}} = N_{\text{sim}} - N_{\text{local}}$.

This result does not correct for the number of periods explored in the GLSP. Therefore, it measures only the significance associated with the candidate period.

5.3.7 Global significance

The global significance evaluates whether the observed power remains exceptional when the full period range explored by the GLSP is considered. In this case, the global null hypothesis, $H_{0,\text{global}}$, states that the observed peak can be produced by a stochastic red noise fluctuation at some period within the search range, without requiring an intrinsic periodicity.

For each simulated light curve, the full GLSP is computed and the maximum power is identified within the same period range used for the observed light curve. A simulation passes the global test if its maximum GLSP power is equal to or greater than P_{obs} , regardless of the period at which it occurs. If N_{global} simulations satisfy this condition, the global p value is defined as

$$p_{\text{global}} = \frac{N_{\text{global}}}{N_{\text{sim}}}. \quad (5.10)$$

In this case, N_{global} corresponds to the number of light curve simulations that produce, anywhere in the explored period range, a peak at least as strong as the observed one. This test incorporates the look elsewhere effect, since examining many frequencies increases the probability of finding a high peak by chance in at least one of them. The number of simulations that do not pass the global test is

given by $N_{\text{global,fail}} = N_{\text{sim}} - N_{\text{global}}$.

If the global range includes the candidate period and the same power threshold is used, then $p_{\text{global}} \geq p_{\text{local}}$. For this reason, p_{global} is the relevant quantity for the final statistical assessment when the candidate period is identified from the same GLSP search. If no simulation exceeds the threshold, the result should be reported as a limit imposed by the finite number of simulations, not as an exact null probability.

5.3.8 Significance for J1309+1154: Results

Table 5.3 summarizes the statistical significance analysis for J1309+1154. The local test gives a significance of 3.12σ , with $p_{\text{local}} = 8.90 \times 10^{-4}$. This indicates that, at the candidate period, only a small fraction of the light curve simulations reproduce a GLSP power equal to or higher than the observed one. However, when the full period range explored by the GLSP is considered, the global probability increases to $p_{\text{global}} = 0.016$, corresponding to 2.14σ . Therefore, under this global test, the null hypothesis cannot be rejected at the 3σ level.

As an additional diagnostic, 53% of the simulations that pass the global test have their strongest peak at periods longer than $T/2$, where T is the total temporal baseline. If these cases are excluded, the remaining fraction gives an approximate value $p_{\text{global,filtered}} \simeq 0.016 \times 0.47 = 0.00752$, equivalent to approximately 2.43σ . This value is still below 3σ and should be interpreted only as a conditional diagnostic, not as a replacement for the global significance, unless the filtering criterion is defined a priori.

Figure 5.16 illustrates the local (left) and global (right) significance tests by comparing the observed GLSP (green) with 100 EMP-simulated periodograms (black) in each panel. These are randomly selected from realizations that reach or exceed the observed power at the candidate period (local), or whose strongest peak has a local significance, evaluated at its own period, at least as high as that of the observed peak (global). This comparison illustrates the look-elsewhere effect discussed by O'Neill et al. (2022).

Table 5.3: Summary of the statistical significance analysis for J1309+1154.

Analysis setup			Significance results		
Quantity	Symbol	Value	Quantity	Symbol	Value
PSD slope	β	1.66	Local pass	N_{local}	267
Number of simulations	N_{sim}	300000	Local p value	p_{local}	8.90×10^{-4}
Number of GLSP bins	N_{freq}	7403	Local significance	σ_{local}	3.12σ
Candidate period	T_{obs}	6547.94 d	Global pass	N_{global}	4829
Observed GLSP power	P_{obs}	0.86473	Global p value	p_{global}	1.61×10^{-2}
GLSP array shape	–	300000×7403	Global significance	σ_{global}	2.14σ

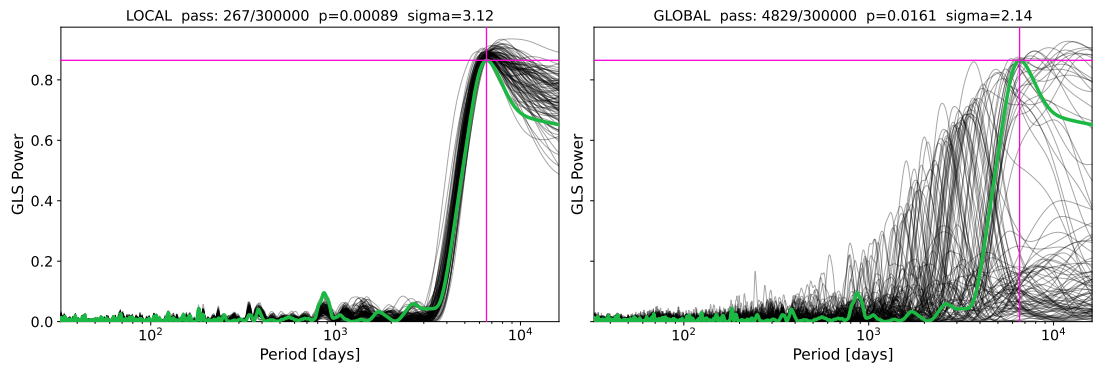


Figure 5.16: Stacked GLSPs of the EMP simulations (black), overlaid with the observed GLSP (green). The comparison illustrates the tests used to determine the local and global significance.

5.3.9 Discussion: Statistical significance in J1309+1154

The global significance obtained for J1309+1154 does not provide sufficient evidence to reject the null hypothesis under the simulation based GLSP test. This result is notable because several independent empirical diagnostics point to a highly structured signal: the GLSP peak is strong, the sinusoidal fit follows the long term light curve, the WWZ analysis shows temporal coherence, the VLBA model fit identifies the relevant emission region, and the SWF analysis supports a well constrained periodic component. In addition, the source shows a possible harmonic relation and an unusually sinusoidal morphology over a multi decade radio baseline.

This apparent tension does not imply that the periodic interpretation is incorrect. Rather, it indicates that the statistical test is strongly conditioned by the properties of the GLSP, the adopted search range, the temporal sampling, and the null distribution generated by the simulations. In particular, the global test penalizes the search over many trial periods through the look elsewhere effect. Therefore, a

strong and visually coherent peak can still yield a modest global significance if comparable peaks arise elsewhere in the simulated periodograms.

For this reason, the next step is to evaluate the behavior and limitations of the GLSP in controlled cases. The goal is not to discard the GLSP, but to determine under which conditions it is reliable and under which conditions it may produce ambiguous or spurious peaks. This requires tests with synthetic light curves built from sinusoidal components with different amplitudes, phases, and periods, both with white noise and red noise; period sweeps over the relevant range; comparisons between period and frequency representations; separate and combined GLSP analyses for different telescopes; tests including artificial flares or localized deviations; and comparisons with real light curves before and after removing dominant flare like structures.

These tests also motivate a more structured treatment of harmonic relations among GLSP peaks. A simple inspection of relations such as $mP_i \simeq nP_j$, where P_i and P_j are periods associated with GLSP peaks ($i \neq j$) and m and n are positive integers ($m \neq n$), is useful as a first diagnostic but is insufficient for a robust classification. A dedicated harmonic counting algorithm is therefore required, with explicit tolerance criteria and consideration of peak independence, relative power, and the number of detected peaks. This would allow a more systematic interpretation of the GLSP results before assigning physical significance to any periodic or harmonic feature.

Chapter 6

Harmonic relations between GLSP peaks

In recent years, periodograms have become fundamental tools for analyzing possible periodicities in astronomical light curves. In the case of blazars, this analysis is especially complex because their light curves are dominated by stochastic red noise variability. Unlike a stellar binary system, where periodicity can be more directly associated with a dynamical timescale, in a blazar it is necessary to evaluate whether the detected period corresponds to a coherent physical process or whether it can be explained as a stochastic fluctuation of the AGN/jet system.

In general terms, the analysis seeks to determine whether a candidate periodicity can be attributed with sufficient confidence to a dynamical, kinematic, or intrinsic process of the system, including a possible SMBHB, and not only to stochastic variability. In this work, a central part of this procedure is based on the GLSP, a tool widely used to study irregularly sampled time series in observational astronomy.

During the development of this work, it was found that the GLSP can be highly sensitive to different aspects of the data and of the analysis, including the temporal baseline, cadence, explored frequency range, oversampling factor ζ , and the presence of flares. In particular, except in cases where the dominant periodicity is very clear, the behavior of secondary peaks can be difficult to anticipate. For this reason, before interpreting relations between peaks as physical evidence, it is necessary to inspect how the GLSP responds under different controlled and observational conditions.

The combined OVRO+UMRAO sample provides a particularly valuable opportunity for this purpose, since it includes light curves with an extended temporal baseline and information obtained at different cadences. In addition, the case of J1309+1154 motivates this methodological review: although the source shows several empirical indicators compatible with a periodic signal, the global significance test does not allow the null hypothesis to be rejected at the 3σ level. This does not discard the periodic interpretation, but it does justify a more detailed examination of the method used and, in particular, of the behavior of the GLSP.

6.1. Controlled Tests of the GLSP

In this section, we present several tests designed to evaluate the response of the GLSP in different scenarios. These tests include:

- The behavior of the GLSP as a function of the oversampling factor ζ .
- The influence of a flare on the power and position of the GLSP peaks.
- The effect of differences in cadence and temporal baseline between data subsets.
- The inspection of special cases in real GLSP of the light curves.
- Synthetic light curves generated from sinusoidal functions with different periods, amplitudes, and phases, including cases with white noise and red noise.

The goal of these tests is to characterize the conditions under which the GLSP produces stable, ambiguous, or potentially spurious peaks. From this inspection, the need to study harmonic relations between periodogram peaks in a more systematic way is introduced. This leads to the development of a harmonic counting algorithm that incorporates tolerance criteria, independence between peaks, relative power, and the number of detected relations.

6.1.1 Effect of the ζ parameter in the GLSP.

In the generalized Lomb–Scargle periodogram (GLSP), the parameter ζ controls the oversampling of the frequency grid. For a time baseline T , the typical frequency spacing can be written as $\Delta f \simeq \frac{1}{\zeta T}$. Thus, larger values of ζ produce a finer grid and allow a more precise numerical localization of the periodogram maximum. However, this does not increase the independent physical information contained in the data, since the effective resolution remains limited by the time baseline, the cadence, and the sampling window function.

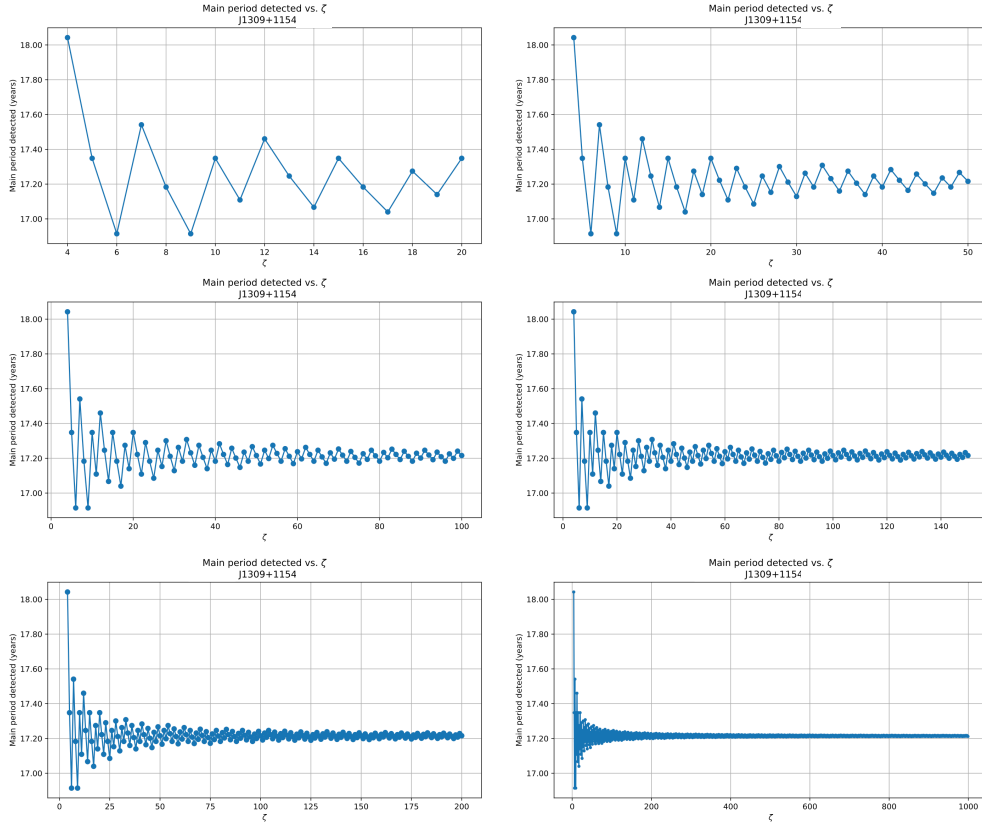


Figure 6.1: Numerical convergence of the dominant GLSP period of J1309+1154 with increasing oversampling parameter ζ , using the combined OVRO+UMRAO light curve.

This behaviour is illustrated for J1309+1154 in Fig. 6.1, using the combined OVRO+UMRAO light curve without detrending or outlier treatment. The panels show the same analysis extended to progressively larger maximum values of

ζ . At low oversampling, the dominant period varies appreciably because of the discretization of the frequency grid; increasing ζ leads to numerical stabilization near 17.2 yr. Figure 6.2 extends this comparison to the full sample of 83 sources.

Each row corresponds to one source, and the color represents the period of the dominant peak in years. Arrows identify changes exceeding 1.5 yr between consecutive values of ζ , while the source-label colors indicate the GLSP power range of the dominant peak.

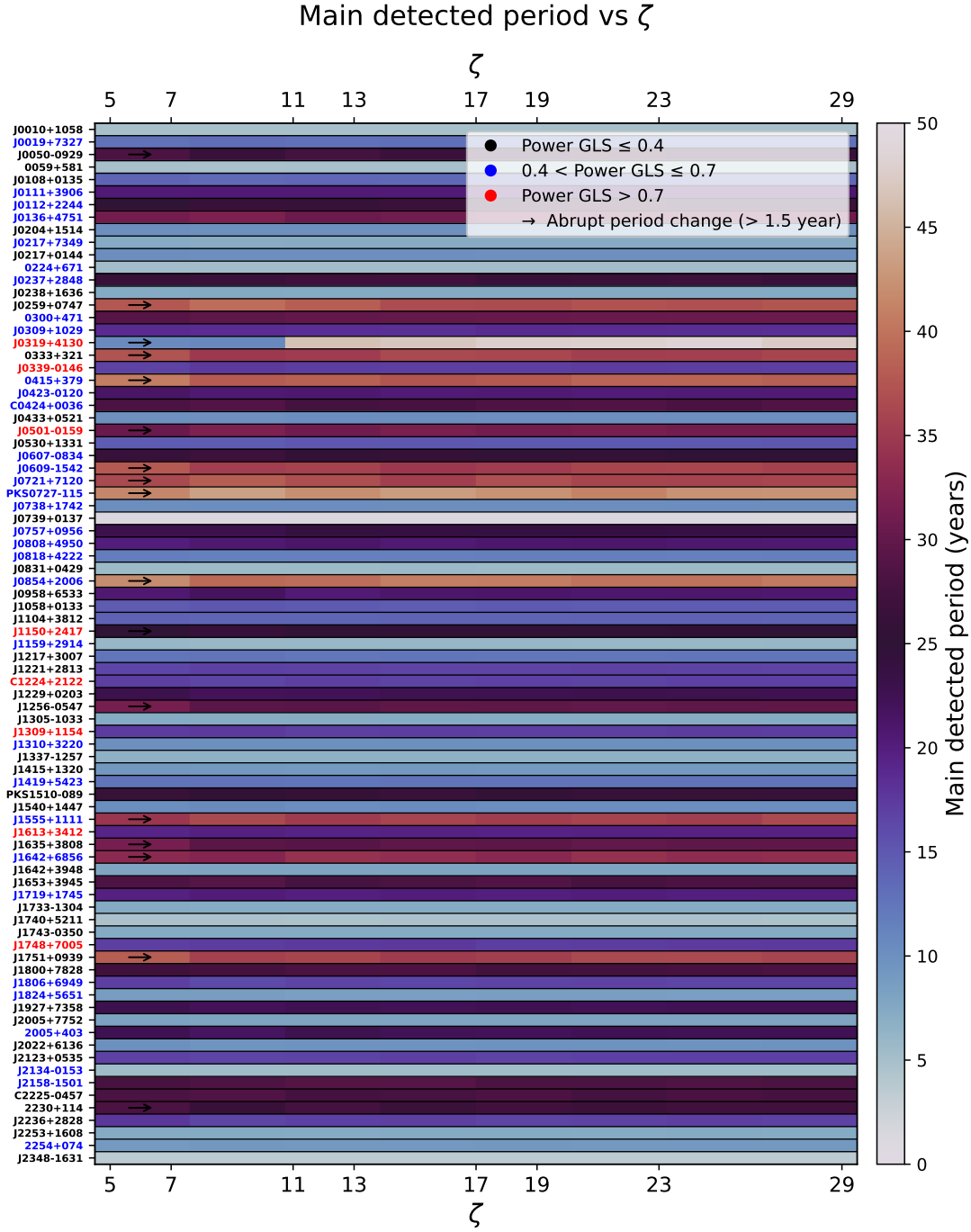


Figure 6.2: Heatmap of the dominant GLSP period as a function of ζ for the 83 sources in the combined OVRO+UMRAO sample.

6.1.2 Transient Flares as a Source of Spurious GLSP Peaks

This test was performed to assess whether a flare can generate harmonic relations between GLSP peaks. The result shows that the persistence of a similar GLSP structure across two different epochs does not necessarily imply a physical origin characteristic of a binary system. It also suggests that a robust GLSP periodicity should be associated with a single, well-defined dominant peak, rather than with a complex set of harmonically related peaks (See Figs. 6.3, 6.4, 6.5).

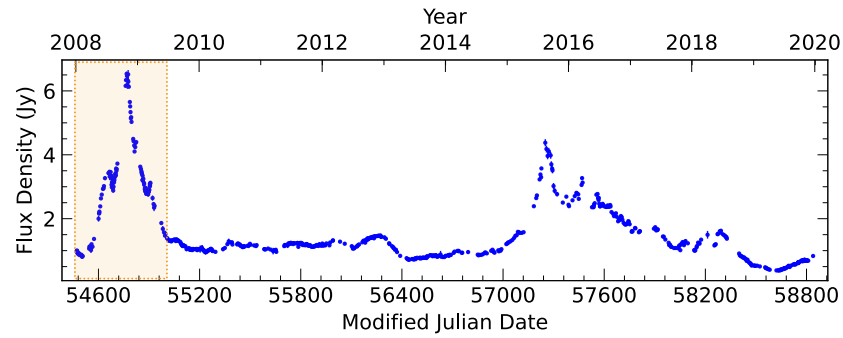


Figure 6.3: Light curve of AO 0235+164 using OVRO 40 m data. The orange box highlights a flare.

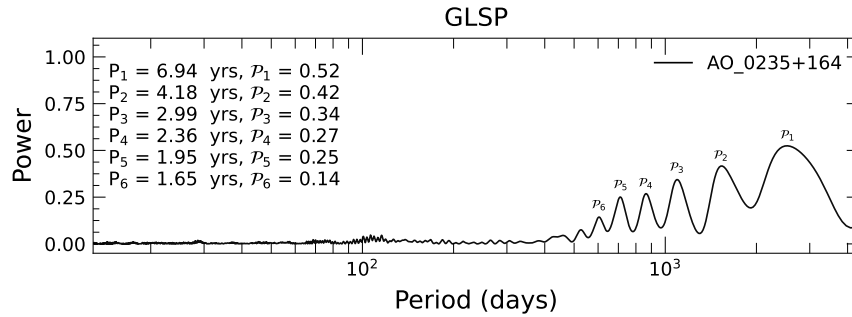


Figure 6.4: GLSP of AO 0235+164 computed using the full OVRO data set spanning 2008–2020, including the flare.

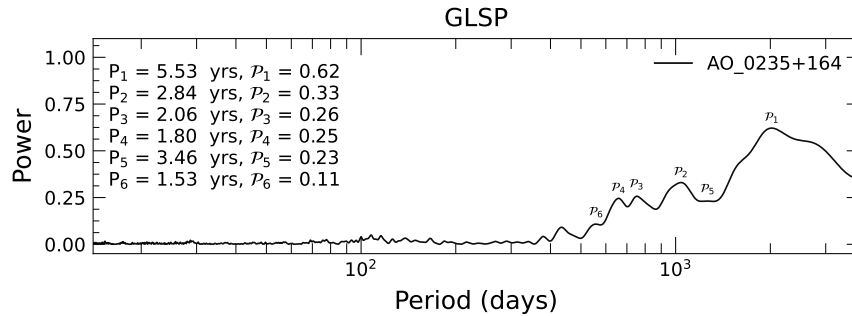


Figure 6.5: GLSP of AO 0235+164 computed after excluding the flare. This example illustrates that similar GLSP shapes in different epochs are not conclusive by themselves, given that the GLSP shape is highly sensitive to transient structures such as flares.

6.1.3 Effect of cadence on apparent harmonic relations

We evaluated whether apparent harmonic relations between GLSP peaks can be affected by the observational cadence, or whether they are mainly dominated by the tail of the red noise distribution. For this purpose, an EMP simulation of AO 0235+164 was used, generated with a PSD slope of $\beta = 1.5$. The EMP method produces light curve simulations that approximately preserve both the PSD and the PDF of the observed light curve. This simulation was resampled using four schemes: the observed cadence, a regular cadence of 3 days, one of 10 days, and one of 30 days (Fig. 6.6, right panel). The GLSP was then computed in each case using $\zeta = 29$ as the oversampling factor. This value was chosen as a prime number to avoid a choice associated with simple subdivisions of the frequency grid, although this does not by itself modify the statistical definition of the periodogram.

The result shows that changes in cadence produce only marginal effects on the global structure of the GLSP (Fig. 6.6, left panel). In this experiment, the apparent harmonic relations do not appear to be mainly controlled by the cadence, but rather by the stochastic structure of the simulation, that is, by the red noise tail. This indicates that a purely stochastic light curve can produce harmonically related peaks without requiring an underlying periodic signal.

Since this case corresponds to a random process, the apparent harmonic relations should be interpreted as possible coincidences within the period range effectively explored. This interpretation is valid only if physically or statistically uninformative periods are excluded, for example periods that are too long compared with the total temporal baseline of the light curve, or too short relative to the temporal resolution of the sampling. In particular, a harmonic relation involving a period comparable to or longer than the duration of the light curve should not be considered robust.

These results motivate the need for a systematic analysis of harmonic relations between GLSP peaks. Later, this idea is formalized through a harmonic counting algorithm that evaluates how many such relations appear in real and simulated light curves, under explicit criteria for tolerance, period range, and peak independence.

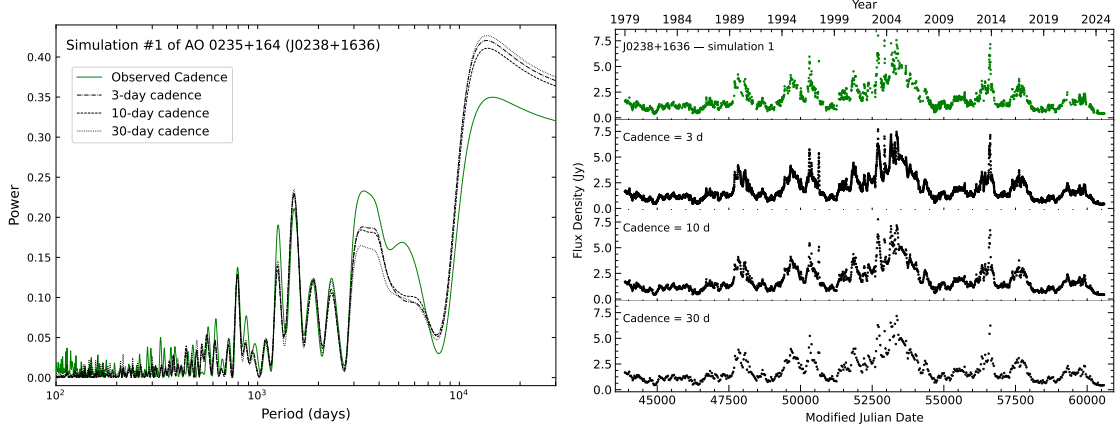


Figure 6.6: Left: GLSPs computed at different cadences. The green curve corresponds to simulation #1 of AO 0235+164 resampled at the observed cadence. Right: the corresponding light curves. Only marginal differences are observed between the GLSPs.

6.1.4 Inspection of special cases in real light curve GLSPs

As part of the methodological inspection, individual GLSPs were computed for each source considering three data sets: OVRO-only, UMRAO-only, and OVRO+UMRAO. The results were plotted in the same panel, using blue for OVRO, red for UMRAO, and green dotted lines for the combined light curve. In all cases, an oversampling factor of $\zeta = 29$ was used. To facilitate the visual comparison between periodograms, the frequency grid was slightly extended with respect to the main analysis range; this extension is only intended for graphical purposes and does not imply interpreting periods outside the physically useful or statistically considered domain. The details of the grid construction and adopted range are presented in Appendix B.

Each figure includes the GLSP representation in both frequency and period, together with a table summarizing the six main peaks in frequency, days, years, and power. This format allows a direct comparison of the position, power, and relative stability of the main periodogram peaks between observational subsets with different cadence and temporal baseline. It also allows identifying cases where the peaks are consistent between telescopes, cases where they change due to sampling or temporal baseline, and cases where apparent harmonic relations appear between GLSP peaks.

The figure 6.7 shows a particularly illustrative case. In the combined OVRO+UMRAO light curve, two peaks of comparable power are observed, with $P_1 = 12.92$ yr and $P_2 = 25.84$ yr. The ratio between both periods is exactly two, producing an exact harmonic relation. This type of configuration should not be automatically interpreted as physical evidence, but it does motivate a more critical inspection of the relations between GLSP peaks. The same exercise was performed for all sources in the sample; the complete results are presented in Appendix B.



Figure 6.7: GLSP of J0019+7327 in frequency and period. OVRO-only is shown in blue, UMRAO-only in red, and OVRO+UMRAO in green dotted lines. The full set of sources is shown in Appendix B.

6.2. Synthetic light curves

The purpose of this test is not to reproduce realistically the full physical complexity of a blazar light curve, but rather to isolate specific aspects that may affect the interpretation of periodograms. These curves make it possible to study the response of the GLSP to known sinusoidal signals, both individual and composite, considering different periods, phases, and amplitudes. Cases with and without additional contributions from white and red noise, added to the flux in arbitrary units, were also explored in order to examine qualitatively how stochastic variability may alter period recovery or produce apparently harmonic structures. The uncertainties of the synthetic light curves were assumed to be constant and equal to the amplitude of the added noise, thereby producing a uniformly weighted GLS and allowing the effects of the aforementioned variables to be isolated.

In the following tests, the observed cadence of J0019+7327 was adopted because this source represents a particularly interesting case: its GLSP contains two peaks of similar power that form an exact 2:1 harmonic relationship. A priori, such a configuration appears to be highly unusual among the GLSPs of blazar light curves. In addition, harmonic relationships between peaks were identified in these tests solely by requiring the relative mismatch to be smaller than 10%.

The implementation of the first test is described in detail below, while the details and corresponding observations for the remaining tests (T2–T6) are presented in Appendix C. After Test 1, a summary of these initial tests is provided, together with a brief discussion of the results.

6.2.1 Test 1: Same amplitude, different periods, random phases.

The following section presents an analytical description of each test performed, followed by the corresponding plots for different values of the simulated period, $P_{1,\text{sim}}$. For simplicity, in Tests 2 through 6, only summaries of the model used are shown. While the first four tests assume Gaussian noise, Tests 5 and 6 incorporate correlated red noise, providing a more realistic representation of stochastic variability.

1. Dual Sinusoidal Components:

The simulated signal consists of the sum of two sinusoids with equal amplitude and independent random phases, but different periods:

$$S_{\text{pure}}(t) = A \sin\left(2\pi \frac{t}{P_1} + \phi_1\right) + A \sin\left(2\pi \frac{t}{P_2} + \phi_2\right) \quad (6.1)$$

where

$$P_2 = \frac{1}{2} P_1 \quad (6.2)$$

The phases ϕ_1 and ϕ_2 are drawn randomly from the interval $[0, 2\pi)$ for each simulation:

$$\phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi) \quad (6.3)$$

Importantly, when running tests for different values of P_1 , the random phases remain fixed within the simulation batch; i.e., the phases are not re-randomized for each new period.

2. White Gaussian Noise:

To the pure signal, we add white (uncorrelated) Gaussian noise:

$$N(t) \sim \mathcal{N}(0, \sigma^2) \quad (6.4)$$

The noise standard deviation is set to be exactly 10% of the sinusoid amplitude:

$$\sigma = 0.1 A \quad (6.5)$$

3. Cadence Matching:

The simulated curve is sampled at the same time points as the original observational data:

$$t_i = \text{MJD}_i \quad (i = 1, 2, \dots, N) \quad (6.6)$$

4. Summary of Full Model (Test 1):

The final simulated data points are:

$$S_{\text{sim}}(t_i) = A \sin \left(2\pi \frac{t_i}{P_1} + \phi_1 \right) + A \sin \left(2\pi \frac{t_i}{P_2} + \phi_2 \right) + N(t_i) \quad (6.7)$$

with

$$\begin{cases} P_2 = \frac{1}{2}P_1 \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi) \\ N(t_i) \sim \mathcal{N}(0, (0.1A)^2) \\ t_i = \text{MJD}_i \end{cases}$$

5. Observations: In this test (Figure 6.8), as the imposed periods increase from 100 to 6000 days, the GLSP peaks become broader and show a slight increase in power. Although secondary peaks appear around the main peaks, the periods imposed on the signal are recovered in all cases. The number of harmonic relationships does not show a clear trend.

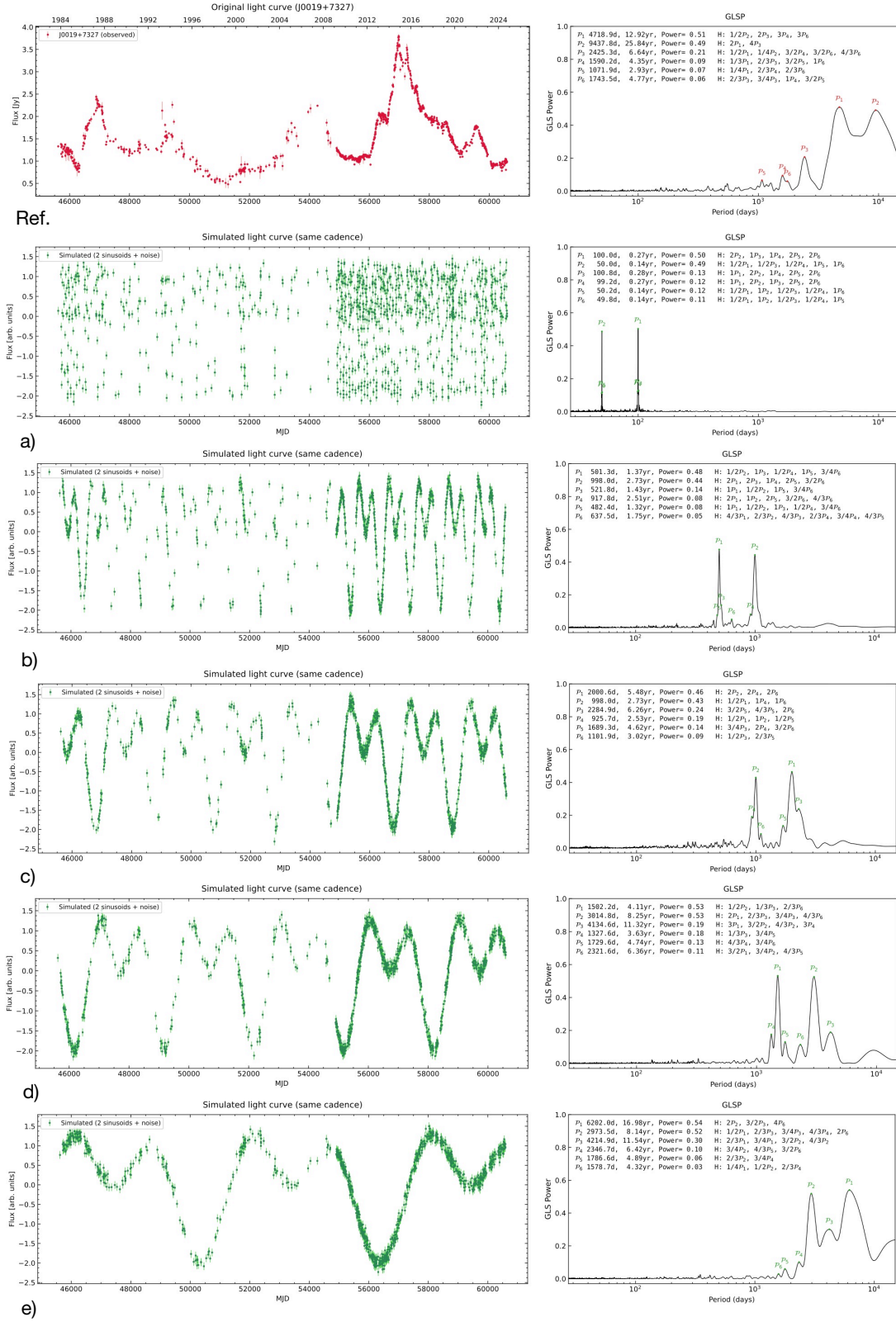


Figure 6.8: Test 1: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 3000, e) 6000.

6.2.2 Summary of Synthetic Light Curve Tests

Below is a summary of Tests 1 through 6, where Equations 6.8 and 6.9 correspond to the general expressions used. Here, $N_i \equiv N(t_i)$ denotes the noise term. For Tests 1–4, it is generated as independent zero-mean Gaussian draws. For Tests 5 and 6, independent Gaussian random terms ϵ_i are recursively filtered through the first-order autoregressive process $N_i = \alpha N_{i-1} + \epsilon_i$. The coefficient α controls the temporal persistence of the noise: when $\alpha = 0$, the process reduces to uncorrelated white noise, whereas positive values approaching unity cause each sample to retain an increasingly large contribution from the preceding sample. This temporal correlation gives greater relative power to slow, low-frequency variations than to rapid, high-frequency fluctuations, thereby producing the characteristic red-noise behavior. The resulting series is finally rescaled to the prescribed rms amplitude.

$$S_{\text{sim}}(t_i) = A_1 \sin\left(2\pi \frac{t_i}{P_1} + \phi_1\right) + A_2 \sin\left(2\pi \frac{t_i}{P_2} + \phi_2\right) + N_i \quad (6.8)$$

$$S_{\text{sim}}(t_i) = A \sin\left(2\pi \frac{t_i}{P} + \phi\right) + N_i \quad (\text{Single Sinusoidal Component}). \quad (6.9)$$

Test 1	Test 2	Test 3
$\left\{ \begin{array}{l} P_2 = \frac{1}{2}P_1, \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi), \\ N_i \sim \mathcal{N}(0, (0.1A)^2), \\ t_i = \text{MJD}_i. \end{array} \right.$	$\left\{ \begin{array}{l} P_2 = \frac{1}{2}P_1, \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi), \\ N_i \sim \mathcal{N}(0, (0.1A)^2), \\ t_i = \text{MJD}_i. \end{array} \right.$ (new draw for every P_1 value)	$\left\{ \begin{array}{l} P_2 = \frac{1}{2}P_1, \\ A_1 = 2A_2, \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi), \\ N_i \sim \mathcal{N}(0, (0.1A_1)^2), \\ t_i = \text{MJD}_i. \end{array} \right.$
Test 4	Test 5	Test 6
$\left\{ \begin{array}{l} P_1 = \frac{5}{7}P_2, \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi), \\ N_i \sim \mathcal{N}(0, (0.1A)^2), \\ t_i = \text{MJD}_i. \end{array} \right.$	$\left\{ \begin{array}{l} \phi \sim \mathcal{U}(0, 2\pi), \\ N_i = \alpha N_{i-1} + \epsilon_i, \\ \epsilon_i \sim \mathcal{N}(0, (0.1A)^2), \\ (\text{red noise}), \\ t_i = \text{MJD}_i, \end{array} \right.$ (Single Sinusoidal Component)	$\left\{ \begin{array}{l} A_1 = 1, \\ A_2 = 0.6A_1, \\ P_1 = 2P_2, \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi), \\ N_i = \alpha N_{i-1} + \epsilon_i, \\ \epsilon_i \sim \mathcal{N}(0, (0.5A_1)^2), \\ (\text{red noise}), \\ t_i = \text{MJD}_i. \end{array} \right.$

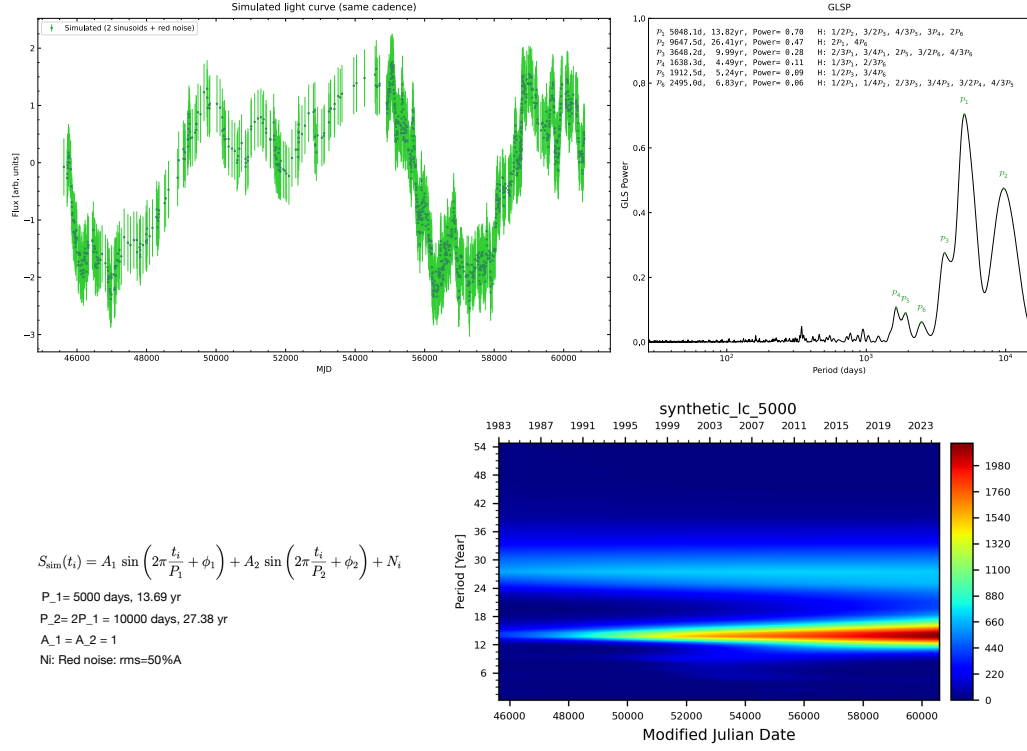
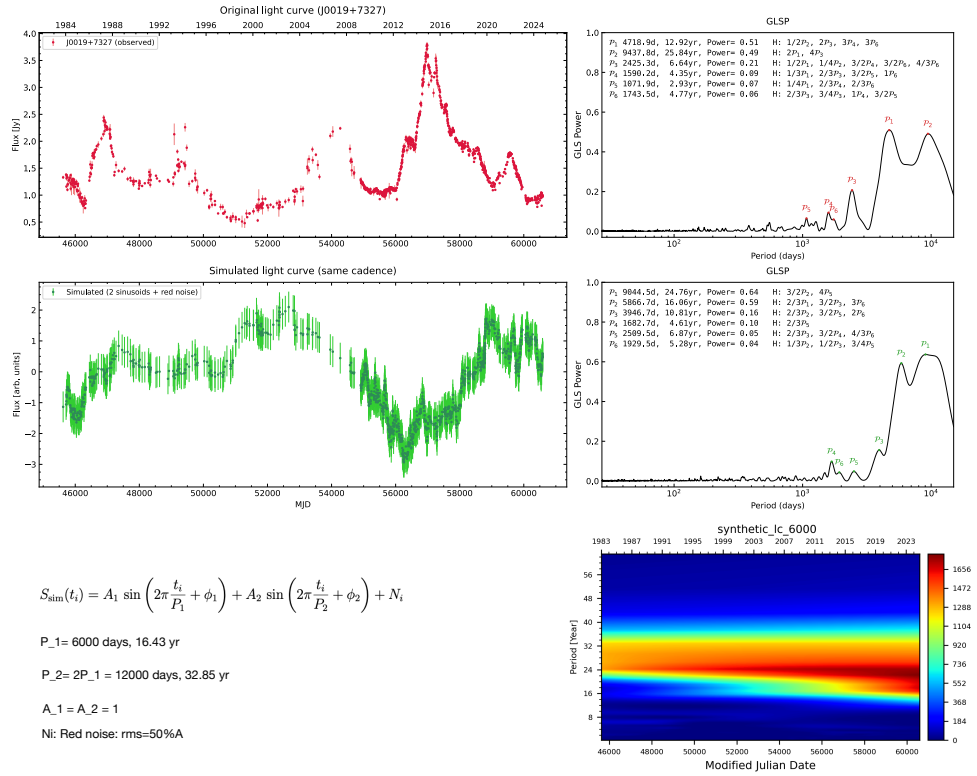
6.2.3 Qualitative Interpretation of the Tests

From a qualitative perspective, the tests performed with synthetic light curves show that the GLSP approximately recovers the imposed periods in most of the analyzed cases. The variation of the phases does not produce relevant differences between Tests 1 and 2, whereas the relative amplitude of the sinusoidal components clearly affects their detectability: the lower-amplitude component can lose definition and be mistaken for secondary GLSP peaks. For longer periods, the GLSP peaks tend to broaden, although they maintain high power in several of the studied cases. The recurrent appearance of secondary peaks around the dominant peaks is also observed. Harmonic relationships do not show a systematic pattern across the different tests, although relationships close to 2:1 are identified in Test 6. Taken together, this qualitative evaluation suggests that the relative amplitude of the components and the period values produce more evident changes in the structure and power of the GLSP than the choice of phases. Finally, two additional tests are carried out based on Test 6, but with an added red noise level of 50% of the amplitude. It is confirmed that when one of the sinusoidal functions has a larger amplitude, the GLSP is represented by a single main peak, and that the added red noise in a higher proportion does not generate significant morphological differences in the GLSP. This is likely because the spectral power is proportional to the squared modulus of the Fourier transform. The WWZ of the tests is included in Figures 6.9b and 6.10b.

For a better visualization of the evolution of the generated GLSPs when the period is varied, you can visit the [GitHub repository](#), where you will find some animations illustrating this:

- `2gls_lc_hr.mp4`: Test 2, double component, period evolution.
- `1gls_lc_hr.mp4`: Test 5, single component, period evolution.
- `1_P_Spectra.mp4`: Single component (frequency).

Through a qualitative description, Tests 1 to 6 reinforce the need to conduct a robust counting of the harmonics produced in the simulations, which will be addressed in the following section.

(a) $P_1 = 5000 \text{ d.}$ (b) $P_1 = 6000 \text{ d.}$ Figure 6.9: Test 6.1 ($A_2 = A_1$), for two values of the imposed primary period (5000 and 6000 days).

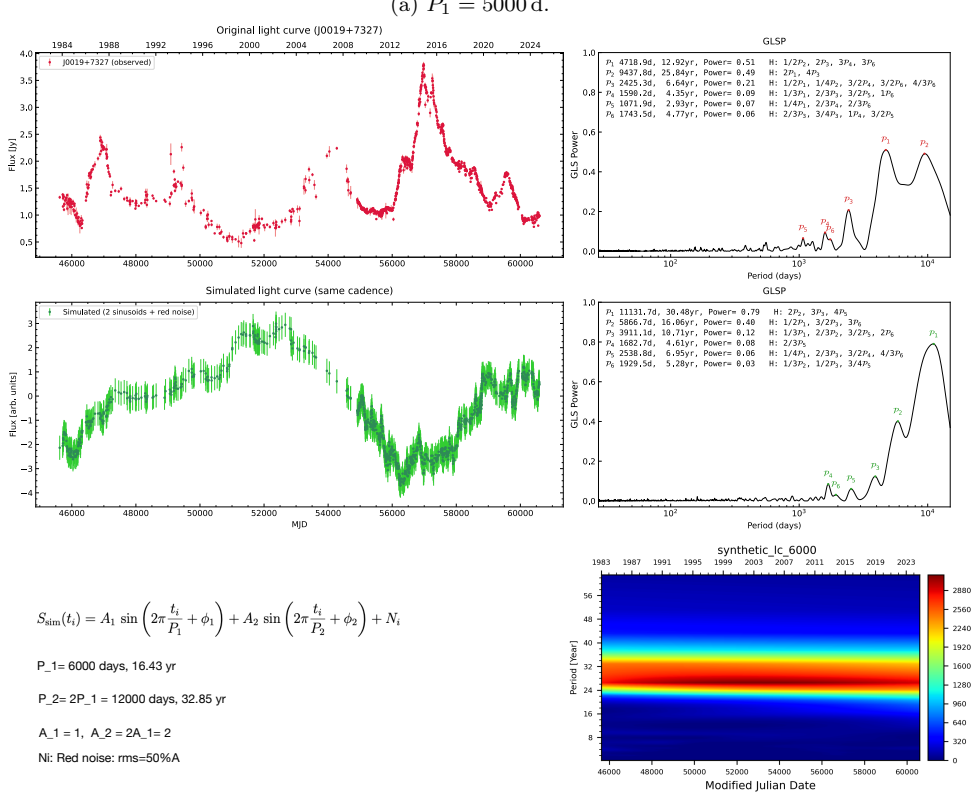
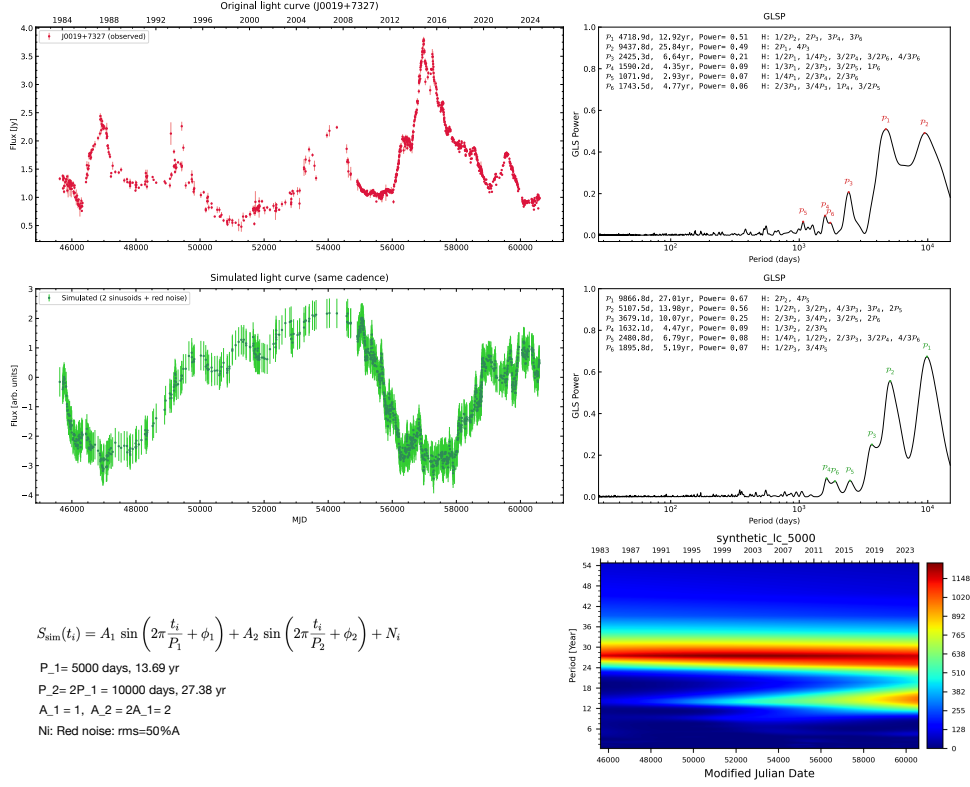


Figure 6.10: Test 6.2 ($A_2 = 2A_1$), for two values of the imposed primary period (5000 and 6000 days).

6.3. Harmonic relations in blazar light curves

Up to this point, the GLSP has been used as a tool to identify dominant sinusoidal periodicities in irregularly sampled light curves. In practical terms, the highest power peak in the periodogram is interpreted as the main periodicity candidate. However, this interpretation is not automatic: the structure of the GLSP can depend on the amount of available data, the temporal baseline, the adopted frequency limits, the sampling cadence, although to a lesser extent, and the presence of transient events such as flares. In particular, a sufficiently strong flare can introduce power over multiple time scales and produce secondary peaks that appear to be harmonically related to the main peak.

For this reason, the presence of harmonic relations between GLSP peaks should not by itself be interpreted as evidence for a physical periodicity, nor as a direct signature of a binary system. A harmonic structure can arise as a consequence of the temporal window, the morphology of the variability, the irregular distribution of the data, or stochastic fluctuations. In this context, it is necessary to perform a systematic count of harmonic relations in the observed light curves and compare them with those obtained from simulations without intrinsic periodicity. This procedure allows us to estimate how often spurious harmonics appear and to evaluate whether the relations detected in the real data exceed what is expected under a scenario dominated by stochastic variability.

To address this problem, we developed an algorithm to systematically detect and count harmonic relations between GLSP peaks. The algorithm is applied to the observed light curves and to simulated light curves generated using the methods of Timmer and Koenig (Timmer & Koenig, 1995) and Emmanoulopoulos (Emmanoulopoulos et al., 2013). The simulated curves are constructed from the properties of each observed curve, including the PSD slope, the flux density PDF, and the observed cadence through the resampling step. These quantities are defined below and constitute the basis for a consistent comparison between the observed and simulated harmonic structures in the GLSP.

Figure 6.12 compares the mean powers of the six strongest GLSP peaks in the 83 combined OVRO+UMRAO light curves with those obtained from simulations. The peaks were ranked by decreasing power, and the mean power was calculated

separately for each rank across the observed and simulated ensembles. For each source, 100 simulations were generated using the EMP method for each of three arbitrarily chosen PSD slopes: $\beta = 1.5$, 1.9, and 2.5, corresponding to panels (a), (b), and (c), respectively. The GLSP analysis used an oversampling factor of $\zeta = 29$, and peak selection was restricted to periods longer than 100 days. The simulated mean powers show the closest overall agreement with the observed values for $\beta = 1.5$. As the assumed PSD slope increases, the discrepancy becomes more pronounced, particularly for the strongest peak, whose simulated mean power increasingly exceeds the observed value.

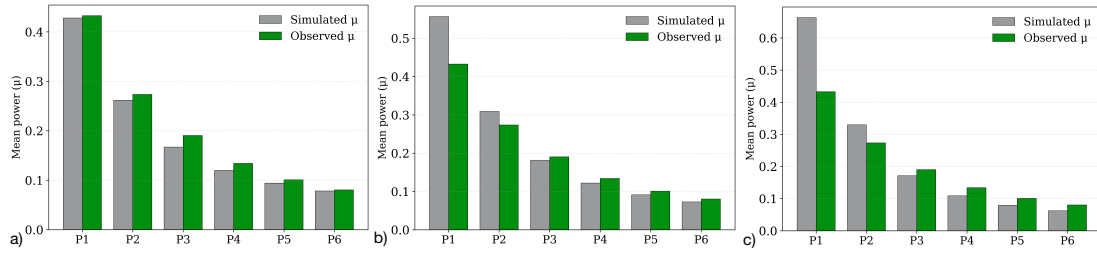


Figure 6.11: Mean powers detected in the six strongest GLSP peaks of the observed light curves, shown in green, compared with those obtained from the simulated light curves, shown in gray, for different PSD slopes. Panels correspond to: (a) $\beta = 1.5$, which shows the closest agreement between the distributions; (b) $\beta = 1.9$; and (c) $\beta = 2.5$.

6.4. Harmonic counting algorithm

To evaluate how frequently apparent harmonic relations arise in light curves dominated by stochastic variability, a harmonic detection and counting algorithm was developed and applied to the main GLSP peaks. The purpose of this procedure is not to identify a physical harmonic relation directly, but to quantify how many relations compatible with simple period ratios can appear in simulated light curves under controlled conditions.

For each source, 100 simulated light curves were generated using the method of (Emmanoulopoulos et al., 2013), considering arbitrary slopes for the power spectral density. In this thesis, the convention adopted is

$$S(f) \propto f^{-\beta}, \quad (6.10)$$

where $S(f)$ is the PSD, f is the temporal frequency, and β controls the PSD slope. The values $\beta = 1.5$, 1.9 , and 2.5 were explored.

A GLSP analysis was then applied to each simulated light curve using an oversampling factor $\zeta = 29$, restricting the search to periods longer than 100 days. For each periodogram, the six peaks with the highest GLSP power were selected and denoted as: $\mathcal{P} = \{P_1, P_2, P_3, P_4, P_5, P_6\}$, where the index indicates decreasing GLSP power. From these six periods, all non-repeated pairs were constructed,

$$N_{\text{pair}} = \binom{6}{2} = 15, \quad (6.11)$$

and for each pair the period ratio was computed as

$$r_{ij} = \frac{P_i}{P_j}, \quad i < j. \quad (6.12)$$

The harmonic families inspected were

$$\mathcal{H} = \left\{ \frac{4}{3}, \frac{3}{2}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1} \right\}, \quad (6.13)$$

together with their reciprocals,

$$\mathcal{H}^{-1} = \left\{ \frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\}. \quad (6.14)$$

For each ratio r_{ij} , the relative disagreement with respect to a candidate harmonic relation q was defined as

$$\epsilon(r_{ij}, q) = 100 \left| \frac{r_{ij} - q}{q} \right|. \quad (6.15)$$

The first step of the algorithm is an admissibility filter. For each pair (P_i, P_j) , the set of admissible harmonic relations is defined as

$$\mathcal{A}_{ij} = \{q \in \mathcal{H} \cup \mathcal{H}^{-1} : \epsilon(r_{ij}, q) < 10\%\}. \quad (6.16)$$

A pair is considered a harmonic candidate if

$$\mathcal{A}_{ij} \neq \emptyset. \quad (6.17)$$

This tolerance criterion is applied before the independence condition. This ordering is intentional, because it allows the algorithm to retain all candidate relations that satisfy the 10% threshold before deciding which ones should contribute to the final count. In particular, this avoids prematurely discarding cases with extremely small disagreement, which are especially relevant when inspecting possible harmonic structure.

After the admissible pairs have been identified, an independence criterion is applied to reduce redundant counting among relations that reuse the same GLSP peaks. The pairs are inspected in the order

$$\frac{P_1}{P_2}, \frac{P_1}{P_3}, \dots, \frac{P_1}{P_6}, \frac{P_2}{P_3}, \dots, \frac{P_5}{P_6}, \quad (6.18)$$

so that relations involving higher-power peaks are examined first. A candidate relation is counted only if at least one of the two periods involved has not been used previously in an already accepted harmonic relation. Thus, the independence criterion acts only after the 10% harmonic tolerance has been satisfied.

For each accepted pair, the harmonic relation assigned to it is the admissible relation with the smallest relative disagreement,

$$q_{\text{best}}(i, j) = \arg \min_{q \in \mathcal{A}_{ij}} \epsilon(r_{ij}, q). \quad (6.19)$$

This step determines which direct or reciprocal relation best represents the accepted pair. The accepted relation is then mapped into a canonical class. The canonical classes used in the counting are

$$\mathcal{C} = \left\{ \frac{4}{3}, \frac{3}{2}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1} \right\}. \quad (6.20)$$

Thus, a reciprocal relation such as $1/3$ is counted in the canonical class $3/1$, and a relation such as $1/2$ is counted in the class $2/1$. The count therefore answers the following question: whether at least one GLSP peak is approximately $4/3$, $3/2$, 2 , 3 , or 4 times another peak, within the adopted tolerance.

The algorithm can be applied in parallel to observed and simulated light curves. In the output summaries, each analyzed object is treated as a record. For the observed sample, one record corresponds to one source. For the simulations, one record corresponds to one source and one simulated realization. In this thesis, however, only the counts from the simulated light curves are used, since the objective is to characterize the expected frequency of apparent harmonic relations produced by red-noise variability.

For a given canonical class c , let $n_{m,c}$ be the number of independent accepted occurrences of that class in record m . The number of records with exactly k independent occurrences of class c is

$$N_{c,k} = \sum_{m=1}^{N_{\text{rec}}} \mathbb{I}(n_{m,c} = k), \quad (6.21)$$

where N_{rec} is the number of records and $\mathbb{I}$ is the indicator function, which takes a value of 1 if $n_{m,c} = k$ and 0 otherwise. The total number of harmonics found in class c , denoted as Tot_c , is

$$\text{Tot}_c = \sum_{k \geq 1} k N_{c,k}. \quad (6.22)$$

The number of unique presences of class c , denoted as Uni_c , is

$$\text{Uni}_c = \sum_{k \geq 1} N_{c,k}. \quad (6.23)$$

The excess count, denoted as Ex_c , measures how many additional occurrences arise from records containing repeated independent relations of the same canonical class,

$$\text{Ex}_c = \text{Tot}_c - \text{Uni}_c = \sum_{k \geq 2} (k - 1) N_{c,k}. \quad (6.24)$$

In this notation, Tot counts all accepted independent occurrences, Uni counts whether a class appears at least once in a record, and Ex quantifies the contribution of repeated occurrences within the same class. The columns labeled $k = 1, \dots, 5$ in the summary tables correspond to the values of $N_{c,k}$, that is, the number of records with exactly k independent occurrences of a given canonical class.

The global number of counted harmonic relations per record can be summarized as

$$\bar{k} = \frac{1}{N_{\text{rec}}} \sum_{c \in \mathcal{C}} \text{Tot}_c. \quad (6.25)$$

This quantity provides a compact measure of how many apparent harmonic relations are typically recovered per analyzed record. For the simulated sample, it gives the expected harmonic count per source-simulation pair under the adopted PSD slope and sampling conditions.

The full flow of the procedure is summarized in Figure 6.12. The algorithm also allows additional configurations for future studies, including the use of power cutoffs on the selected GLSP peaks. In particular, the cutoff can be set to zero, so that the six strongest peaks are always considered, or it can be defined relative to the GLSP power of the sixth peak through a user-defined scaling factor. This flexibility allows the harmonic count to be tested as a function of the periodogram power threshold.

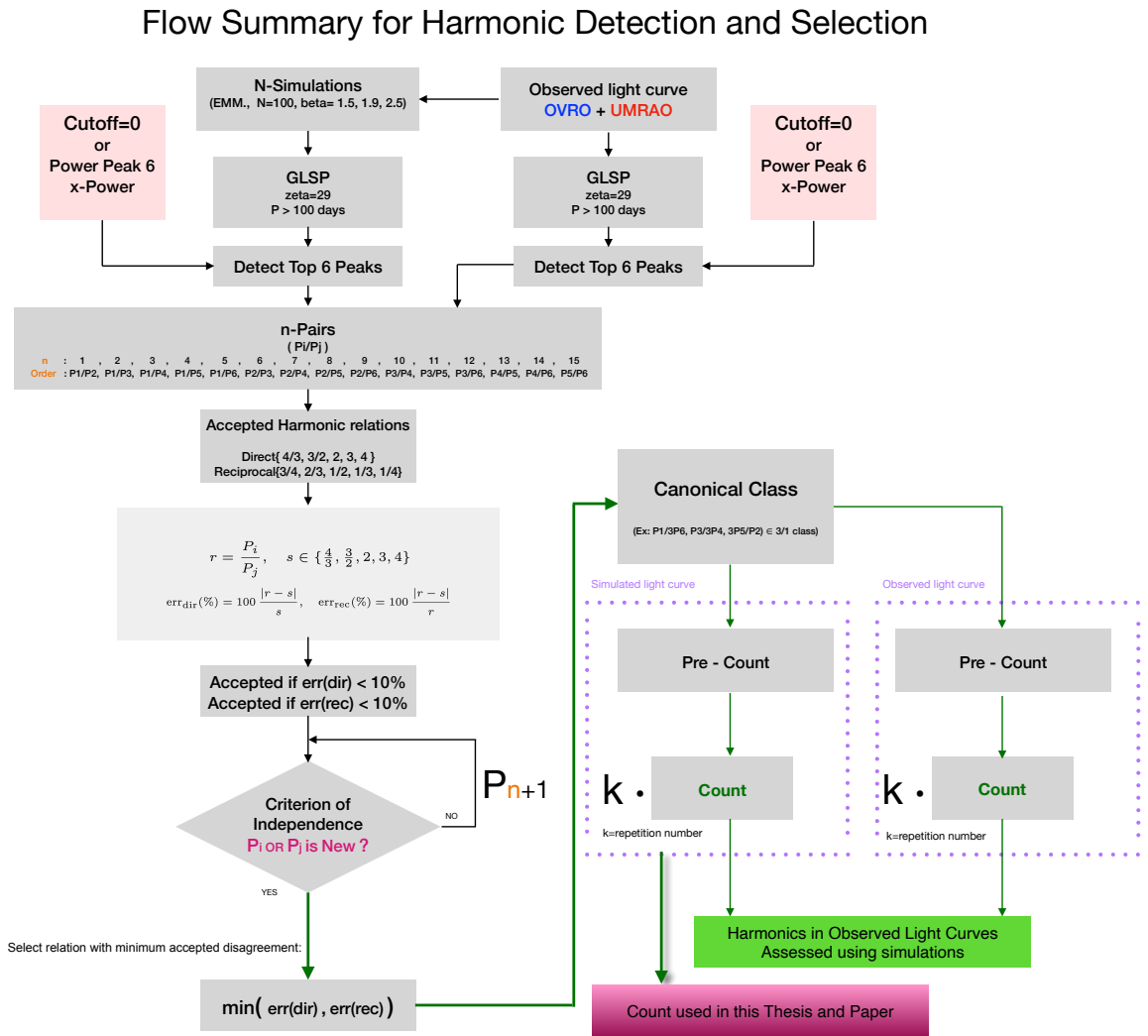


Figure 6.12: Flow diagram of the harmonic counting algorithm. For each light curve, the GLSP is computed with $\zeta = 29$ and $P > 100$ days, the six strongest peaks are selected, all 15 period ratios are tested against the adopted harmonic families, and the accepted independent relations are assigned to canonical classes.

6.5. Results of the harmonic relation counts

The results of the harmonic-relation counting analysis are presented in Tables 6.1 and 6.2. In both tables, the PSD slopes are expressed using the convention $S(f) \propto f^{-\beta}$, with $\beta > 0$. The values of β apply only to the simulated light curves; therefore, the observed counts are reported only once.

For a canonical class c , $N_{c,k}$ is defined as the number of records with exactly k independent occurrences of that class. For the observed sample, one record corresponds to one source; for the simulated sample, one record corresponds to one source–simulation pair. From $N_{c,k}$, we define

$$\text{Tot}_c = \sum_{k \geq 1} k N_{c,k}, \quad (6.26)$$

$$\text{Uni}_c = \sum_{k \geq 1} N_{c,k}, \quad (6.27)$$

and

$$\text{Ex}_c = \text{Tot}_c - \text{Uni}_c = \sum_{k \geq 2} (k - 1) N_{c,k}. \quad (6.28)$$

Here, Tot_c is the total number of accepted harmonic relations in class c , Uni_c counts how many records contain that class at least once, and Ex_c measures the excess produced by repeated occurrences within the same record. The canonical classes considered are 4/3, 3/2, 2/1, 3/1, and 4/1.

In total,

$$83 \times 100 \times 3 = 24900 \quad (6.29)$$

simulated light curves were analyzed. Of these, 24879 produced at least one accepted harmonic relation, while only 21 produced none. Therefore, the fraction of simulations with at least one apparent harmonic relation is

$$\frac{24879}{24900} \times 100 \simeq 99.92\%. \quad (6.30)$$

This result shows that, under the adopted criteria, apparent harmonic relations among the main GLSP peaks are extremely frequent in simulated light curves.

The observed sample yields $N_h^{\text{obs}} = 371$ accepted relations in 83 sources, corresponding to

$$\bar{k}_{\text{obs}} = \frac{371}{83} \simeq 4.47 \quad (6.31)$$

relations per source. In the simulations, the total counts are 31720, 31675, and 31258 for $\beta = 1.5$, 1.9, and 2.5, respectively, with mean values of $\bar{k}_{\text{sim}} \simeq 3.8$ relations per source and simulation pair. The observed-to-simulated ratio remains close to unity, with values between 1.17 and 1.19.

The uncertainties reported in Table 6.2 should be interpreted as nominal counting uncertainties. Once the observed sample is defined and the algorithm is applied, the resulting counts are fixed quantities. However, the $\sqrt{N}$ approximation provides an estimate of the expected fluctuation scale for a finite count of accepted harmonic relations. Under this interpretation, for a total count N_h , we adopt

$$\sigma_{N_h} = \sqrt{N_h}. \quad (6.32)$$

The uncertainties in the mean values are obtained by propagating this counting uncertainty. For the observed sample,

$$\sigma_{\bar{k}_{\text{obs}}} = \frac{\sqrt{N_h^{\text{obs}}}}{N_{\text{src}}}, \quad (6.33)$$

whereas for the simulations,

$$\sigma_{\bar{k}_{\text{sim}}} = \frac{\sqrt{N_h^{\text{sim}}}}{N_{\text{pair}}}. \quad (6.34)$$

The ratio between the observed and simulated mean values is defined as

$$R = \frac{\bar{k}_{\text{obs}}}{\bar{k}_{\text{sim}}}. \quad (6.35)$$

Its nominal uncertainty is computed using standard error propagation for a ratio,

$$\sigma_R = R \sqrt{\left(\frac{\sigma_{\bar{k}_{\text{obs}}}}{\bar{k}_{\text{obs}}}\right)^2 + \left(\frac{\sigma_{\bar{k}_{\text{sim}}}}{\bar{k}_{\text{sim}}}\right)^2}. \quad (6.36)$$

This propagation assumes that the observed and simulated counts represent estimates of mean occurrence rates in two distinct sets: the observed sample and the EMP simulations. Therefore, these uncertainties should not be interpreted as measurement errors on deterministic quantities, but rather as a simple estimate of the scale of the counting fluctuations.

Table 6.1: Harmonic counts for observed and simulated light curves.

β	Set	Class	Tot	Uni	Ex	N_k				
						$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
1.5	Sim.	4/3	6270	4763	1507	3408	1206	146	3	0
1.5	Sim.	3/2	6417	4786	1631	3358	1232	190	5	1
1.5	Sim.	2/1	6964	5159	1805	3585	1355	207	12	0
1.5	Sim.	3/1	6138	4656	1482	3360	1121	164	11	0
1.5	Sim.	4/1	5931	4590	1341	3402	1039	145	4	0
1.5	Sim.	Total	31720	23954	7766	17113	5953	852	35	1
1.9	Sim.	4/3	6322	4875	1447	3577	1153	141	4	0
1.9	Sim.	3/2	6495	4816	1679	3349	1269	184	14	0
1.9	Sim.	2/1	6701	5012	1689	3537	1268	200	7	0
1.9	Sim.	3/1	6022	4637	1385	3419	1058	153	7	0
1.9	Sim.	4/1	6135	4704	1431	3443	1098	156	7	0
1.9	Sim.	Total	31675	24044	7631	17325	5846	834	39	0
2.5	Sim.	4/3	6367	4851	1516	3479	1230	140	2	0
2.5	Sim.	3/2	6326	4729	1597	3309	1251	162	6	1
2.5	Sim.	2/1	6461	4848	1613	3414	1267	155	12	0
2.5	Sim.	3/1	6031	4591	1440	3321	1110	150	10	0
2.5	Sim.	4/1	6073	4689	1384	3473	1056	152	8	0
2.5	Sim.	Total	31258	23708	7550	16996	5914	759	38	1
–	Obs.	4/3	53	41	12	30	10	1	0	0
–	Obs.	3/2	76	58	18	41	16	1	0	0
–	Obs.	2/1	82	54	28	31	18	5	0	0
–	Obs.	3/1	85	57	28	35	16	6	0	0
–	Obs.	4/1	75	54	21	34	19	1	0	0
–	Obs.	Total	371	264	107	171	79	14	0	0

Note. Definitions of Tot, Uni, Ex, N_k , canonical classes, and records are given in the main text. The observed counts are listed only once because the adopted PSD slopes apply only to the simulated light curves.

Overall, the results do not show significant differences among the adopted values of β . The simulated counts and the mean values $\bar{k}_{\text{sim}}$ remain very similar for $\beta = 1.5$, 1.9, and 2.5. The disagreement between the observed and simulated samples is also small, with Obs./sim. ratios close to unity. Thus, the observed light curves

show only a modest excess of harmonic relations relative to the simulations.

This result supports the interpretation that steep PSDs can generate random flares responsible for most of the apparent periodicities and harmonic relations seen in blazar GLSP spectra (Molina et al., 2026). Therefore, the presence of simple period ratios, even when they involve the strongest GLSP peaks (see fig. 6.7), is not by itself evidence for a physical harmonic. Such cases require additional evidence, including phase coherence, temporal persistence, multifrequency consistency, and direct agreement with the morphology of the light curve.

The PSD slopes $\beta = 1.5, 1.9$, and 2.5 adopted in the simulations were selected to span a representative range of red-noise behaviours observed in radio blazar light curves. It should also be noted that the simulations assume a power-law dependence, whereas the actual shapes of the observed PSDs may be more complex. Consequently, this introduces an additional source of uncertainty that is not included in the uncertainty associated with the number of harmonic relationships reported below (Table 6.2).

Table 6.2: Global harmonic counts for observed and simulated light curves.

β	N_{src}	N_{cl}	N_{pair}	N_0	$N_{\text{h}}^{\text{obs}}$	$\bar{k}_{\text{obs}}$	$N_{\text{h}}^{\text{sim}}$	$\bar{k}_{\text{sim}}$	Obs./sim.
1.5	83	8300	8294	6	371 ± 19	4.47 ± 0.23	31720 ± 178	3.82 ± 0.02	1.17 ± 0.06
1.9	83	8300	8295	5	371 ± 19	4.47 ± 0.23	31675 ± 178	3.82 ± 0.02	1.17 ± 0.06
2.5	83	8300	8290	10	371 ± 19	4.47 ± 0.23	31258 ± 177	3.77 ± 0.02	1.19 ± 0.06

Note. Definitions of N_{src} , N_{cl} , N_{pair} , N_0 , N_{h} , and $\bar{k}$ are given in the main text. Uncertainties on N_{h} are assumed to be $\sqrt{N}$.

The harmonic counts obtained for the three adopted PSD slopes are similar. Therefore, within the explored range of β , the occurrence of harmonic relationships appears to be largely independent of the power-law slope. This result indicates that stochastic flares may play an important role in producing the apparent harmonic relationships observed in the GLS periodograms.

Chapter 7

Summary, Conclusions, and Outlook

This final chapter summarizes the main results obtained in this thesis, discusses their interpretation in the context of blazar variability and SMBHB candidates, and outlines the main conclusions and future directions that follow from this work.

7.1. Summary and Discussion

7.1.1 Periodicity search and candidate selection

The first step in interpreting a periodicity as possible evidence of an SMBHB system is to establish whether the detected signal can be regarded as intrinsic to the blazar light curve or whether it can be reproduced by the stochastic red-noise variability characteristic of this type of source. In this thesis, multiple periodicity-analysis methods were applied to the combined OVRO+UMRAO sample, comprising 83 predominantly blazar sources.

The initial inspection combined the GLSP, least-squares sinusoidal fitting, the WWZ, and SWF estimates, applied both to the individual light curves from each telescope and to the combined OVRO+UMRAO light curves. This set of tools made it possible to identify sources with prominent periodicities, evaluate the temporal consistency of the signals, and compare the relationship between the periodogram peaks and the morphology of the light curves.

Within the sample, at least three sources stood out as preliminary candidates:

J2134–0153, also known as PKS 2131–021, J0818+4222, and J1309+1154. Among them, J1309+1154 exhibited the most dominant periodicity in the GLSP and a more persistent temporal structure in the WWZ, with a characteristic period of approximately 18 years. For this reason, a substantial part of the thesis focused on the detailed analysis of this source.

7.1.2 Evidence supporting the periodicity of J1309+1154

In addition to the preliminary analyses, J1309+1154 was subjected to a set of complementary tests aimed at evaluating the coherence of the detected periodicity. First, the GLSP and WWZ analyses were repeated after applying a detrending correction, which provided a cleaner characterization of the dominant periodicity.

Second, the MOJAVE/VLBA observations at 15 GHz enabled a comparison between the UMRAO+OVRO light curve and the flux-density evolution of the modeled components. In J1309+1154, the stationary component #5 is located very close in angular separation to the core and exhibits a sinusoidal modulation similar to that observed in the OVRO+UMRAO light curve. In contrast, the non-stationary components located at larger angular separations do not follow the long-term modulation of the light curve. It is important to note that no absolute physical scale was assigned to these separations because the redshift of the source is unknown.

Third, complementary observations, including data from UMRAO at 4.8 and 8 GHz, ALMA Band 3, Metsähovi at 37 GHz, and the MOJAVE program at 15 GHz, provided additional evidence for the modulation. Although the temporal coverage and sampling density are not homogeneous across the different instruments, these observations show qualitative consistency with the periodicity detected in the combined OVRO+UMRAO light curve.

Finally, the presence of a possible 2 : 1 harmonic relation, associated with a timescale close to 9 years, was identified as a relevant feature of J1309+1154. This structure was subsequently analyzed in greater detail by [Readhead et al. \(2026\)](#), further strengthening the interest of the source as an SMBHB candidate.

7.1.3 Statistical significance and harmonic relations

The significance analysis was performed under the null hypothesis that the detected periodicity could be reproduced by stochastic red-noise variability. Under this criterion, the dominant signal in J1309+1154 did not reach the conventional 3σ threshold, yielding a formal significance of approximately 2.14σ . This result represents an important point of the thesis, as it reveals a discrepancy between the observational evidence supporting the periodicity and the purely statistical interpretation based on red-noise simulations.

This discrepancy motivated a more detailed evaluation of the behavior of the GLSP and the occurrence of harmonic relations in simulated and observed light curves. In particular, the GLSP was found to be highly sensitive to the presence of flares, the available time baseline, and the power distribution introduced by stochastic variability. Using an algorithm developed in this research to count harmonic relations, it was shown that 99.9% of the light curves simulated using the EMP method produce apparent harmonic relations under the adopted criteria, with an average of ~ 4 harmonics per simulation. Furthermore, the number of harmonic relations per source in the observed sample is modestly higher than that obtained in the simulations, with a ratio of ~ 1.17 .

This result indicates that simple harmonic relations among GLSP peaks cannot, by themselves, be interpreted as evidence of a physical harmonic. Their interpretation requires additional information, including phase coherence, temporal persistence, multifrequency consistency, and a direct correspondence with the morphology of the light curve.

7.1.4 PSD estimation and limitations

A central aspect of periodicity analyses of light curves is the estimation of the power spectral density and, in particular, of its intrinsic slope, since this is a fundamental parameter for generating simulations and evaluating the statistical significance of the detected signals.

A direct approach based on the GLSP can characterize the observed power distribution. However, this estimate depends on both the intrinsic variability

of the source and the temporal sampling pattern of the observations, and may therefore be affected by gaps, irregular sampling, spurious peaks, and spectral leakage. As an alternative, several studies have estimated the intrinsic slope using procedures that include an initial interpolation of the light curve to regularize the sampling, followed by steps designed to reduce the impact of extended temporal gaps.

The latter approach may be appropriate for monitoring programs with relatively regular cadences, such as OVRO, with a cadence of approximately 4 days. However, its application to combined OVRO and UMRAO light curves is more complex because of differences in cadence and temporal coverage, as well as the presence of gaps within the UMRAO data and between the two datasets. Under these conditions, interpolation may introduce a non-negligible number of artificial data points, particularly during the initial estimation of the PSD, which must be considered when interpreting the recovered slope.

Furthermore, representing the PSD with a single power law may be insufficient when the light curve contains a dominant periodicity or exhibits frequency-dependent changes in slope. In such cases, more flexible models could be considered, such as a broken power law, with different slopes on either side of a break frequency, or a bending power law, in which the transition occurs gradually around a characteristic frequency. These models could describe the effective shape of the PSD more accurately and reduce the dependence on a single global slope. Implementing these models would involve additional parameters and, therefore, a higher computational cost, which is an important consideration for future work.

7.1.5 Light-curve simulations and shape effects

The Timmer and Koenig (TK) and Emmanoulopoulos (EMP) methods have been fundamental for simulating light curves with controlled statistical properties. The TK method reproduces a prescribed PSD but generates gaussian flux distributions. In contrast, the EMP method can simulate light curves that preserve both the observed PSD and PDF, as well as isolated flares when these are present in the original light curve (Fig. 5.15, top right).

However, EMP simulations may have relevant limitations when applied to light curves dominated by a strong sinusoidal component. In such cases, the PSD, the PDF, and the resampling procedure, as inputs to the simulator, may be overly restrictive, producing light curves with morphologies that are uncommon for this type of source. Future simulation methods could therefore benefit from incorporating additional metrics related to the shape of the light curve.

Given the limitations discussed in PSD estimation and light-curve simulations, targeted searches with pulsar timing arrays (PTAs) constitute an independent approach for assessing the SMBHB nature of the most robust candidates. In this context, PKS 2131–021 represents the most relevant case, while PKS J0805–0111 provides an additional example under favorable conditions of mass, geometry, and observational sensitivity (O’Neill et al., 2022; Kiehlmann et al., 2025; de la Parra et al., 2025).

7.2. Conclusions

In this thesis, the combined light curves of 83 blazars monitored by UMRAO and OVRO at 14.5/15 GHz were analyzed, with time baselines extending up to approximately 46–50 years. This dataset, unprecedented in both duration and cadence, enabled a systematic search for long-term periodicities using the generalized Lomb–Scargle periodogram (GLSP), the weighted wavelet Z transform (WWZ), and sinusoidal fitting. The main results of this research were published in Molina et al. (2026).

Although apparent periodicities were detected in all 83 sources, the comparison among the UMRAO data, the OVRO data, and the combined light curve showed that, among the new cases studied, only J1309+1154 retains a dominant and coherent signal throughout the entire time baseline. Its best-fit period is

$$P = 6551 \pm 26 \text{ days} = 17.9 \text{ yr}, \quad (7.1)$$

corresponding to approximately 2.5 observed cycles. The signal is consistent across the GLSP, WWZ, and sinusoidal analyses; however, its global significance is

$$p_{\text{global}} = 1.16 \times 10^{-2}, \quad (7.2)$$

equivalent to 2.14σ , below the adopted threshold of

$$p = 1.35 \times 10^{-3}, \quad (7.3)$$

corresponding to 3σ . Therefore, the available evidence supports the classification of J1309+1154 as a candidate supermassive black hole binary system, but not as a confirmed detection. Observational coherence and statistical significance are not contradictory results: the former describes the persistence of the signal in the data, whereas the latter quantifies the probability that stochastic variability could produce a feature of equal or greater relevance.

After subtracting the main sinusoidal component, an additional residual component with a period of approximately 9.2 years was identified, close to the expected first harmonic at 8.95 years. This component, subsequently developed and analyzed in greater depth by [Readhead et al. \(2026\)](#), provides additional observational evidence for a periodic structure in J1309+1154. Nevertheless, its interpretation must also account for the possibility that harmonic relationships can arise spuriously in processes dominated by red noise.

The central methodological result of this thesis is that the GLSP can produce multiple spurious peaks and apparent harmonic relationships when applied to light curves dominated by stochastic variability. To quantify this effect,

$$83 \times 100 \times 3 = 24\,900 \quad (7.4)$$

simulated light curves were analyzed, corresponding to power spectral density slopes $\beta = \{1.5, 1.9, 2.5\}$. Of these, only 21 did not exhibit multiple periodicities with apparent harmonics (Table 6.2). Consequently, these patterns appeared in

$$\frac{24\,879}{24\,900} = 99.916\% \quad (7.5)$$

of the simulations. This result demonstrates that the presence of multiple spectral peaks or harmonic period ratios does not, by itself, constitute evidence of a

physical periodicity. In most cases, these structures can be reproduced by the combination of a steep PSD and the stochastic nature of blazar flaring activity.

The identification of periodic candidates therefore requires a joint assessment of spectral power, global significance, temporal coherence, and signal stability across independent observing intervals. As a conservative criterion, [Molina et al. \(2026\)](#) proposed considering signals dominated by a peak satisfying

$$\mathcal{P}_{\text{GLSP}} \geq 0.75 \quad (7.6)$$

and observed over at least 2.5 cycles. Even under these conditions, the classification should remain at the candidate level unless the null hypothesis of stochastic variability can be rejected with sufficient significance.

The main contribution of this thesis is not limited to the identification of J1309+1154 as a new candidate. The analysis of an unprecedented multidecadal sample and the tests performed using 24 900 simulated light curves made it possible to quantitatively contrast the observational evidence supporting a periodicity with one of the most widely used significance criteria in this field. The apparent discrepancy between these results motivated a critical evaluation of the GLSP and its response to stochastic red-noise processes. Thus, J1309+1154 is not only an astrophysically relevant candidate, but also a case study for examining the limitations of current methods and establishing a basis for improving their robustness and further refining them in future work.

7.2.1 Publications arising from this research

1. A Search for Supermassive Black Hole Binary Candidates in 46 yr Radio Light Curves of 83 Blazars. ([Molina et al., 2026](#))
2. Compelling Evidence for a Harmonic in the Light Curve of the Supermassive Black Hole Binary Candidate PKS J1309+1154. ([Readhead et al., 2026](#))

7.3. Outlook and Future Work

Future work on the OVRO+UMRAO sample could consider the following short-term objectives:

- A stationarity analysis of the OVRO+UMRAO light curves, since a single observed PDF can be clearly interpreted as a parent distribution only if the process generating the flux densities preserves, at least approximately, its statistical properties over time. This analysis would make it possible to justify the use of the observed PDF in the simulation algorithms or, alternatively, to minimize the impact of departures from stationarity.
- The development of a more flexible PSD estimation method that avoids interpolation and applies more comprehensive models when estimating the intrinsic PSD slope.
- The exploration of simulations that incorporate morphological information from the observed light curve. For example, the current constraints the PSD slope, the PDF, and resampling at the observed cadence could be combined with an additional shape factor or morphological function, with an appropriate balance among their relative weights.

In the medium term, the following approaches could be explored and/or developed:

- New algorithms, for example based on the WWZ, that can be parallelized on a supercomputer and employ self-calibrating metrics.
- The Double Lomb–Scargle Periodogram proposed by [Scargle & Wagner \(2026\)](#) could be explored as a complementary tool for the simultaneous characterization of a dominant periodicity and a possible 2 : 1 harmonic. Its application would allow their frequencies, amplitudes, and relative phases to be estimated jointly. However, a dedicated significance analysis based on light-curve simulations would still need to be developed.

References

- Abraham, Z. (2018, May). Jet precession in binary black holes. *Nature Astronomy*, 2, 443-444. doi: [doi:10.1038/s41550-018-0484-2](https://doi.org/10.1038/s41550-018-0484-2)
- Ackermann, M., et al. (2015). The Third Catalog of Active Galactic Nuclei Detected by the Fermi Large Area Telescope. *ApJ*, 810, 14. doi: [doi:10.1088/0004-637X/810/1/14](https://doi.org/10.1088/0004-637X/810/1/14)
- Agarwal, N., Agazie, G., Anumalapudi, A., Archibald, A. M., Arzoumanian, Z., Baier, J. G., ... Collaboration, T. N. (2026, feb). The nanograv 15 yr dataset: Targeted searches for supermassive black hole binaries. *The Astrophysical Journal Letters*, 998(1), L11. doi: [doi:10.3847/2041-8213/ae3719](https://doi.org/10.3847/2041-8213/ae3719)
- Agazie, G., Anumalapudi, A., Archibald, A. M., Arzoumanian, Z., Baker, P. T., Bécsy, B., ... Nanograv Collaboration (2023, July). The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background. *ApJL*, 951(1), L8. doi: [doi:10.3847/2041-8213/acdac6](https://doi.org/10.3847/2041-8213/acdac6)
- Aharonian, F., et al. (2007). An Exceptional Very High Energy Gamma-Ray Flare of PKS 2155-304. *ApJL*, 664, L71-L74. doi: [doi:10.1086/520635](https://doi.org/10.1086/520635)
- Aller, H. D., Aller, M. F., & Hughes, P. A. (1985). Polarized radio outbursts in BL Lacertae. i. polarized emission from a compact jet. *The Astrophysical Journal*, 298, 296-300. doi: [doi:10.1086/163610](https://doi.org/10.1086/163610)
- Aller, H. D., Aller, M. F., Latimer, G. E., & Hodge, P. E. (1985, December). Spectra and linear polarizations of extragalactic variable sources at centimeter wavelengths. *ApJS*, 59, 513-768. doi: [doi:10.1086/191083](https://doi.org/10.1086/191083)
- Aller, M. F., Aller, H. D., & Hughes, P. A. (2017). The University of Michigan Centimeter-Band All Stokes Blazar Monitoring Program: Single-Dish Polarimetry as a Probe of Parsec-Scale Magnetic Fields. *Galaxies*, 5(4), 75. doi: [doi:10.3390/galaxies5040075](https://doi.org/10.3390/galaxies5040075)
- Aller, M. F., Aller, H. D., Hughes, P. A., & Latimer, G. E. (1999). Cm-Wavelength Total Flux and Linear Polarization Properties of Radio-Loud BL Lacertae Objects. *The Astrophysical Journal*, 512(2), 601. doi: [doi:10.1086/306799](https://doi.org/10.1086/306799)
- Baars, J. W. M., Genzel, R., Pauliny-Toth, I. I. K., & Witzel, A. (1977, October). The absolute spectrum of Cas A: an accurate flux density scale and a set of secondary calibrators. *A&A*, 61(1), 99 - 106. doi: [doi:10.1051/A&A/1977A6199B](https://doi.org/10.1051/A&A/1977A6199B)

- Beckmann, V., & Shrader, C. R. (2013, February). The AGN phenomenon: open issues. *arXiv e-prints*, arXiv:1302.1397. doi: [doi:10.48550/arXiv.1302.1397](https://doi.org/10.48550/arXiv.1302.1397)
- Begelman, M. C., Blandford, R. D., & Rees, M. J. (1980, September). Massive black hole binaries in active galactic nuclei. *Nature*, *287*(5780), 307-309. doi: [doi:10.1038/287307a0](https://doi.org/10.1038/287307a0)
- Blandford, R. D., & Königl, A. (1979, August). Relativistic jets as compact radio sources. *ApJ*, *232*, 34-48. doi: [doi:10.1086/157262](https://doi.org/10.1086/157262)
- Blandford, R. D., & Payne, D. G. (1982, June). Hydromagnetic flows from accretion disks and the production of radio jets. *MNRAS*, *199*, 883-903. doi: [doi:10.1093/mnras/199.4.883](https://doi.org/10.1093/mnras/199.4.883)
- Blandford, R. D., & Rees, M. J. (1974, December). A "twin-exhaust" model for double radio sources. *Monthly Notices of the Royal Astronomical Society*, *169*, 395-415. doi: [doi:10.1093/mnras/169.3.395](https://doi.org/10.1093/mnras/169.3.395)
- Blandford, R. D., & Znajek, R. L. (1977, May). Electromagnetic extraction of energy from Kerr black holes. *MNRAS*, *179*, 433-456. doi: [doi:10.1093/mnras/179.3.433](https://doi.org/10.1093/mnras/179.3.433)
- Bloom, S. D., & Marscher, A. P. (1996). An Analysis of the Synchrotron Self-Compton Model for the Multi-Wave Band Spectra of Blazars. *ApJ*, *461*, 657. doi: [doi:10.1086/177092](https://doi.org/10.1086/177092)
- Britzen, S., Fendt, C., Witzel, G., Qian, S. J., Pashchenko, I. N., Kurtanidze, O., ... Witzel, A. (2018, August). OJ287: deciphering the 'Rosetta stone of blazars. *MNRAS*, *478*(3), 3199-3219. doi: [doi:10.1093/mnras/sty1026](https://doi.org/10.1093/mnras/sty1026)
- Cackett, E. M., Horne, K., & Winkler, H. (2007). Testing thermal reprocessing in active galactic nuclei accretion discs. *MNRAS*, *380*, 669-682. doi: [doi:10.1111/j.1365-2966.2007.12098.x](https://doi.org/10.1111/j.1365-2966.2007.12098.x)
- Caproni, A., Mosquera Cuesta, H. J., & Abraham, Z. (2004, December). Observational Evidence of Spin-induced Precession in Active Galactic Nuclei. *ApJL*, *616*(2), L99-L102. doi: [doi:10.1086/426863](https://doi.org/10.1086/426863)
- Chartas, G., Agol, E., Eracleous, M., Garmire, G., Bautz, M. W., & Morgan, N. D. (2002, apr). Caught in the act: Chandra observations of microlensing of the radio-loud quasar mg j0414+0534. *The Astrophysical Journal*, *568*(2), 509. doi: [doi:10.1086/339162](https://doi.org/10.1086/339162)
- Cohen, M. H., Kellermann, K. I., Shaffer, D. B., Linfield, R. P., Moffet, A. T., Romney, J. D., ... Geldzahler, B. J. (1977, August). Radio sources with superluminal velocities. *Nature*, *268*, 405-409. doi: [doi:10.1038/268405a0](https://doi.org/10.1038/268405a0)
- de la Parra, P. V., Kiehlmann, S., Mróz, P., Readhead, A. C. S., Synani, A., Begelman, M. C., ... Zensus, J. A. (2025, July). PKS J0805-0111: A Second Owens Valley Radio Observatory Blazar Showing Highly Significant

- Sinusoidal Radio Variability—The Tip of the Iceberg. *ApJ*, 987(2), 191. doi: [doi:10.3847/1538-4357/addc60](https://doi.org/10.3847/1538-4357/addc60)
- Dorigo Jones, J., Johnson, S. D., Muzahid, S., Charlton, J., Chen, H.-W., Narayanan, A., ... Wijers, N. A. (2022). Improving blazar redshift constraints with the edge of the $\text{Ly}\alpha$ forest: 1es 1553+113 and implications for observations of the whim. *Monthly Notices of the Royal Astronomical Society*, 509(3), 4330–4343. doi: [doi:10.1093/mnras/stab3331](https://doi.org/10.1093/mnras/stab3331)
- Emmanoulopoulos, D., McHardy, I. M., & Papadakis, I. E. (2013, August). Generating artificial light curves: revisited and updated. *MNRAS*, 433(2), 907–927. doi: [doi:10.1093/mnras/stt764](https://doi.org/10.1093/mnras/stt764)
- EPTA Collaboration. (2023, October). The second data release from the European Pulsar Timing Array. III. Search for gravitational wave signals. *A&A*, 678, A50. doi: [doi:10.1051/0004-6361/202346844](https://doi.org/10.1051/0004-6361/202346844)
- Event Horizon Telescope Collaboration, Akiyama, K., Alberdi, A., Alef, W., Asada, K., Azulay, R., ... Younsi, Z. (2019, April). First M87 Event Horizon Telescope Results. IV. Imaging the Central Supermassive Black Hole. *ApJL*, 875(1), L4. doi: [doi:10.3847/2041-8213/ab0e85](https://doi.org/10.3847/2041-8213/ab0e85)
- Foster, G. (1996, October). Wavelets for period analysis of unevenly sampled time series. *AJ*, 112, 1709–1729. doi: [doi:10.1086/118137](https://doi.org/10.1086/118137)
- Haardt, F., & Maraschi, L. (1991). A two-phase model for the X-ray emission from Seyfert galaxies. *ApJL*, 380, L51–L54. doi: [doi:10.1086/186171](https://doi.org/10.1086/186171)
- Hincks, A. D., Ma, X., Mróz, P., Naess, S. K., Kiehlmann, S., Blandford, R. D., ... Vargas, C. (2026, February). The Atacama Cosmology Telescope: Observations of supermassive black hole binary candidates: Strong sinusoidal variations at 95, 147, and 225 GHz in PKS 2131–021 and PKS J0805–0111. *Astronomy & Astrophysics*, 706, A206. doi: [doi:10.1051/0004-6361/202555047](https://doi.org/10.1051/0004-6361/202555047)
- Hughes, P. A., Aller, H. D., & Aller, M. F. (1985). Polarized radio outbursts in BL Lacertae. ii. the flux and polarization of a piston-driven shock. *The Astrophysical Journal*, 298, 301–315. doi: [doi:10.1086/163611](https://doi.org/10.1086/163611)
- Hughes, P. A., Aller, H. D., & Aller, M. F. (1992). The university of michigan radio astronomy data base. i. structure function analysis and the relation between bl lacertae objects and quasi-stellar objects. *The Astrophysical Journal*, 396, 469–486. doi: [doi:10.1086/171734](https://doi.org/10.1086/171734)
- Johnson, S. D., Mulchaey, J. S., Chen, H.-W., Wijers, N. A., Connor, T., Muzahid, S., ... Walth, G. L. (2019). The physical origins of the identified and still missing components of the warm-hot intergalactic medium: Insights from deep surveys in the field of blazar 1es1553+113. *The Astrophysical Journal Letters*, 884(2), L31. doi: [doi:10.3847/2041-8213/ab479a](https://doi.org/10.3847/2041-8213/ab479a)

- Kellermann, K. I., & Pauliny-Toth, I. I. K. (1969). The spectra of opaque radio sources. *The Astrophysical Journal Letters*, 155, L71–L78. doi: [doi:10.1086/180305](https://doi.org/10.1086/180305)
- Kiehlmann, S., de la Parra, P. V., Sullivan, A. G., Synani, A., Liodakis, I., Mróz, P., ... Zensus, J. A. (2025, May). PKS 2131-021—Discovery of Strong Coherent Sinusoidal Variations from Radio to Optical Frequencies: Compelling Evidence for a Blazar Supermassive Black Hole Binary. *ApJ*, 985(1), 59. doi: [doi:10.3847/1538-4357/adc567](https://doi.org/10.3847/1538-4357/adc567)
- Kiehlmann, S., Lister, M. L., Readhead, A. C. S., Liodakis, I., O'Neill, S., Pearson, T. J., ... Wilkinson, P. N. (2024, February). Compact Symmetric Objects. I. Toward a Comprehensive Bona Fide Catalog. *ApJ*, 961(2), 240. doi: [doi:10.3847/1538-4357/ad0c56](https://doi.org/10.3847/1538-4357/ad0c56)
- Lind, K. R., & Blandford, R. D. (1985, August). Semidynamical models of radio jets: relativistic beaming and source counts. *The Astrophysical Journal*, 295, 358–367. doi: [doi:10.1086/163380](https://doi.org/10.1086/163380)
- Lister, M. L., Aller, H. D., Aller, M. F., Cohen, M. H., Homan, D. C., Kadler, M., ... Vermeulen, R. C. (2009). MOJAVE: monitoring of jets in active galactic nuclei with VLBA experiments. V. multi-epoch VLBA images. *The Astronomical Journal*, 137(3), 3718–3729. doi: [doi:10.1088/0004-6256/137/3/3718](https://doi.org/10.1088/0004-6256/137/3/3718)
- Lister, M. L., Homan, D. C., Kellermann, K. I., Kovalev, Y. Y., Pushkarev, A. B., Ros, E., & Savolainen, T. (2021). Monitoring of jets in active galactic nuclei with VLBA experiments. xviii. kinematics and inner jet evolution of bright radio-loud active galaxies. *The Astrophysical Journal*, 923, 30. doi: [doi:10.3847/1538-4357/ac230f](https://doi.org/10.3847/1538-4357/ac230f)
- Lister, M. L., Homan, D. C., Kellermann, K. I., Kovalev, Y. Y., Pushkarev, A. B., Ros, E., & Savolainen, T. (2021, November). Monitoring of Jets in Active Galactic Nuclei with VLBA Experiments. XVIII. Kinematics and Inner Jet Evolution of Bright Radio-loud Active Galaxies. *ApJ*, 923(1), 30. doi: [doi:10.3847/1538-4357/ac230f](https://doi.org/10.3847/1538-4357/ac230f)
- Lobanov, A. P. (1998, February). Ultracompact jets in active galactic nuclei. *Astronomy & Astrophysics*, 330, 79–89. Retrieved from <https://ui.adsabs.harvard.edu/abs/1998A&A...330...79L>
- Lobanov, A. P., & Roland, J. (2005, March). A supermassive binary black hole in the quasar 3C 345. *A&A*, 431(3), 831–846. doi: [doi:10.1051/0004-6361:20041831](https://doi.org/10.1051/0004-6361:20041831)
- Maraschi, L., Ghisellini, G., & Celotti, A. (1992). A jet model for the gamma-ray emitting blazar 3C 279. *ApJL*, 397, L5–L9. doi: [doi:10.1086/186531](https://doi.org/10.1086/186531)
- Marscher, A. P., & Gear, W. K. (1985). Models for high-frequency radio outbursts in extragalactic sources, with application to the early 1983 millimeter-to-infrared flare of 3C 273. *The Astrophysical Journal*, 298, 114–127. doi: [doi:10.1086/163592](https://doi.org/10.1086/163592)

- Marscher, A. P., Jorstad, S. G., D’Arcangelo, F. D., Smith, P. S., Larionov, V. M., Gear, W. K., . . . Thompson, D. J. (2008). The inner jet of an active galactic nucleus as revealed by a radio-to- γ -ray outburst. *Nature*, *452*(7190), 966–969. doi: [doi:10.1038/nature06895](https://doi.org/10.1038/nature06895)
- Max-Moerbeck, W., Richards, J. L., Hovatta, T., Pavlidou, V., Pearson, T. J., & Readhead, A. C. S. (2014, November). A method for the estimation of the significance of cross-correlations in unevenly sampled red-noise time series. *MNRAS*, *445*(1), 437–459. doi: [doi:10.1093/mnras/stu1707](https://doi.org/10.1093/mnras/stu1707)
- McHardy, I. M., Koerding, E., Knigge, C., Uttley, P., & Fender, R. P. (2006, December). Active galactic nuclei as scaled-up Galactic black holes. *Nature*, *444*(7120), 730–732. doi: [doi:10.1038/nature05389](https://doi.org/10.1038/nature05389)
- Miles, M. T., Shannon, R. M., Bailes, M., Reardon, D. J., Keith, M. J., Cameron, A. D., . . . Venkatraman Krishnan, V. (2023, March). The MeerKAT Pulsar Timing Array: first data release. *MNRAS*, *519*(3), 3976–3991. doi: [doi:10.1093/mnras/stac3644](https://doi.org/10.1093/mnras/stac3644)
- Molina, B., Mróz, P., De la Parra, P. V., Readhead, A. C. S., Surti, T., Aller, M. F., . . . Zensus, J. A. (2026, mar). A search for supermassive black hole binary candidates in 46 yr radio light curves of 83 blazars. *The Astrophysical Journal*, *1000*(1), 72. doi: [doi:10.3847/1538-4357/ae3141](https://doi.org/10.3847/1538-4357/ae3141)
- Morgan, C. W., Kochanek, C. S., Morgan, N. D., & Falco, E. E. (2010). The Quasar Accretion Disk Size-Black Hole Mass Relation. *ApJ*, *712*, 1129–1136. doi: [doi:10.1088/0004-637X/712/2/1129](https://doi.org/10.1088/0004-637X/712/2/1129)
- O’Neill, S., Kiehlmann, S., Readhead, A. C. S., Aller, M. F., Blandford, R. D., Lioudakis, I., . . . Zensus, J. A. (2022, February). The Unanticipated Phenomenology of the Blazar PKS 2131-021: A Unique Supermassive Black Hole Binary Candidate. *ApJL*, *926*(2), L35. doi: [doi:10.3847/2041-8213/ac504b](https://doi.org/10.3847/2041-8213/ac504b)
- Padovani, P., Alexander, D. M., Assef, R. J., De Marco, B., Giommi, P., Hickox, R. C., . . . Salvato, M. (2017, August). Active galactic nuclei: what’s in a name? *Astronomy and Astrophysics Review*, *25*(1), 2. doi: [doi:10.1007/s00159-017-0102-9](https://doi.org/10.1007/s00159-017-0102-9)
- Peterson, B. M. (1993). Reverberation Mapping of Active Galactic Nuclei. *PASP*, *105*, 247. doi: [doi:10.1086/133140](https://doi.org/10.1086/133140)
- Ramos Almeida, C., & Ricci, C. (2017, October). Nuclear obscuration in active galactic nuclei. *Nature Astronomy*, *1*, 679–689. doi: [doi:10.1038/s41550-017-0232-z](https://doi.org/10.1038/s41550-017-0232-z)
- Readhead, A. C. S., Aller, M. F., Sullivan, A. G., Blandford, R. D., Mróz, P., De la Parra, P. V., . . . Zensus, J. A. (2026, jan). Compelling evidence for a harmonic in the light curve of the supermassive black hole binary candidate pks j1309+1154. *The Astrophysical Journal Letters*, *996*(2), L39. doi: [doi:10.3847/2041-8213/ae2656](https://doi.org/10.3847/2041-8213/ae2656)

- Readhead, A. C. S., Lawrence, C. R., Myers, S. T., Sargent, W. L. W., Hardebeck, H. E., & Moffet, A. T. (1989, November). A Limit on the Anisotropy of the Microwave Background Radiation on Arcminute Scales. *ApJ*, *346*, 566. doi: [doi:10.1086/168039](https://doi.org/10.1086/168039)
- Rees, M. J. (1966, July). Appearance of Relativistically Expanding Radio Sources. *Nature*, *211*, 468-470. doi: [doi:10.1038/211468a0](https://doi.org/10.1038/211468a0)
- Rees, M. J. (1984). Black hole models for active galactic nuclei. *Annual Review of Astronomy and Astrophysics*, *22*(1), 471-506. doi: [doi:10.1146/annurev.aa.22.090184.002351](https://doi.org/10.1146/annurev.aa.22.090184.002351)
- Remillard, R. A., & McClintock, J. E. (2006, September). X-Ray Properties of Black-Hole Binaries. *ARA&A*, *44*(1), 49-92. doi: [doi:10.1146/annurev.astro.44.051905.092532](https://doi.org/10.1146/annurev.astro.44.051905.092532)
- Richards, J. L., Max-Moerbeck, W., Pavlidou, V., King, O. G., Pearson, T. J., Readhead, A. C. S., ... Bustos, R. (2011, June). Blazars in the Fermi Era: The OVRO 40 m Telescope Monitoring Program. *ApJS*, *194*(2), 29. doi: [doi:10.1088/0067-0049/194/2/29](https://doi.org/10.1088/0067-0049/194/2/29)
- Scargle, J. D., & Wagner, S. M. (2026, feb). Studies in astronomical time series analysis. viii. the double lomb-scargle periodogram and super resolution. *The Astrophysical Journal*, *999*(1), 84. doi: [doi:10.3847/1538-4357/ae40b9](https://doi.org/10.3847/1538-4357/ae40b9)
- Schmidt, K. B., Marshall, P. J., Rix, H.-W., Jester, S., Hennawi, J. F., & Dobler, G. (2010). Selecting Quasars by their Intrinsic Variability. *ApJ*, *714*, 1194-1208. doi: [doi:10.1088/0004-637X/714/2/1194](https://doi.org/10.1088/0004-637X/714/2/1194)
- Shakura, N. I., & Sunyaev, R. A. (1973). Black holes in binary systems. observational appearance. *Astronomy and Astrophysics*, *24*, 337-355.
- Shepherd, M. C. (1997, January). Difmap: an Interactive Program for Synthesis Imaging. In G. Hunt & H. Payne (Eds.), *Astronomical data analysis software and systems vi* (Vol. 125, p. 77). Retrieved from <https://ui.adsabs.harvard.edu/abs/1997ASPC..125...77S>
- Shepherd, M. C., Pearson, T. J., & Taylor, G. B. (1994, May). DIFMAP: an interactive program for synthesis imaging. In *Bulletin of the american astronomical society* (Vol. 26, p. 987-989). Retrieved from <https://ui.adsabs.harvard.edu/abs/1994BAAS...26..987S>
- Sikora, M., Begelman, M. C., & Rees, M. J. (1994). Comptonization of diffuse ambient radiation by a relativistic jet: The source of gamma rays from blazars? *ApJ*, *421*, 153-162. doi: [doi:10.1086/173633](https://doi.org/10.1086/173633)
- Sobacchi, E., Sormani, M. C., & Stamerra, A. (2017, February). A model for periodic blazars. *MNRAS*, *465*(1), 161-172. doi: [doi:10.1093/mnras/stw2684](https://doi.org/10.1093/mnras/stw2684)
- Sołtan, A. (1982). Masses of quasars. *Monthly Notices of the Royal Astronomical Society*, *200*(1), 115-122. doi: [doi:10.1093/mnras/200.1.115](https://doi.org/10.1093/mnras/200.1.115)

- Sullivan, A. G., Blandford, R. D., Synani, A., & de la Parra, P. V. (2026). Relativistic Jets and Winds in Radio-identified Supermassive Black Hole Binary Candidates. *The Astrophysical Journal*, 997(1), 85. doi: [doi:10.3847/1538-4357/ae2613](https://doi.org/10.3847/1538-4357/ae2613)
- Tchekhovskoy, A., Narayan, R., & McKinney, J. C. (2011, November). Efficient generation of jets from magnetically arrested accretion on a rapidly spinning black hole. *MNRAS*, 418(1), L79-L83. doi: [doi:10.1111/j.1745-3933.2011.01147.x](https://doi.org/10.1111/j.1745-3933.2011.01147.x)
- Timmer, J., & Koenig, M. (1995, August). On generating power law noise. *A&A*, 300, 707. Retrieved from <https://ui.adsabs.harvard.edu/abs/1995A&A...300..707T>
- Ulrich, M.-H., Maraschi, L., & Urry, C. M. (1997). Variability of active galactic nuclei. *Annual Review of Astronomy and Astrophysics*, 35(1), 445–502. doi: [doi:10.1146/annurev.astro.35.1.445](https://doi.org/10.1146/annurev.astro.35.1.445)
- Urry, C. M., & Padovani, P. (1995, September). Unified Schemes for Radio-Loud Active Galactic Nuclei. *PASP*, 107, 803. doi: [doi:10.1086/133630](https://doi.org/10.1086/133630)
- Uttley, P., McHardy, I. M., & Papadakis, I. E. (2002, May). Measuring the broadband power spectra of active galactic nuclei with RXTE. *MNRAS*, 332(1), 231–250. doi: [doi:10.1046/j.1365-8711.2002.05298.x](https://doi.org/10.1046/j.1365-8711.2002.05298.x)
- Vaughan, S. (2005). A simple test for periodic signals in red noise. *Astronomy & Astrophysics*, 431(1), 391–403. doi: [doi:10.1051/0004-6361:20041453](https://doi.org/10.1051/0004-6361:20041453)
- Wilhelm, A. (2021). *Stochastic re-acceleration of particles in supernova remnants* (Doctoral dissertation, Universität Potsdam). doi: [doi:10.25932/publishup-51291](https://doi.org/10.25932/publishup-51291)
- Wilkins, D. R., & Fabian, A. C. (2012). Understanding X-ray reflection and the corona in the Seyfert 1 galaxy 1H 0707-495. *MNRAS*, 424, 1284–1296. doi: [doi:10.1111/j.1365-2966.2012.21308.x](https://doi.org/10.1111/j.1365-2966.2012.21308.x)
- Wilkinson, P. N., Readhead, A. C. S., Anderson, B., & Purcell, G. H. (1979, September). VLBI observations of compact radio sources at 609 megahertz. *ApJ*, 232, 365–381. doi: [doi:10.1086/157296](https://doi.org/10.1086/157296)

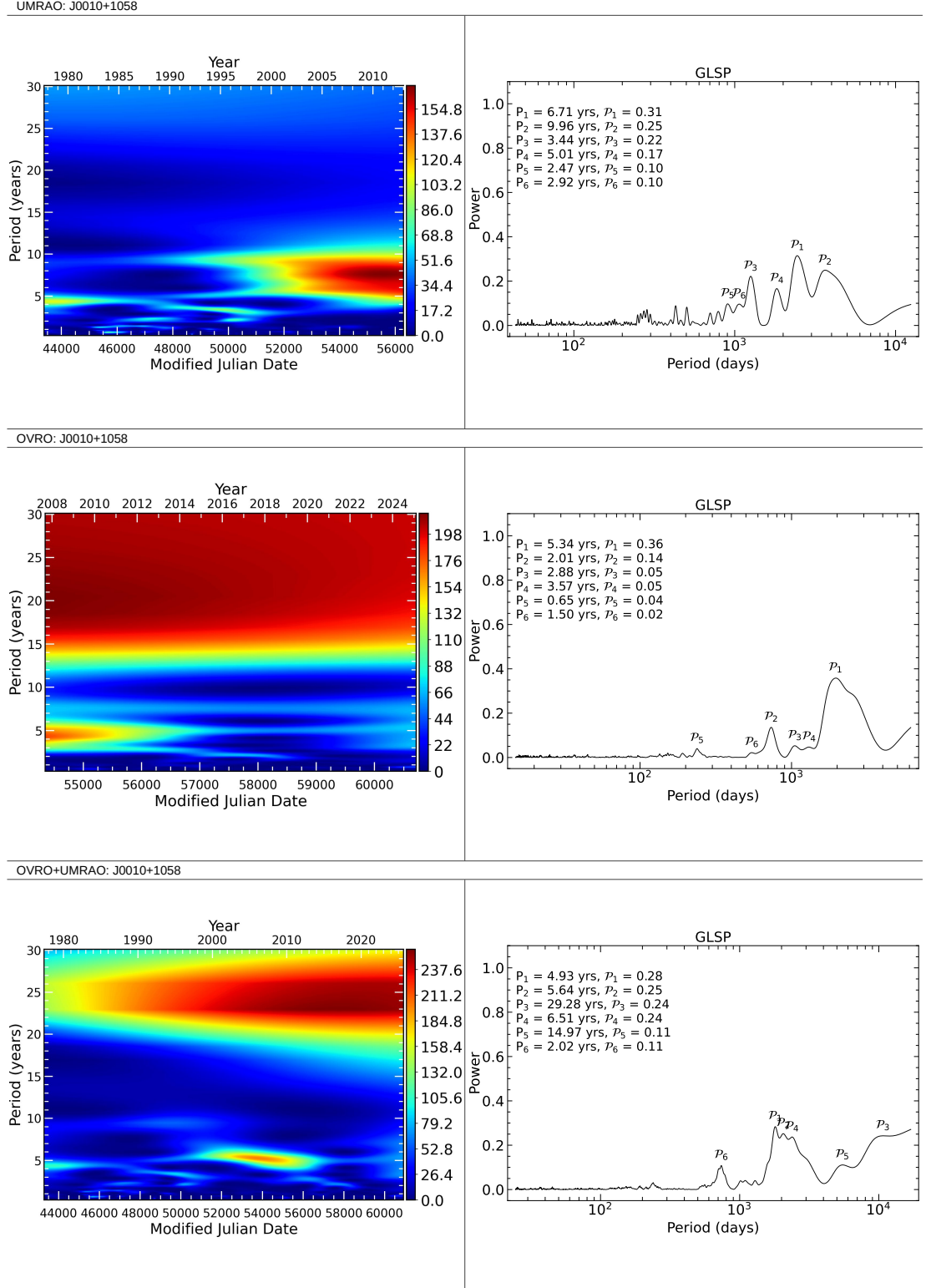
Appendix A

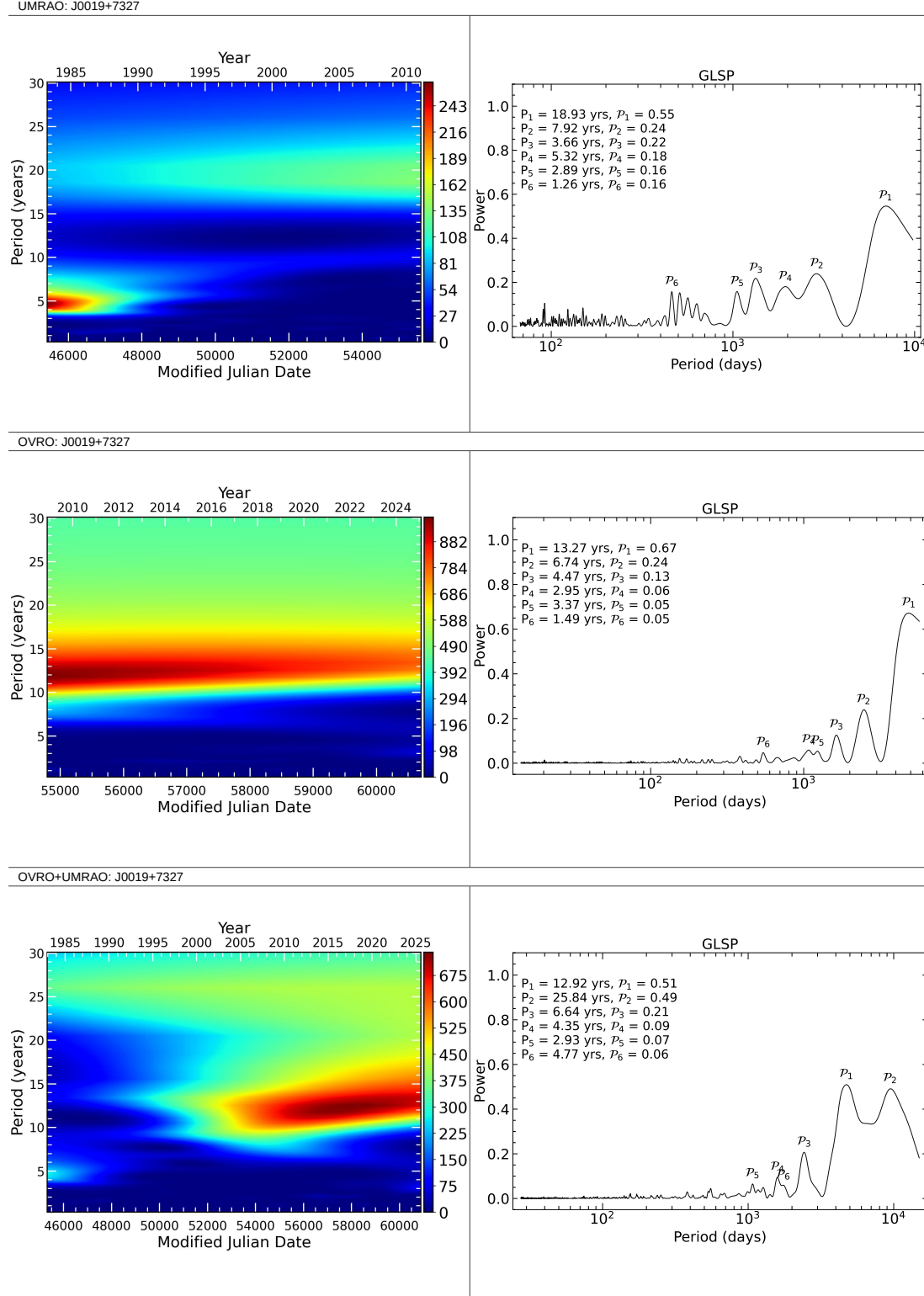
GLSP and WWZ analysis for the OVRO+UMRAO sample

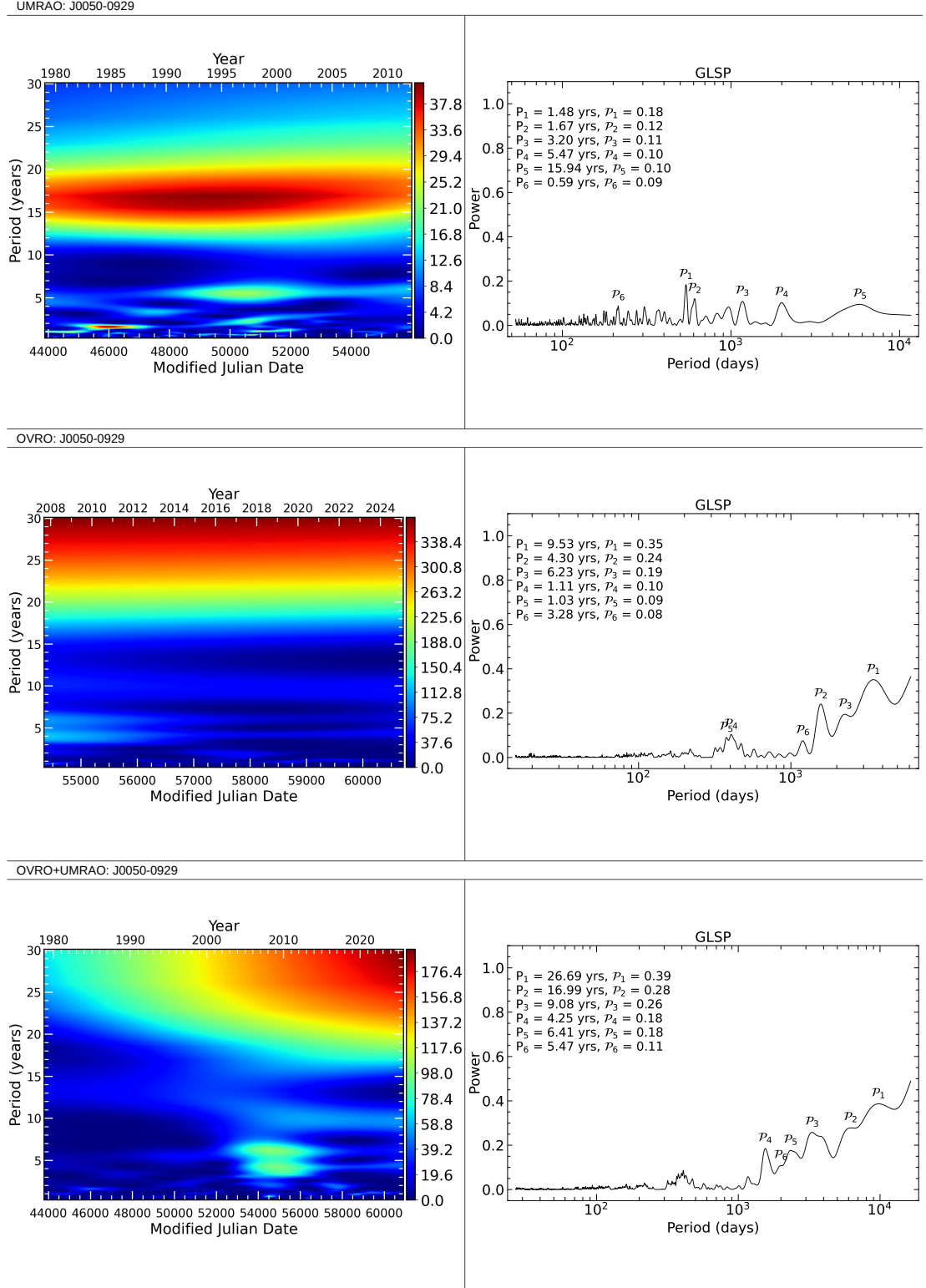
Below are the figures showing the GLSP and WWZ analyses of the 83 sources, which are considered preliminary. This stage enables an inspection to assess periodicity consistency across the UMRAO (26 m), OVRO (40 m), and combined datasets, for each of the 83 sources shown here. It is worth noting that periodicity studies often require treatment of outliers and, in some cases, the correction of trends. In this appendix, however, the analyses are performed on the calibrated data, without additional modifications.

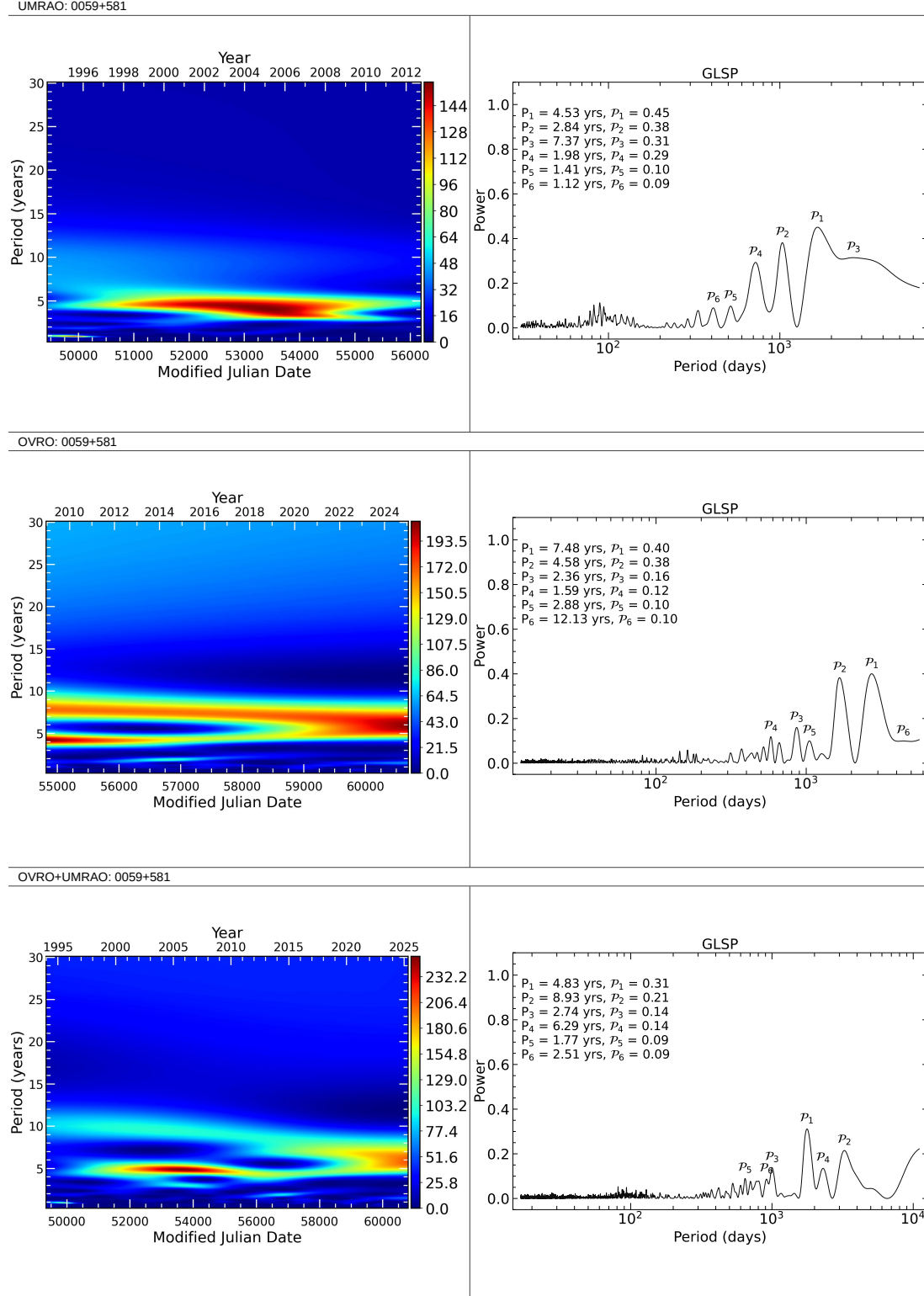
Each page is organized into three sections (from top to bottom): UMRAO, OVRO, and OVRO+UMRAO. Within each section, the left-hand panel corresponds to the WWZ analysis, where the horizontal axis represents time in Modified Julian Date (MJD) and the vertical axis shows the period in years.

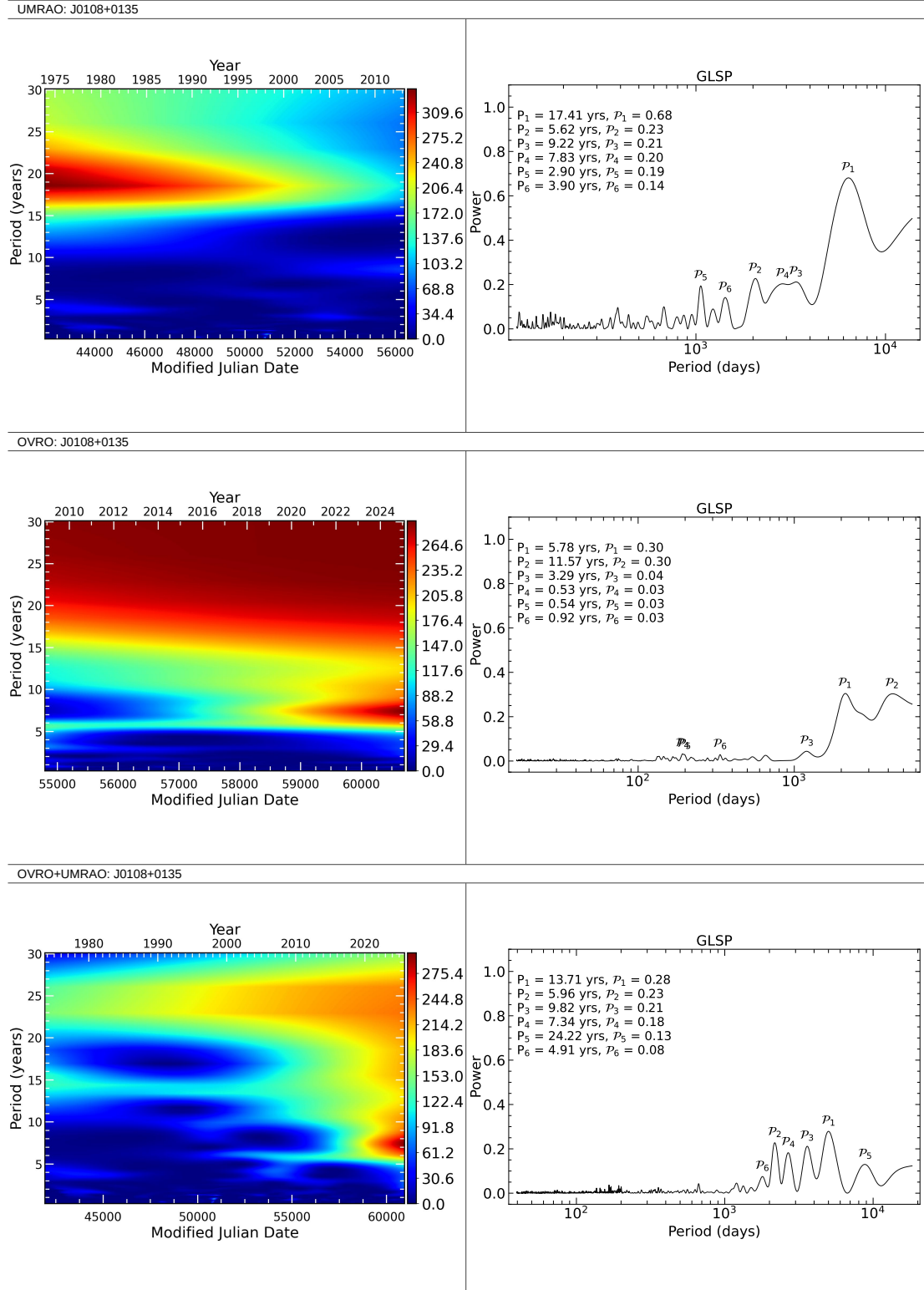
On the right, the GLSP spectra display the six peaks, defined as those with the highest power for periods $P \geq 100$ days. The vertical axis corresponds to the normalized power (0 to 1), while the horizontal axis, shown on a logarithmic scale, represents the period in days. To facilitate readability especially in the long period regime, the text box also reports the periods in years.

Figure A.1: J0010+1058, (RA = 2.629°, DEC = 10.975°), $z = 0.089$.

Figure A.2: J0019+7327, (RA = 4.941° , DEC = 73.458°), $z = 1.781$.

Figure A.3: J0050-0929, (RA = 12.672°, DEC = −9.485°), $z = 0.635$.

Figure A.4: J0102+5824, (RA = 15.691°, DEC = 58.403°), $z = 0.644$.

Figure A.5: J0108+0135, (RA = 17.162°, DEC = 1.583°), $z = 2.109$.

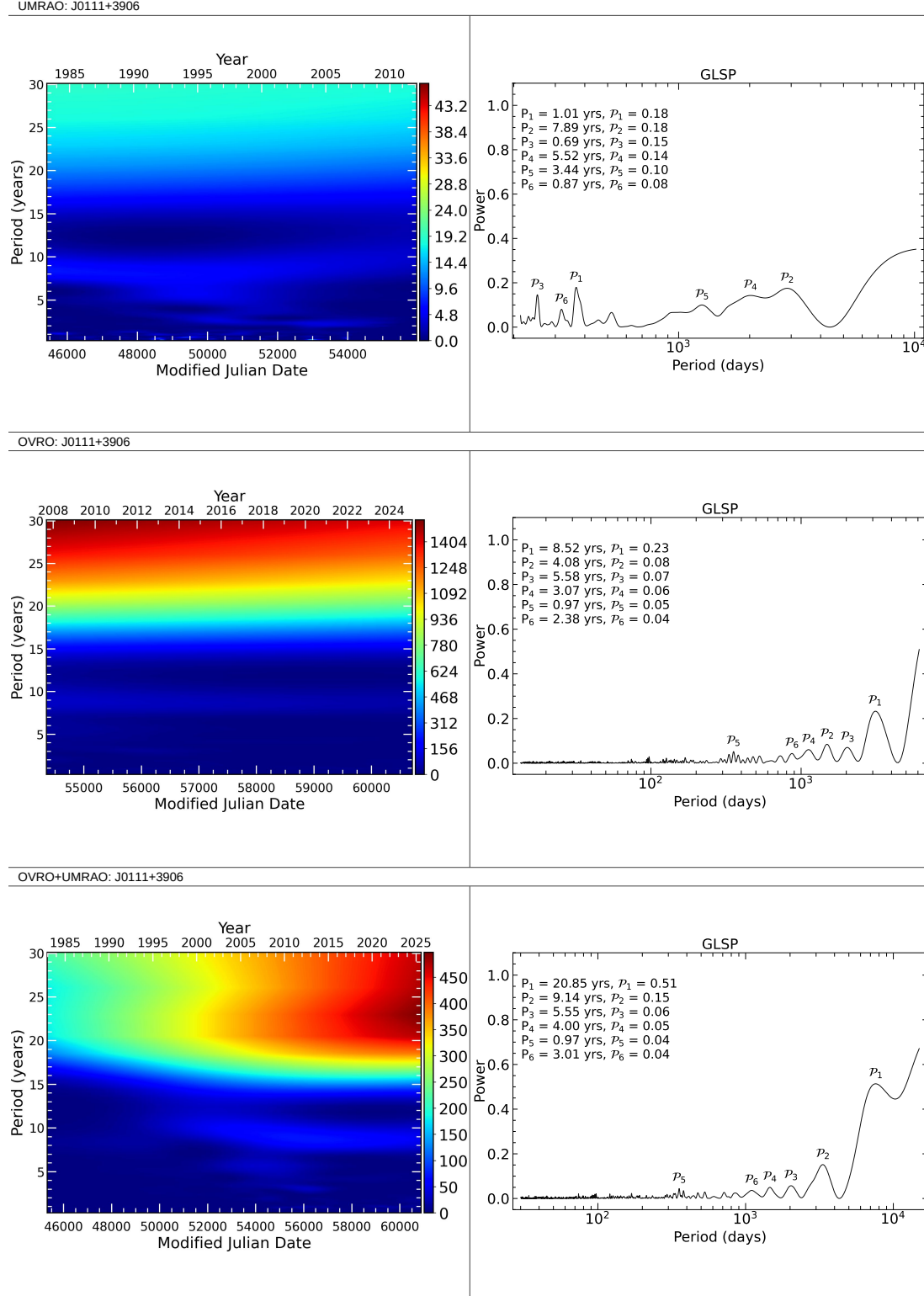
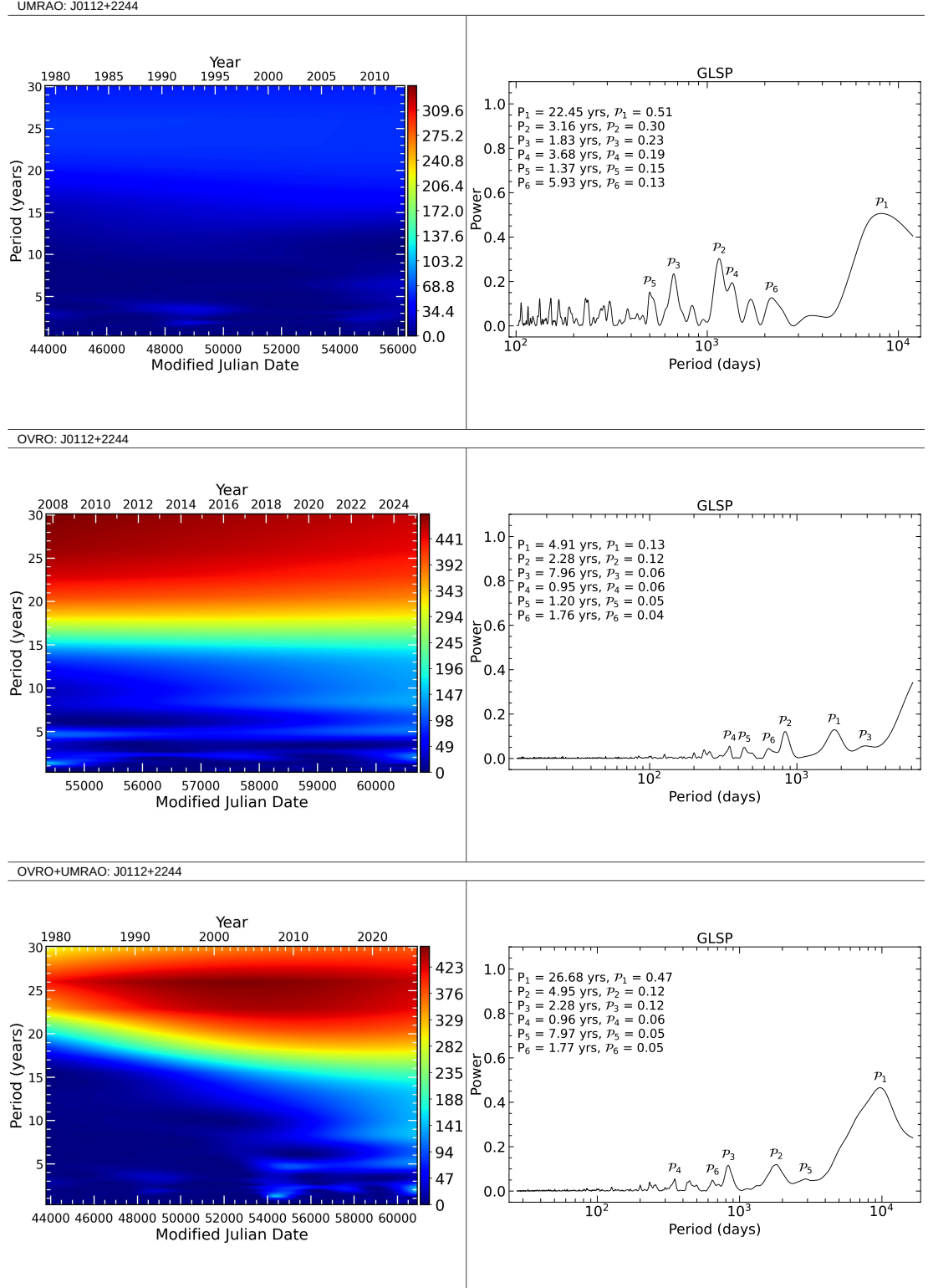


Figure A.6: J0111+3906, (RA = 17.905°, DEC = 39.108°), $z = 0.668$.

Figure A.7: J0112+2244, (RA = 18.024°, DEC = 22.744°), $z = 0.265$.

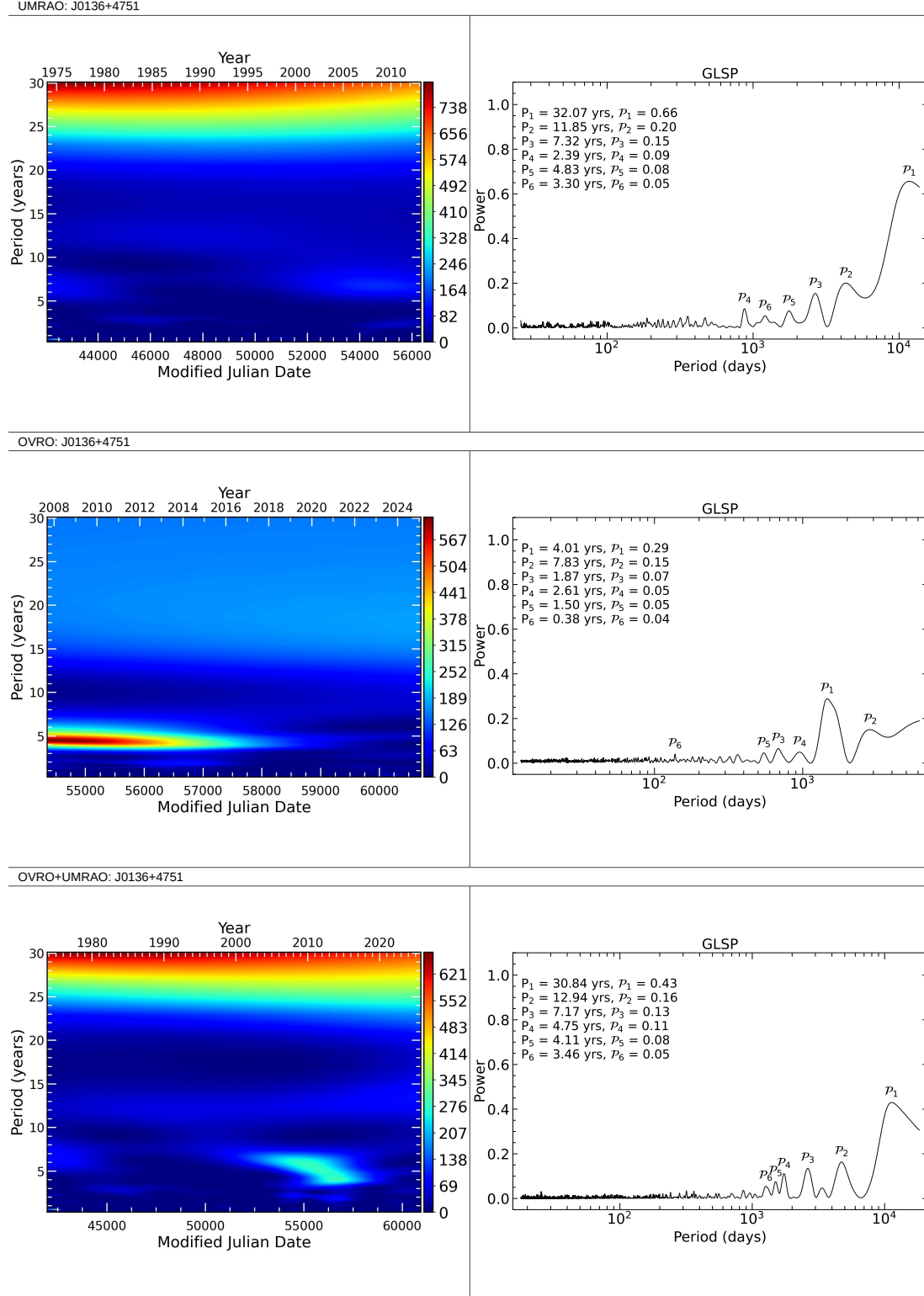
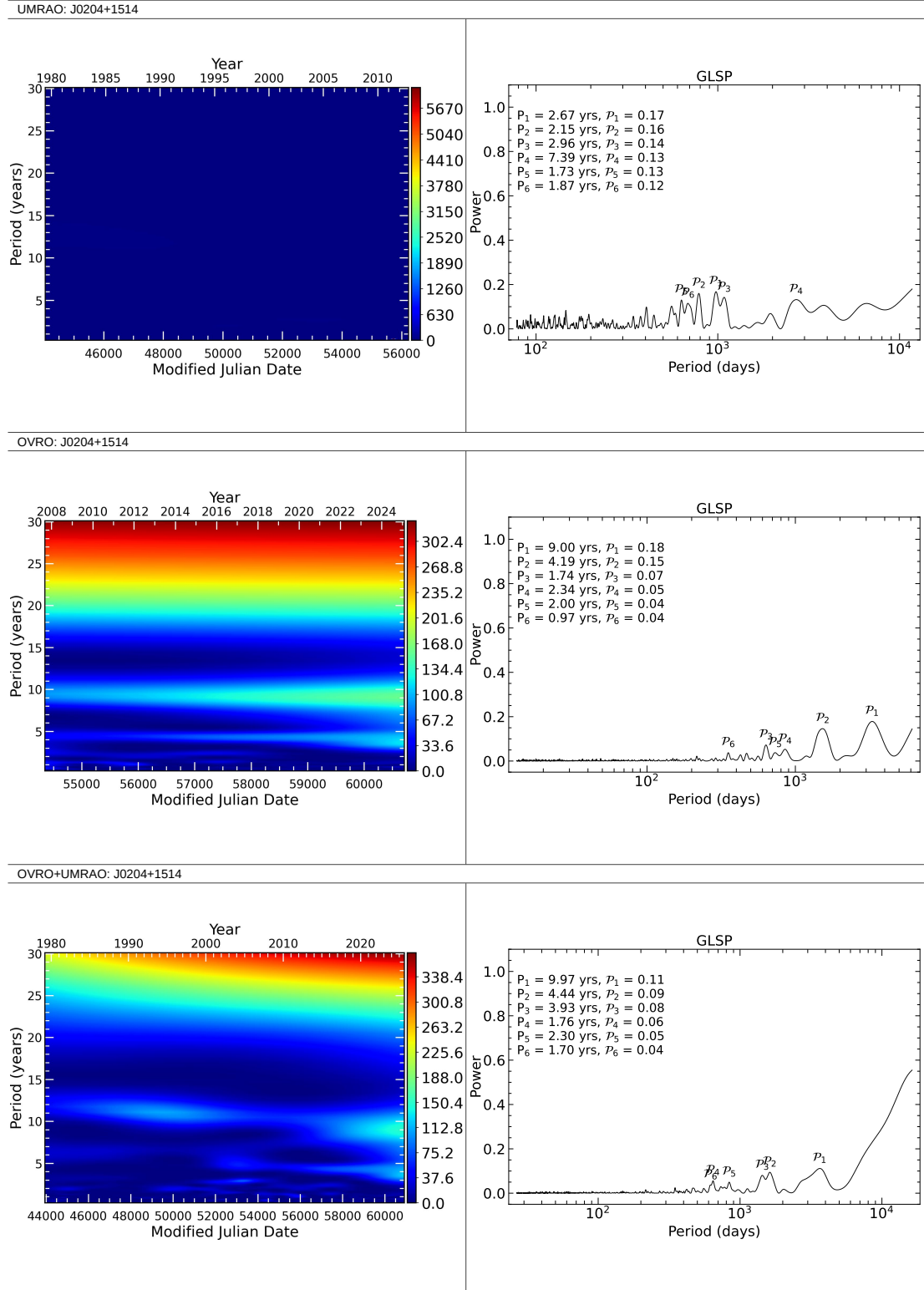
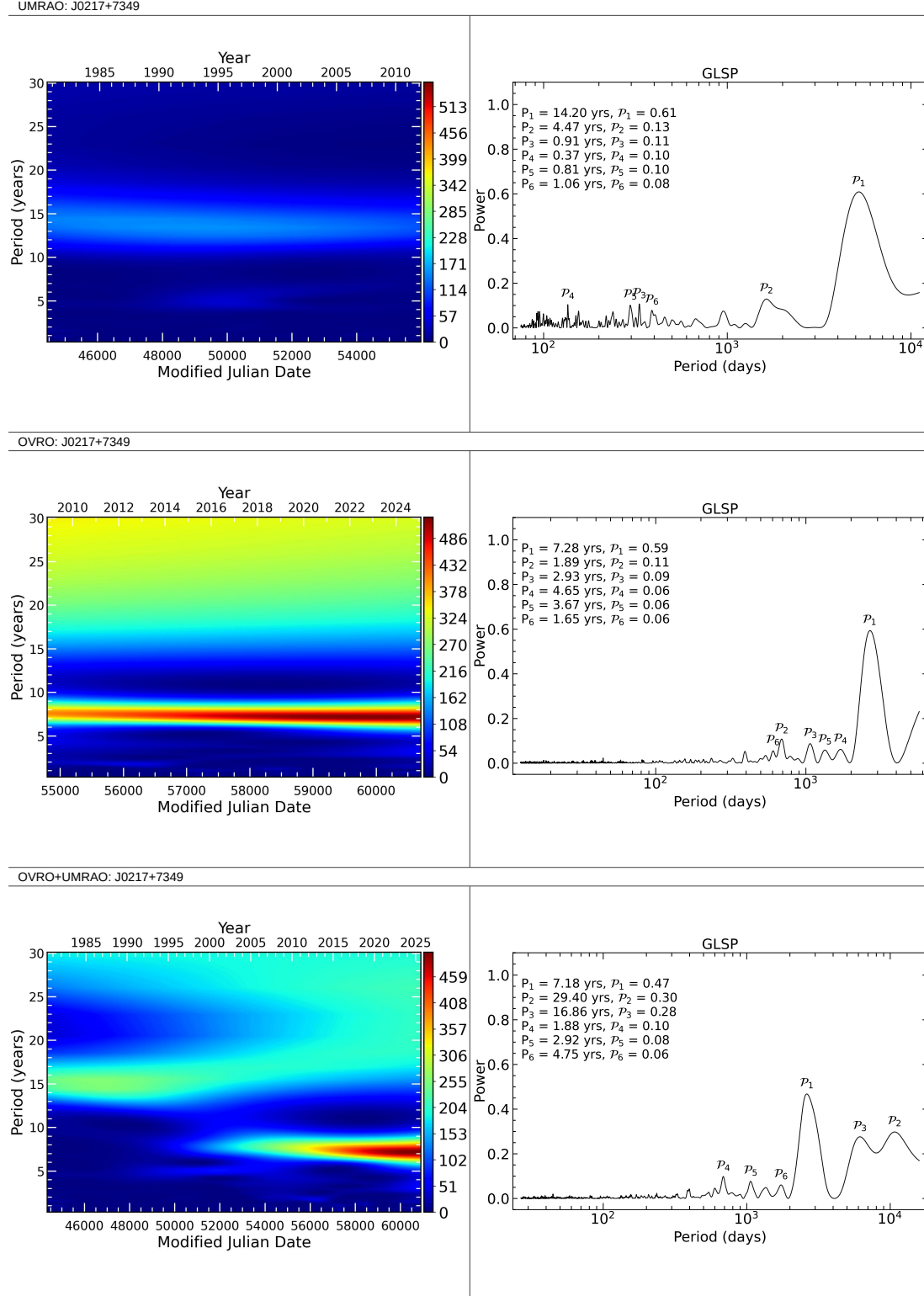
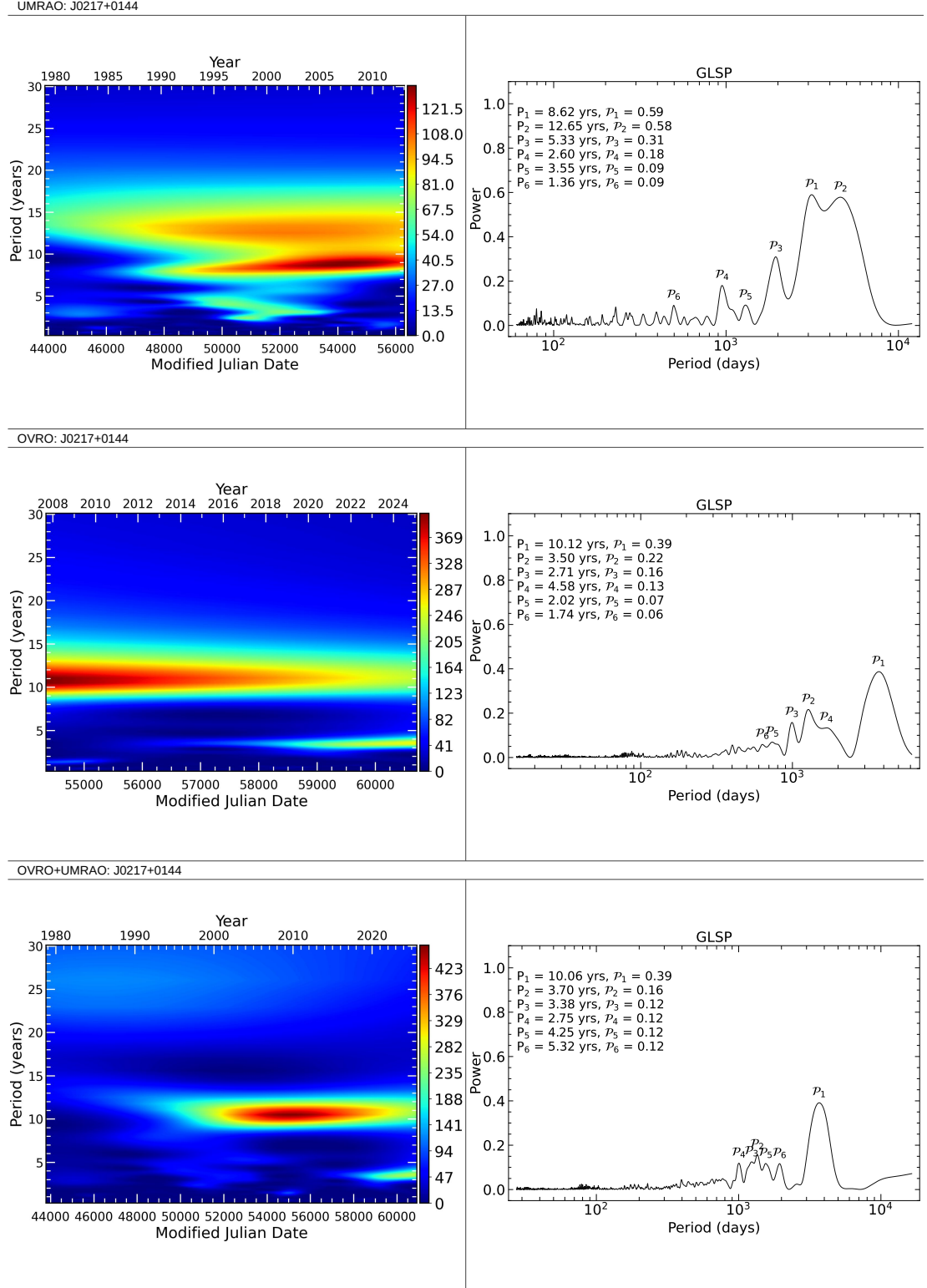
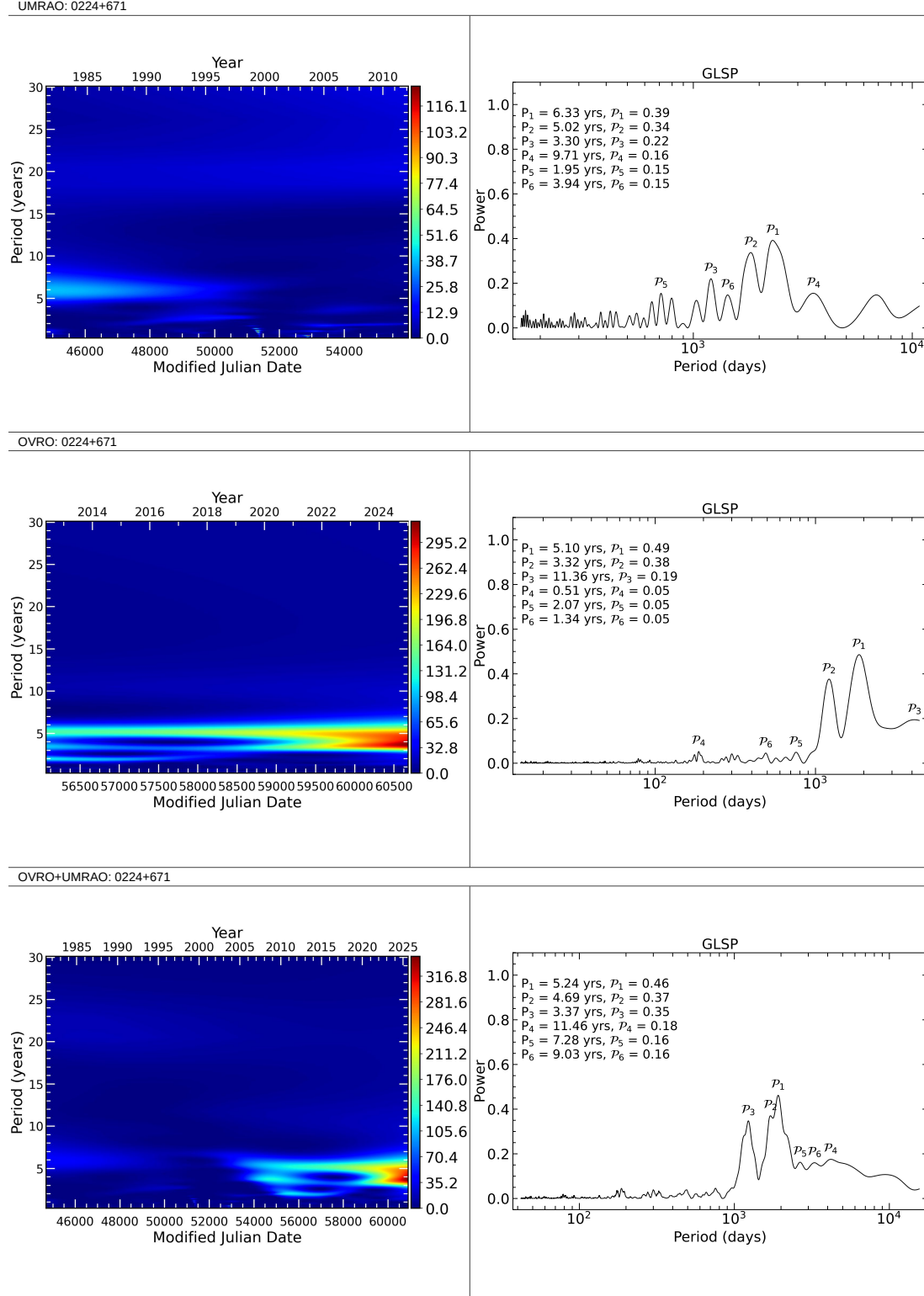


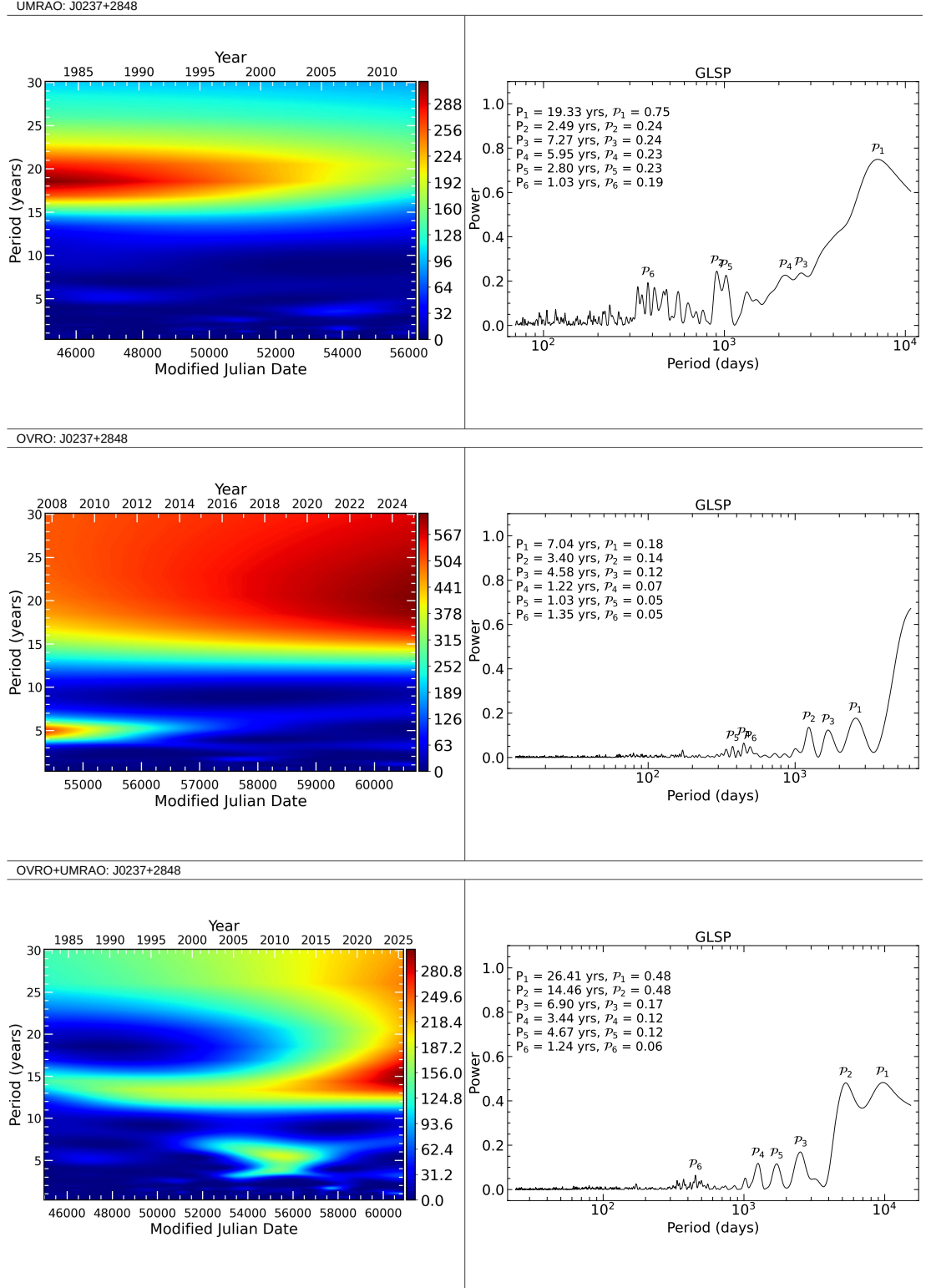
Figure A.8: J0136+4751, (RA = 24.244°, DEC = 47.858°), $z = 0.859$.

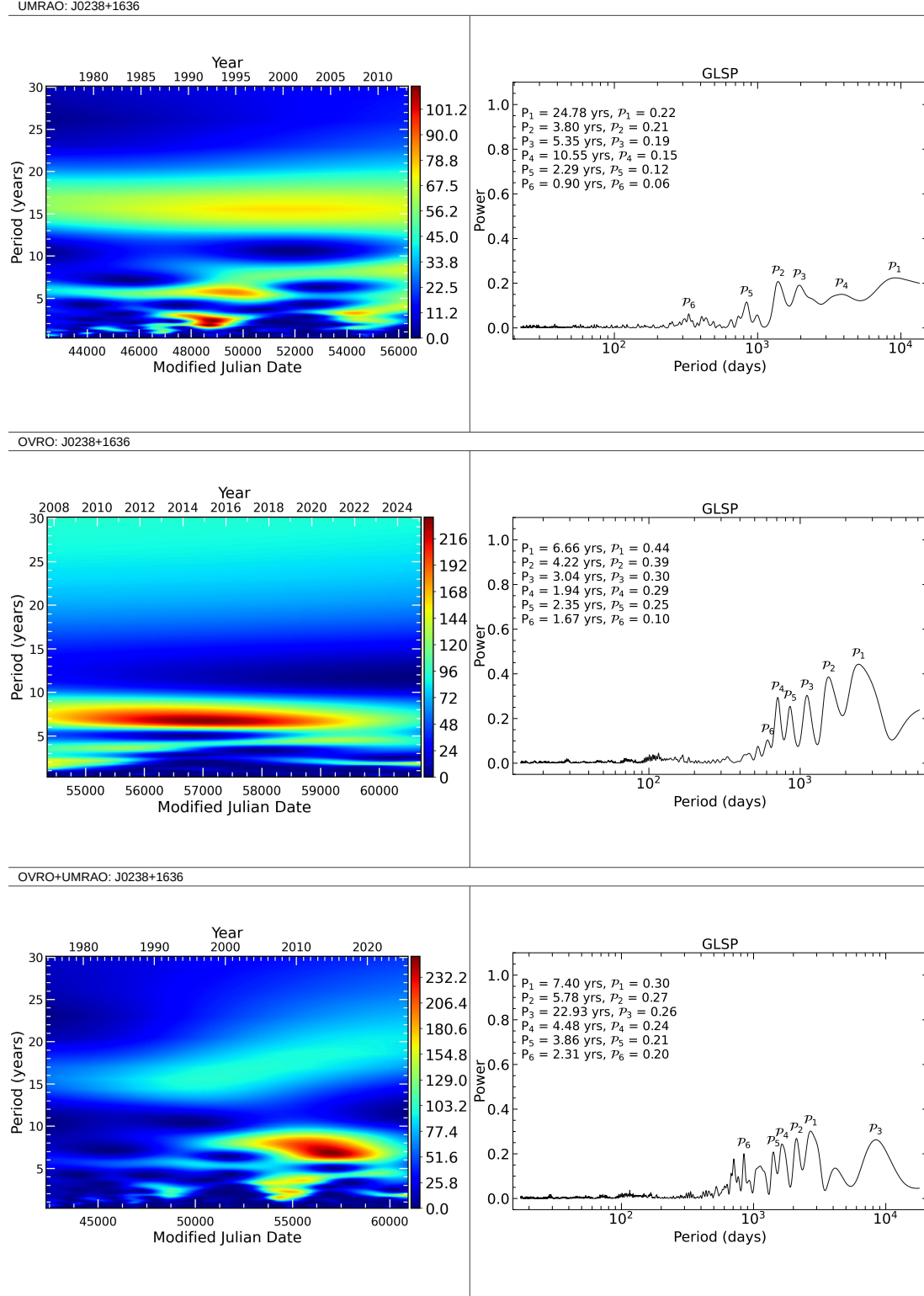
Figure A.9: J0204+1514, (RA = 31.210°, DEC = 15.236°), $z = 0.405$.

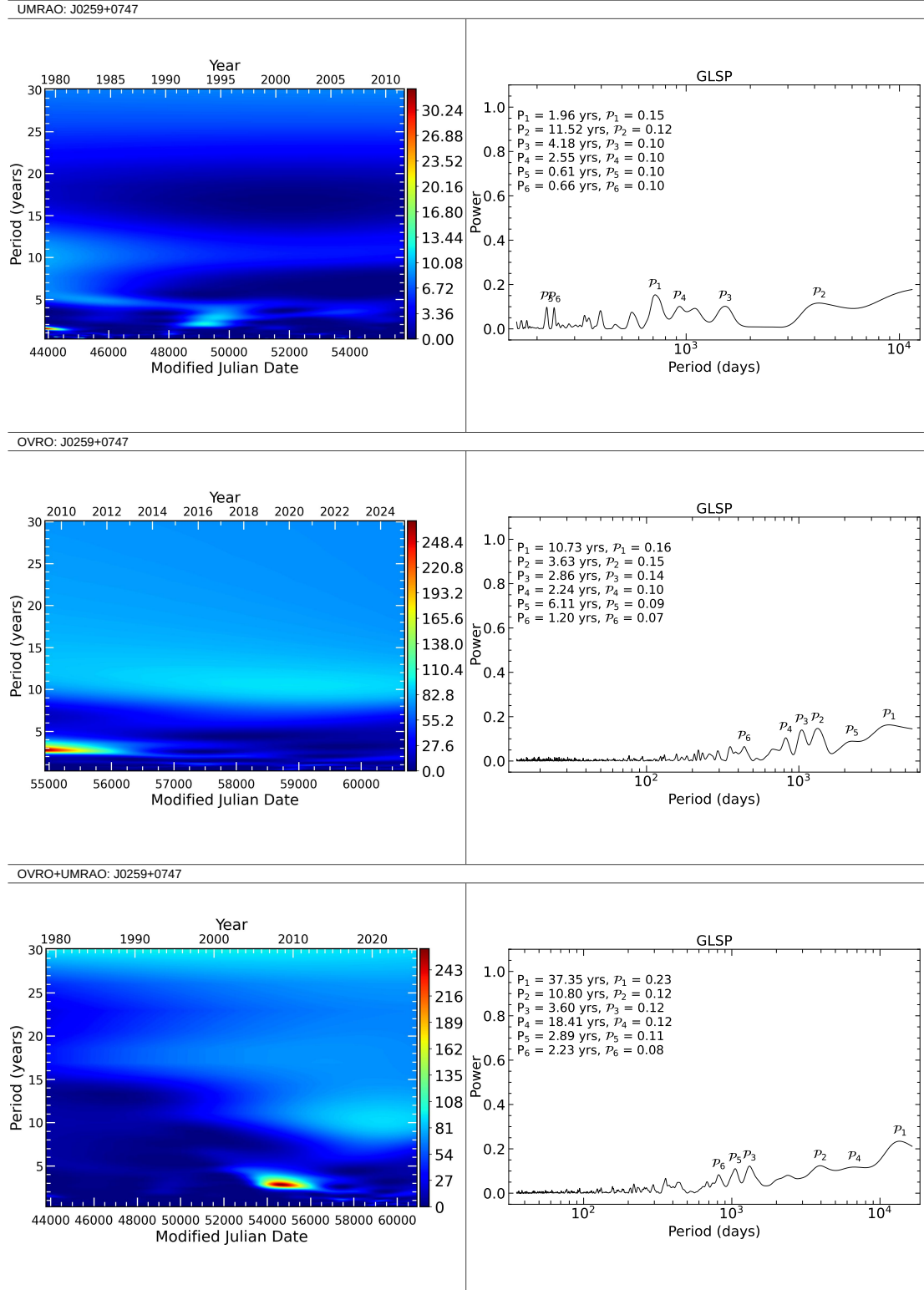
Figure A.10: J0217+7349, (RA = 34.378°, DEC = 73.826°), $z = 2.346$.

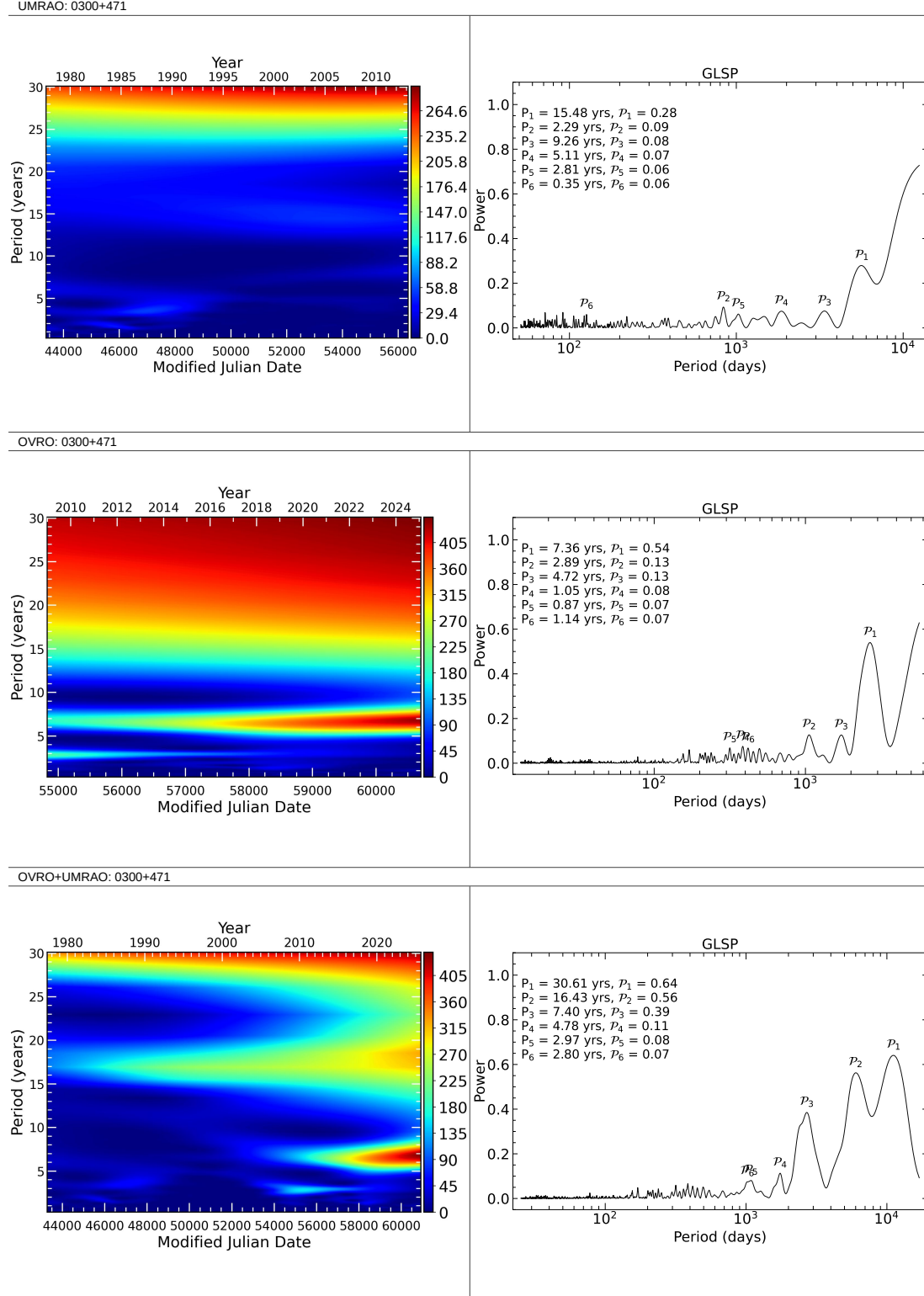
Figure A.11: J0217+0144, (RA = 34.454°, DEC = 1.747°), $z = 1.715$.

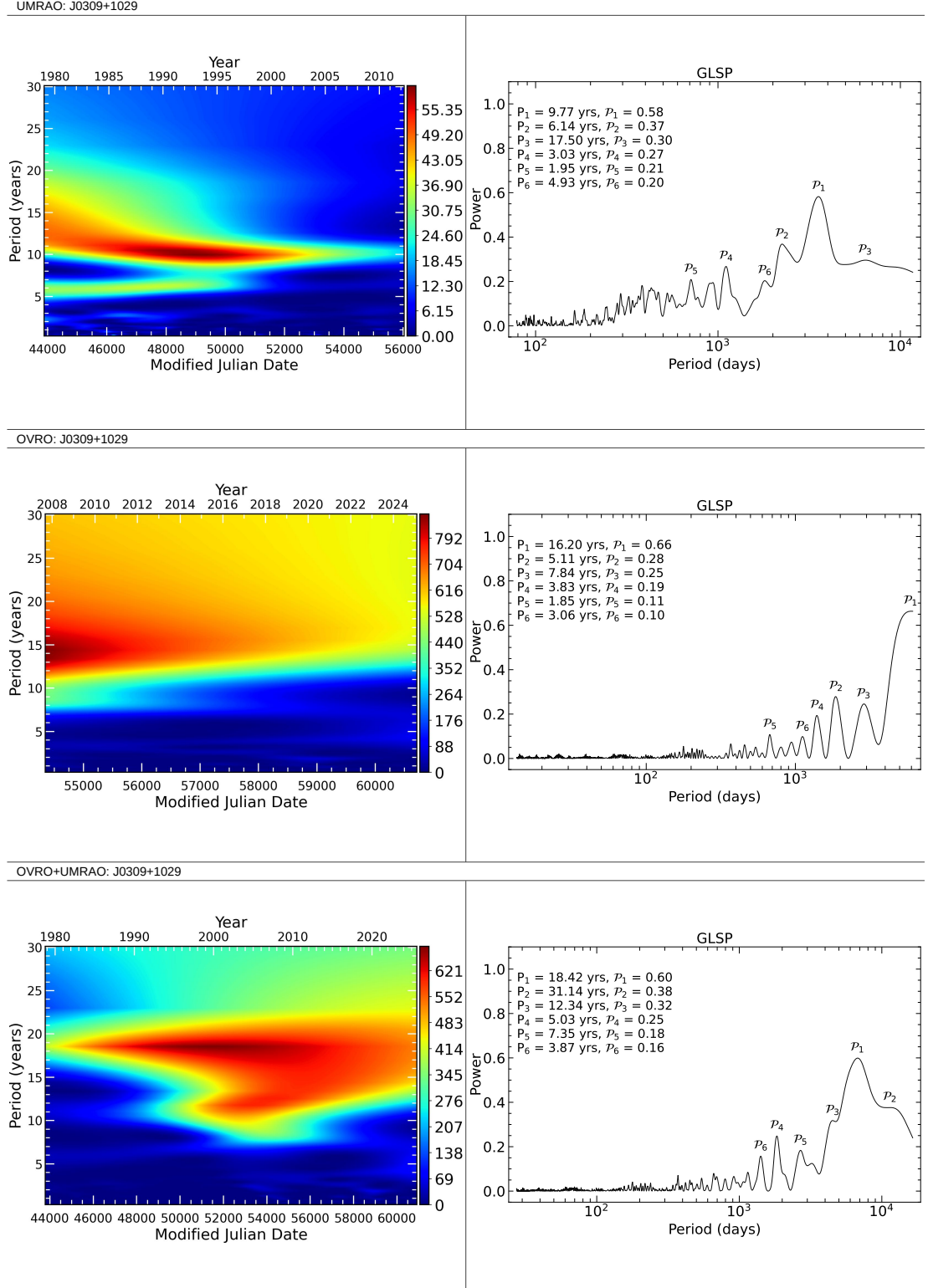
Figure A.12: J0228+6721, (RA = 37.209°, DEC = 67.351°), $z = 0.523$.

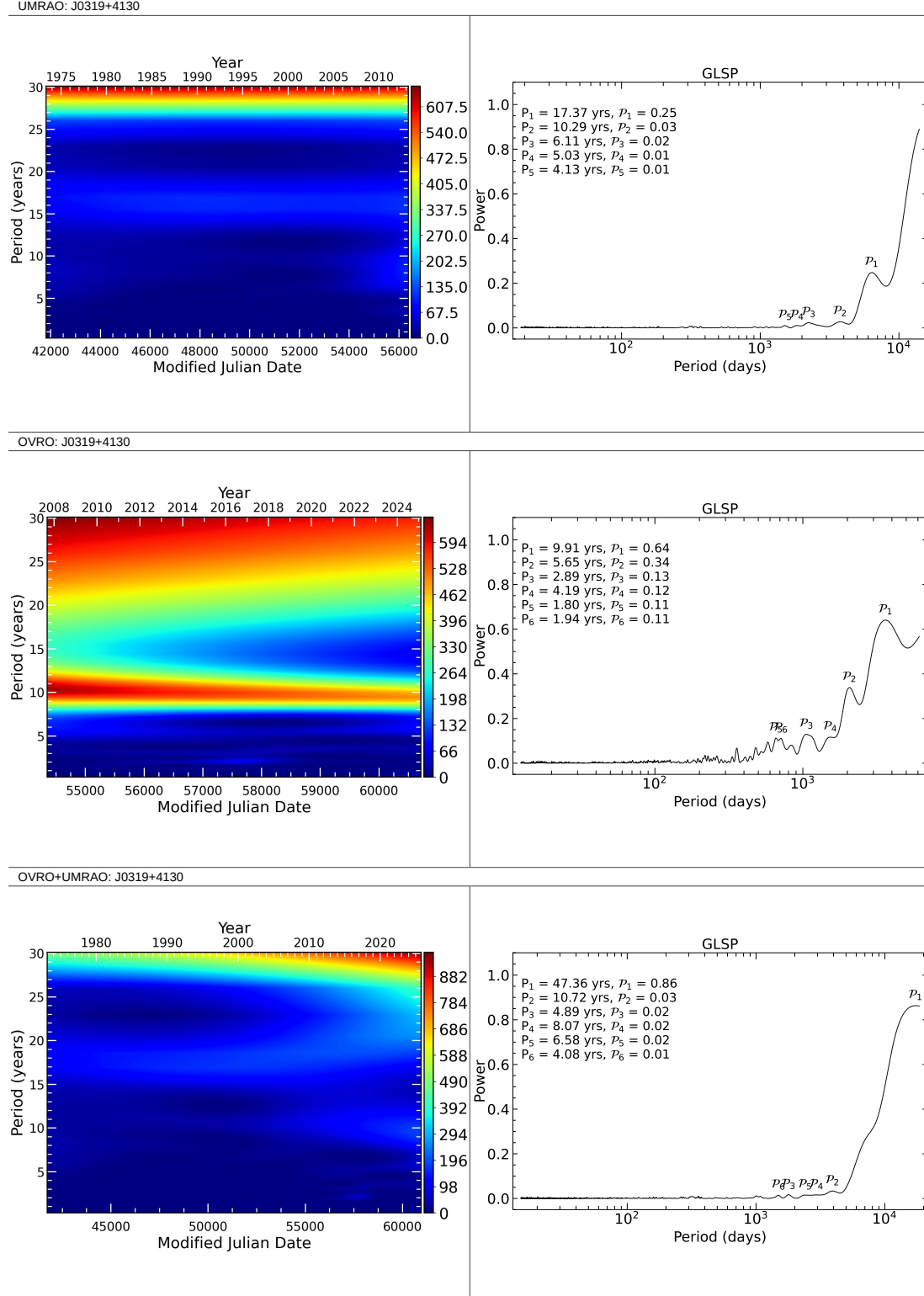
Figure A.13: J0237+2848, (RA = 39.468°, DEC = 28.802°), $z = 1.206$.

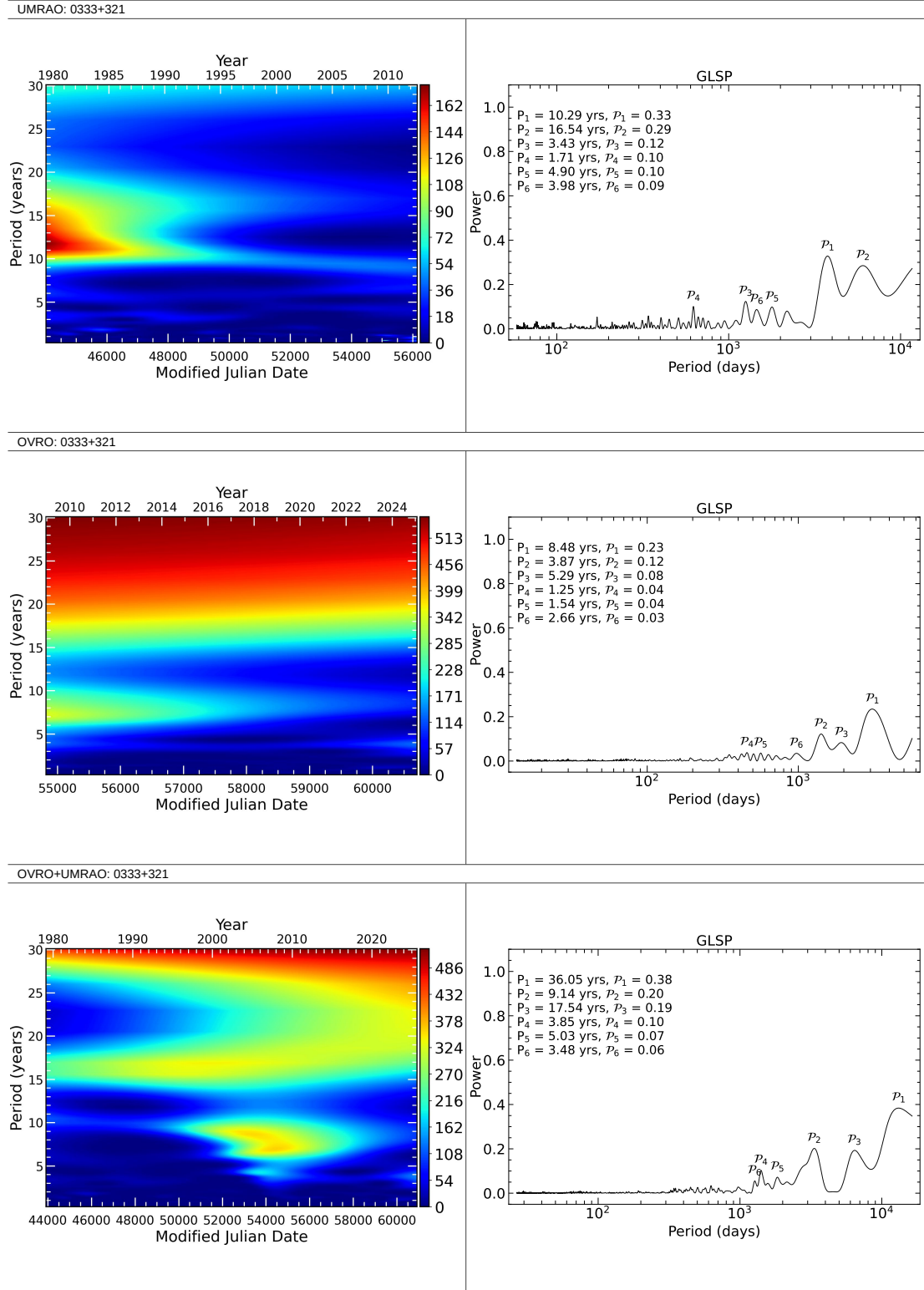
Figure A.14: J0238+1636, (RA = 39.662°, DEC = 16.616°), $z = 0.940$.

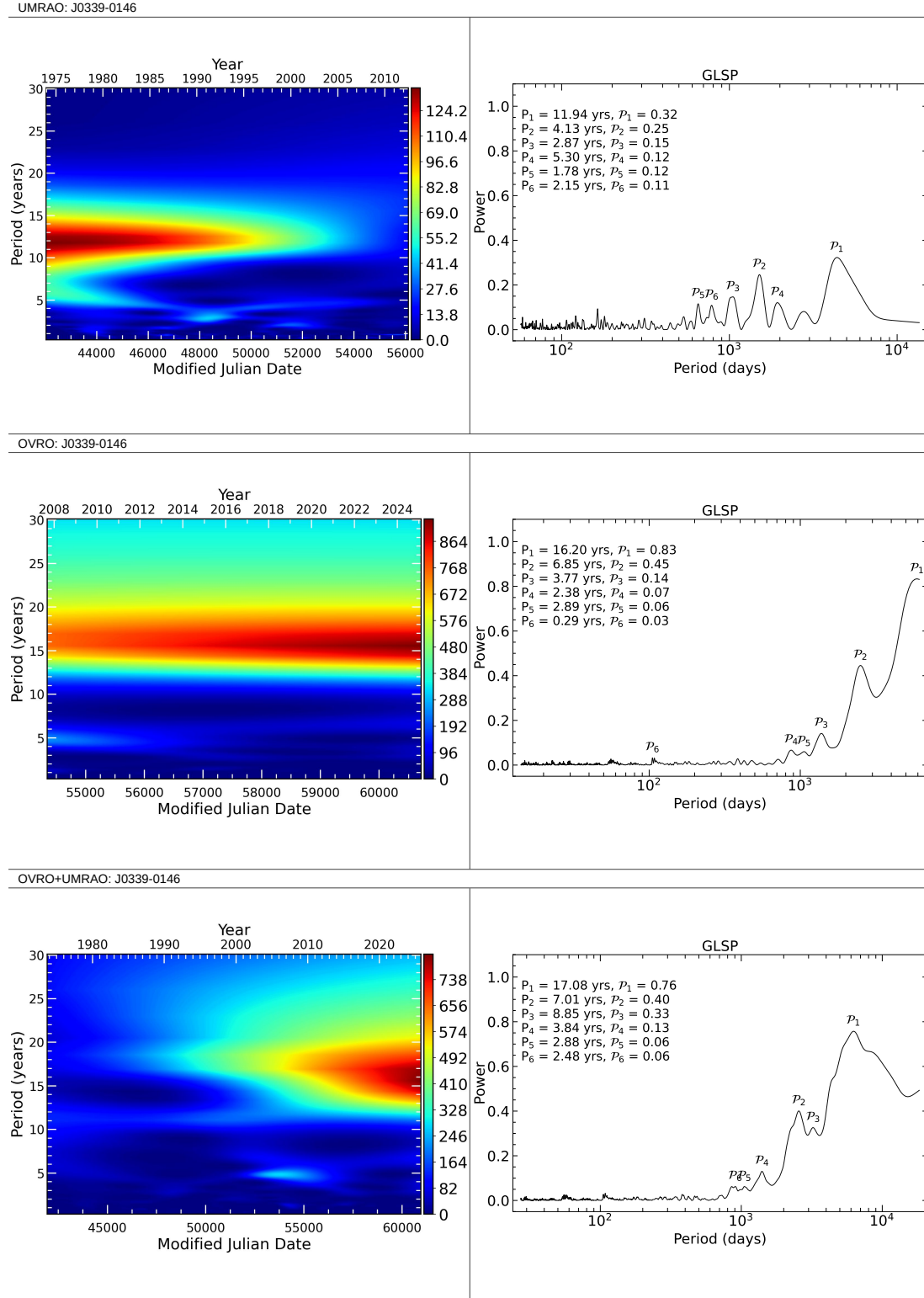
Figure A.15: J0259+0747, (RA = 44.863°, DEC = 7.794°), $z = 0.893$.

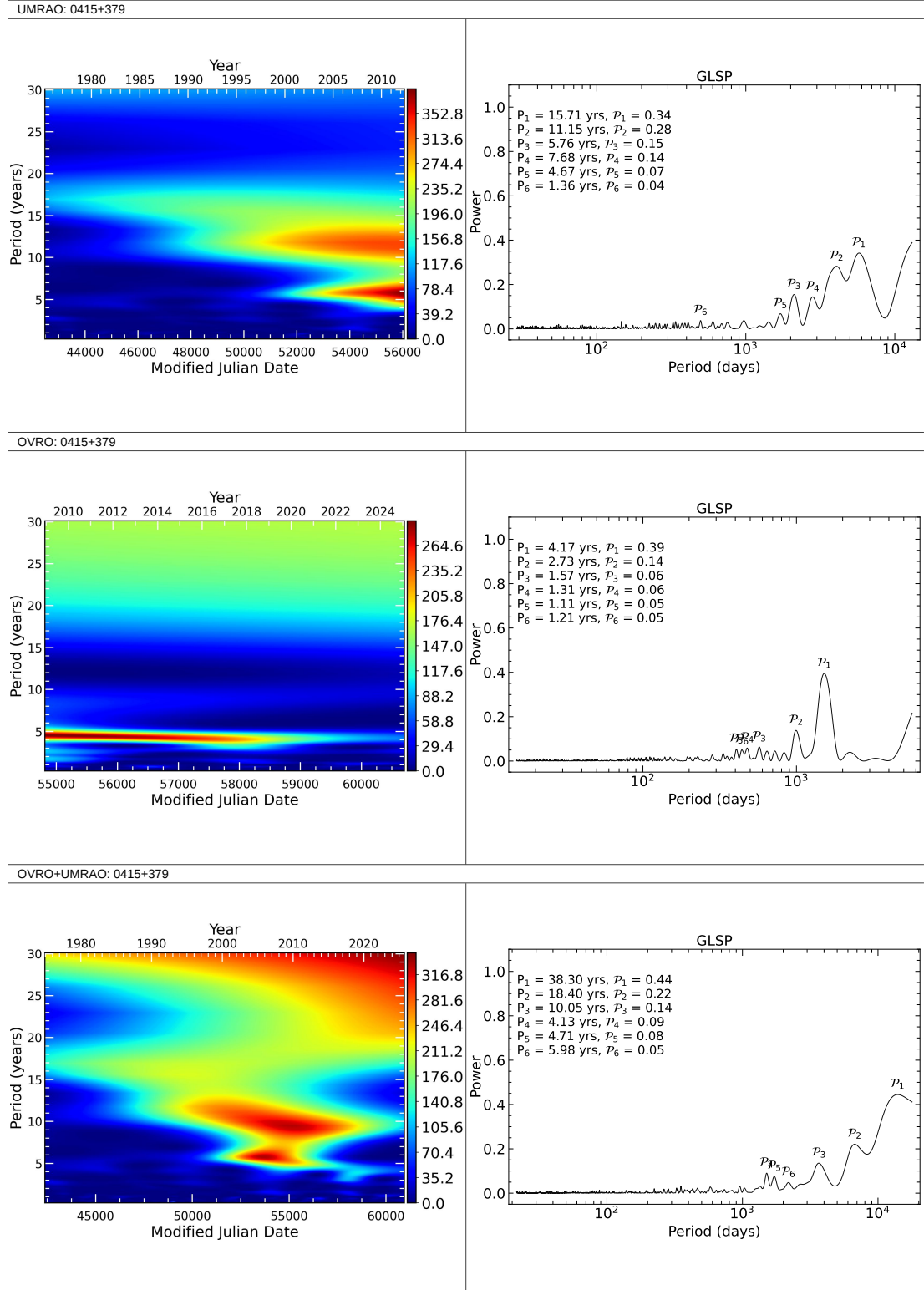
Figure A.16: J0303+4716, (RA = 45.897°, DEC = 47.271°), $z = \text{U}$.

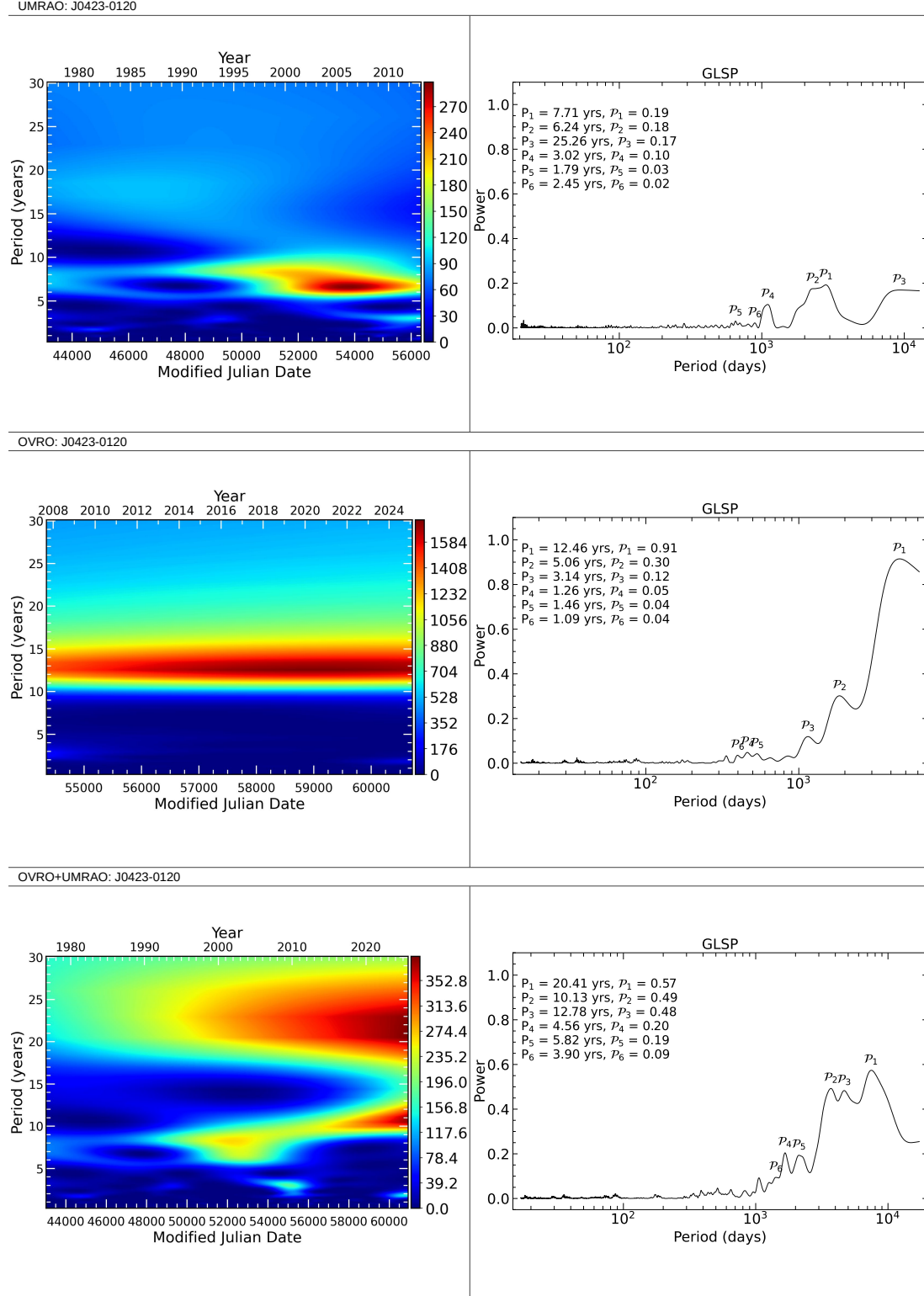
Figure A.17: J0309+1029, (RA = 47.265°, DEC = 10.488°), $z = 0.862$.

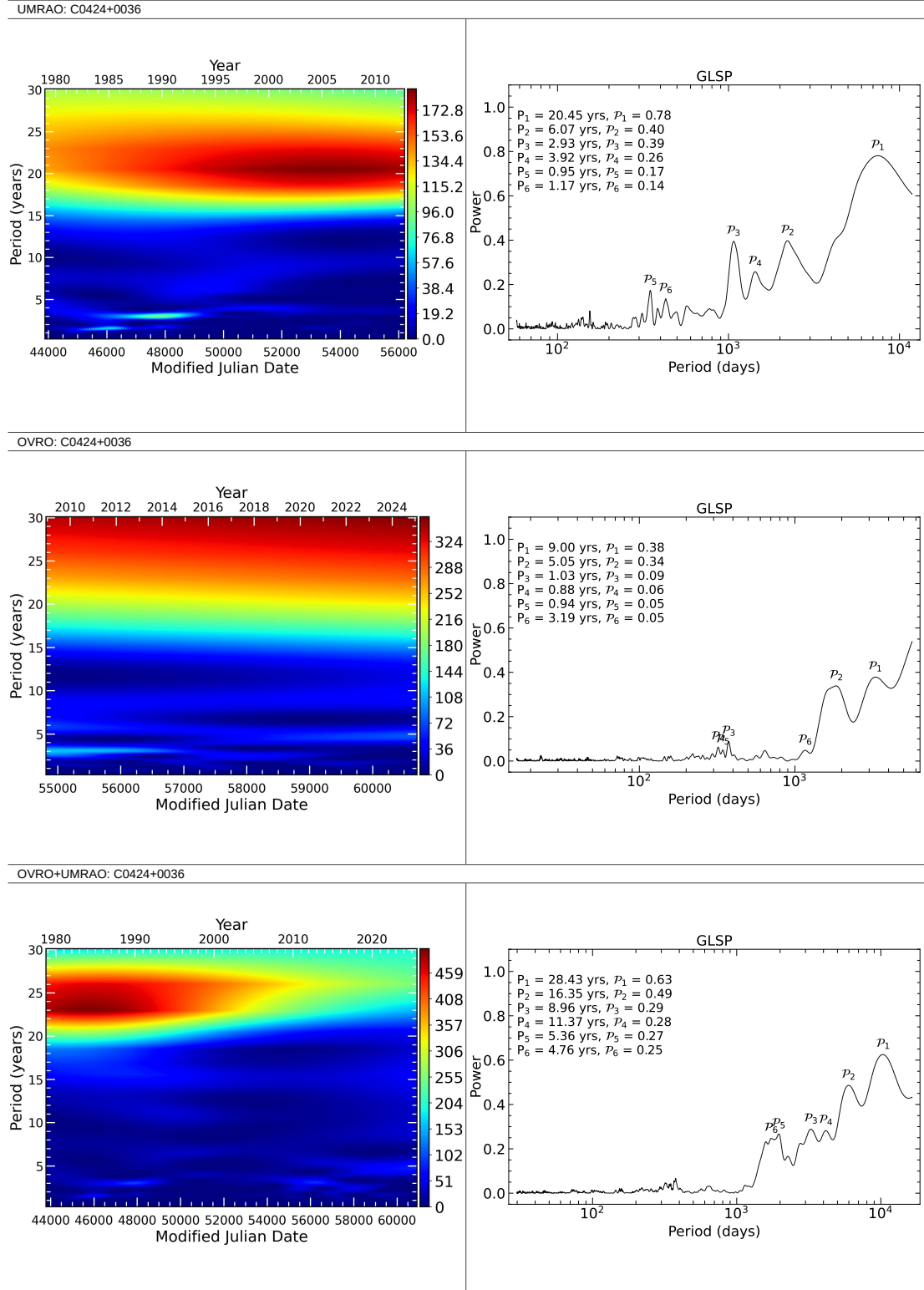
Figure A.18: J0319+4130, (RA = 49.951°, DEC = 41.512°), $z = 0.018$.

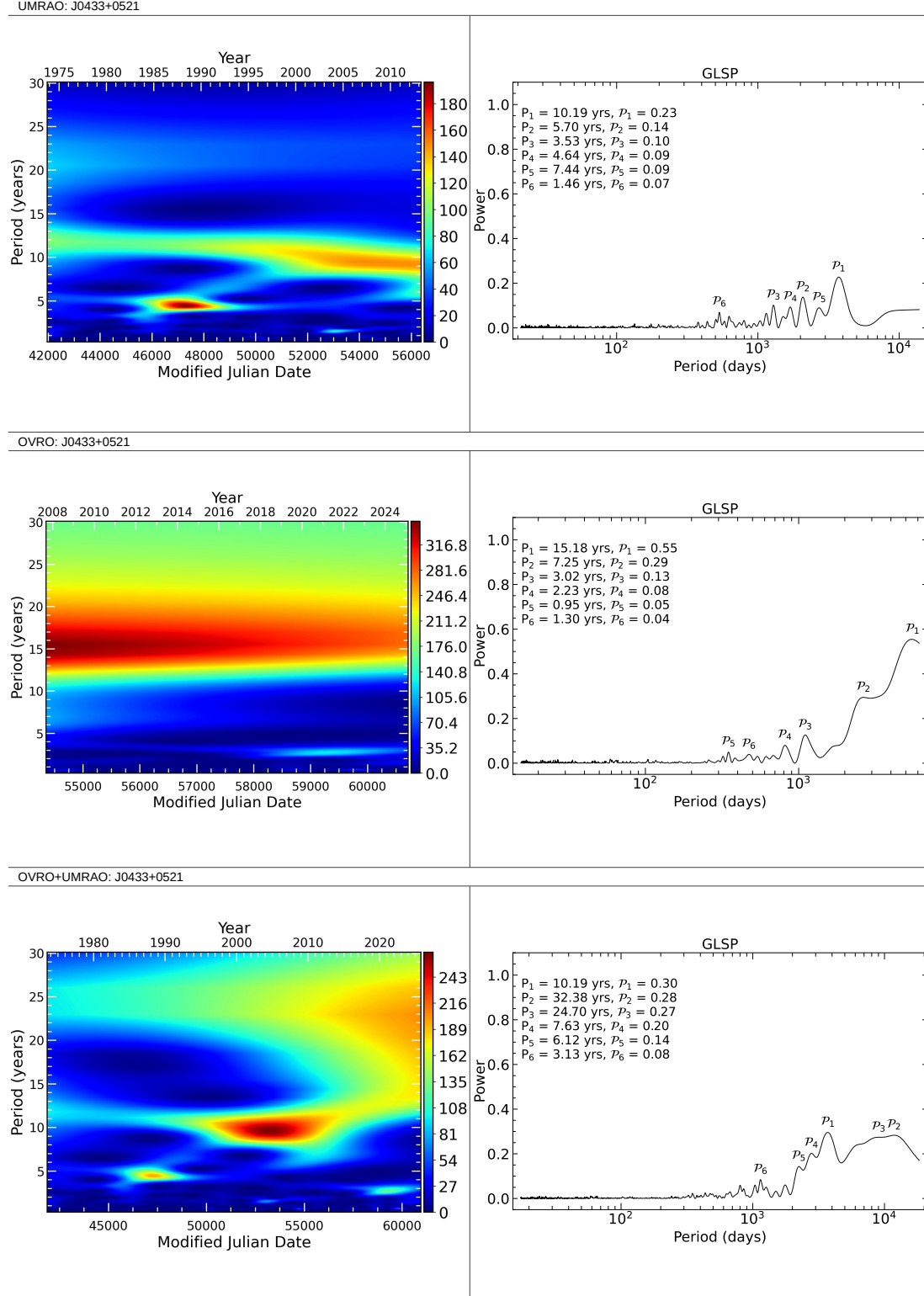
Figure A.19: J0336+3218, (RA = 54.125°, DEC = 32.308°), $z = 1.259$.

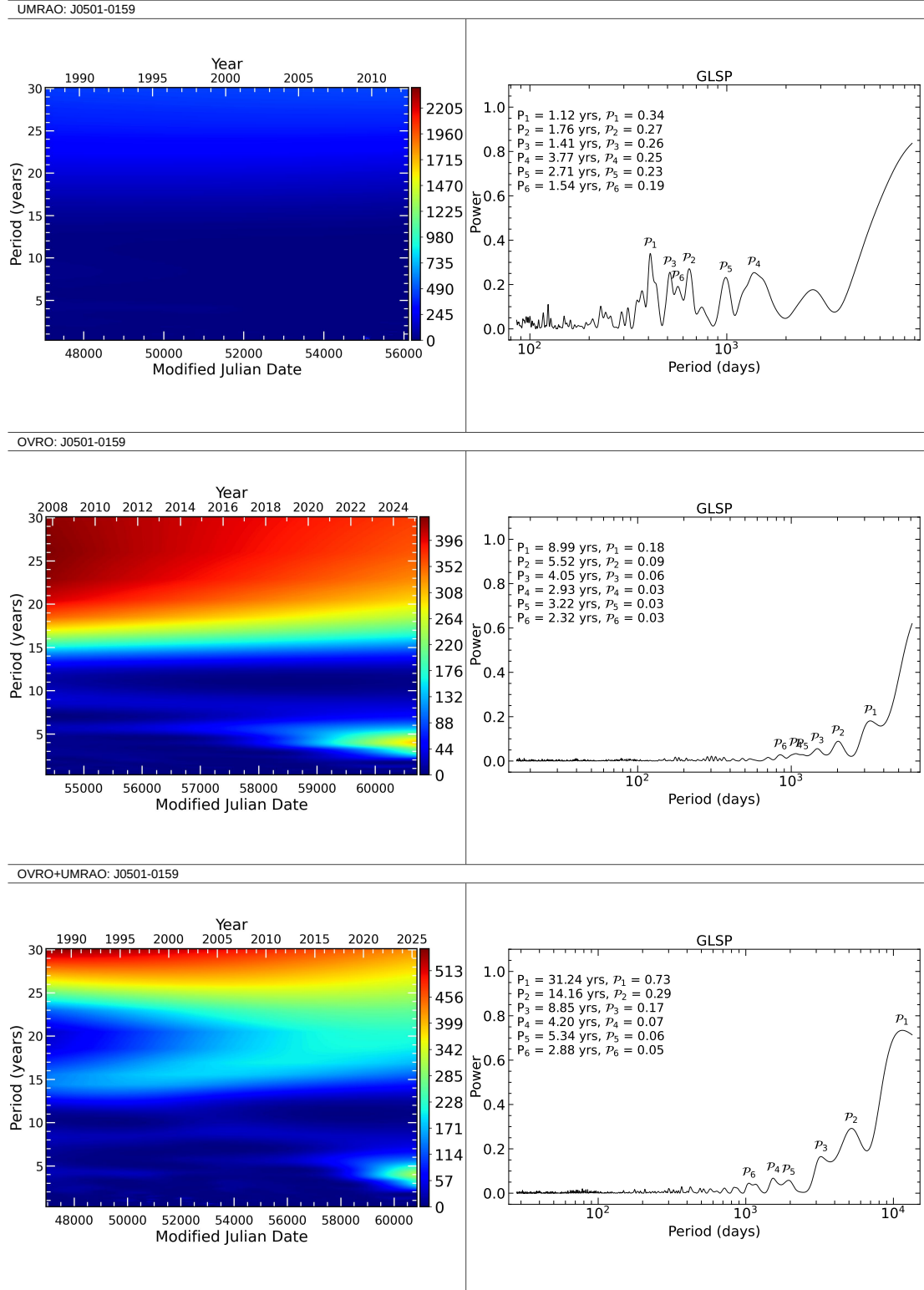
Figure A.20: J0339-0146, (RA = 54.879°, DEC = −1.776°), $z = 0.847$.

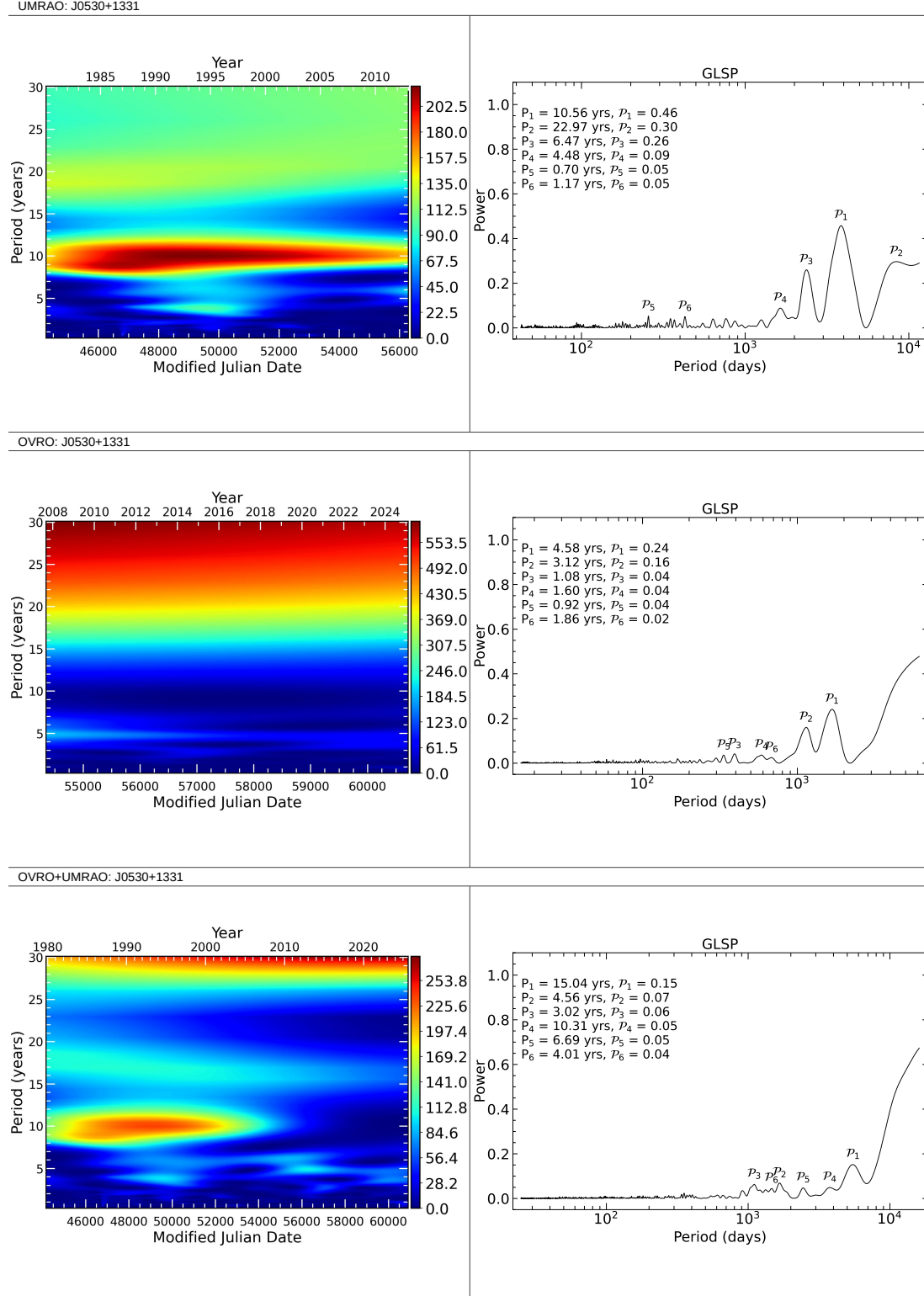
Figure A.21: J0418+3801, (RA = 64.589°, DEC = 38.027°), $z = 0.049$.

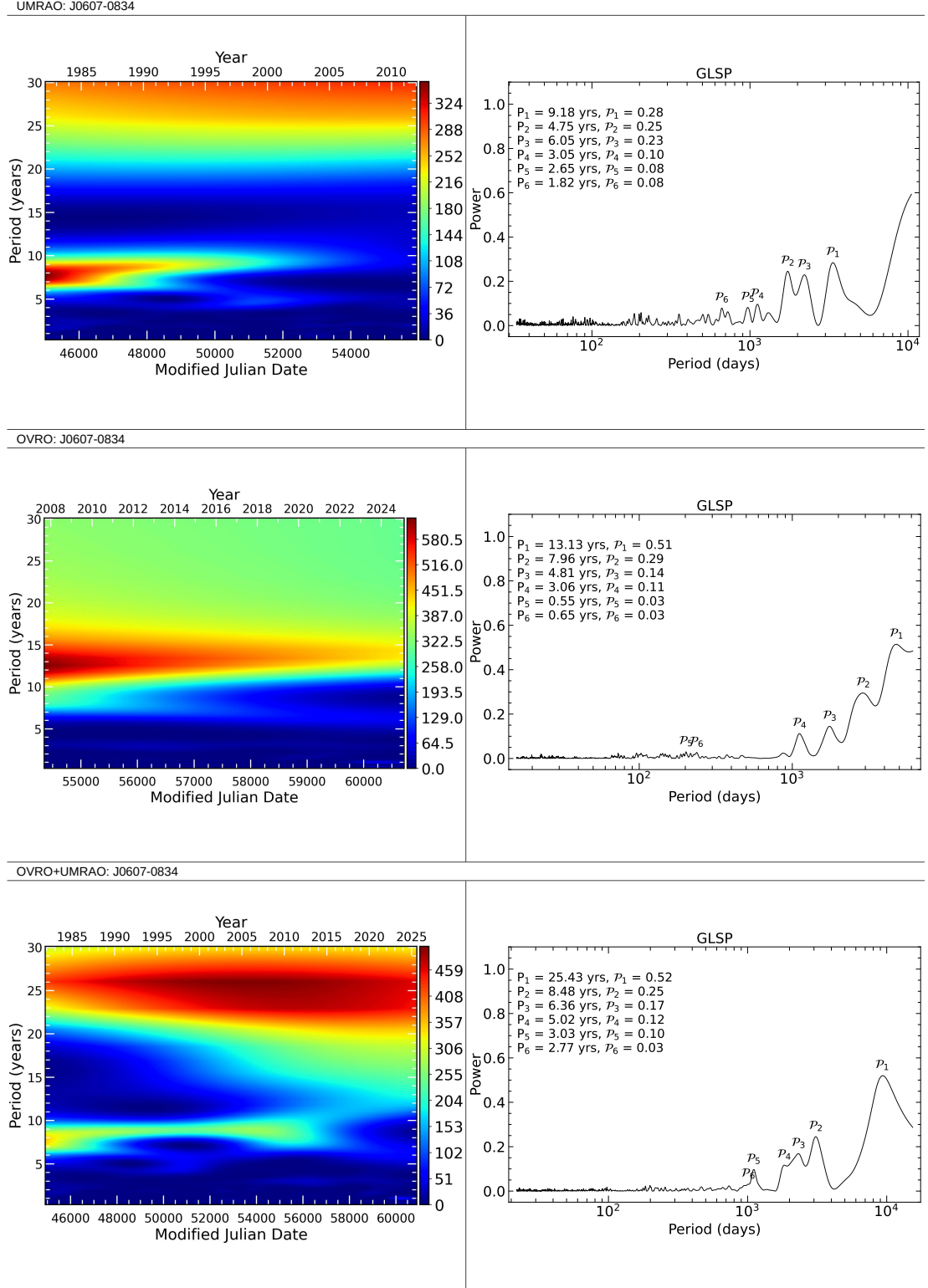
Figure A.22: J0423-0120, (RA = 65.816°, DEC = -1.343°), $z = 0.913$.

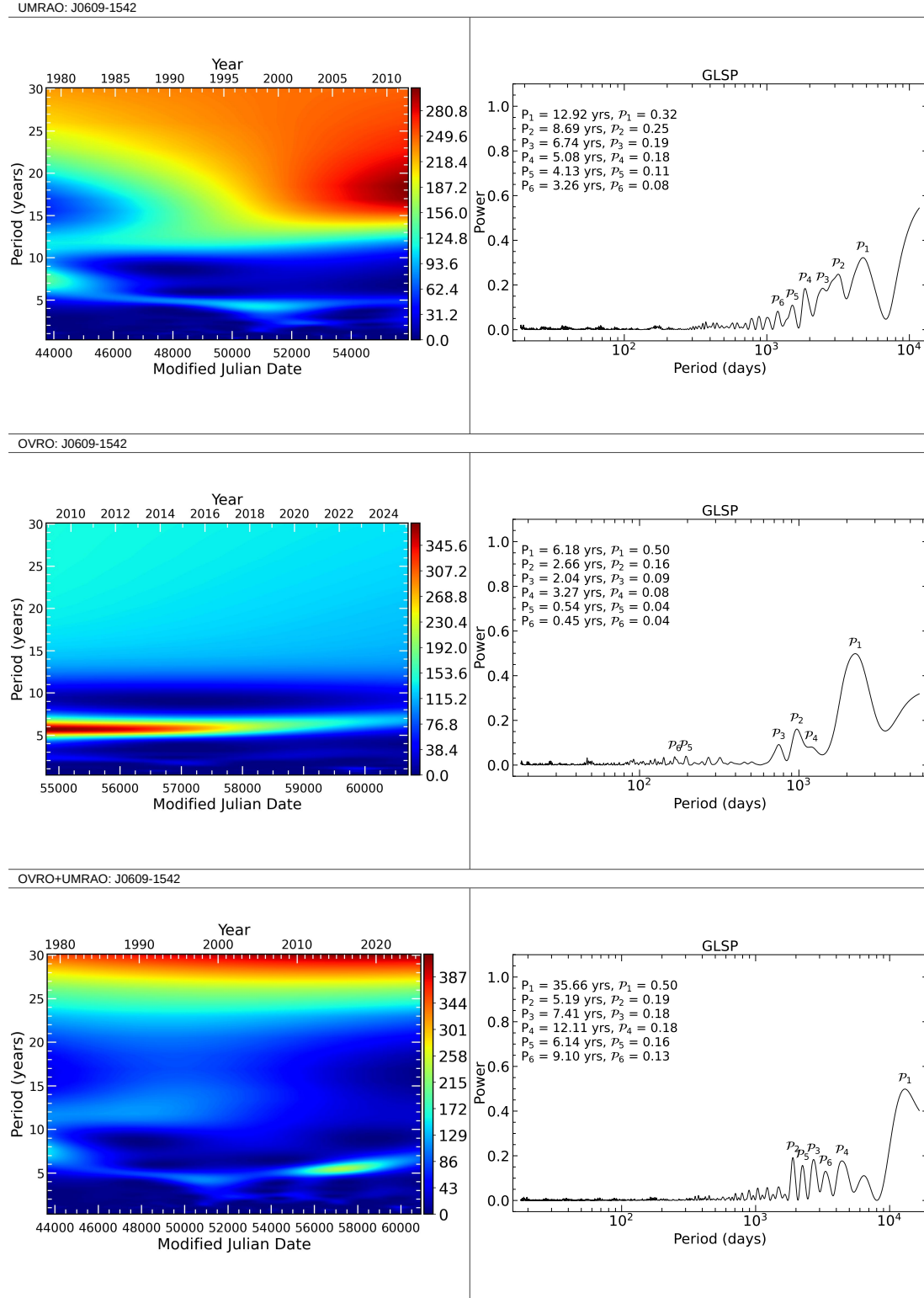
Figure A.23: J0424+0036, (RA = 66.195°, DEC = 0.602°), $z = 0.268$.

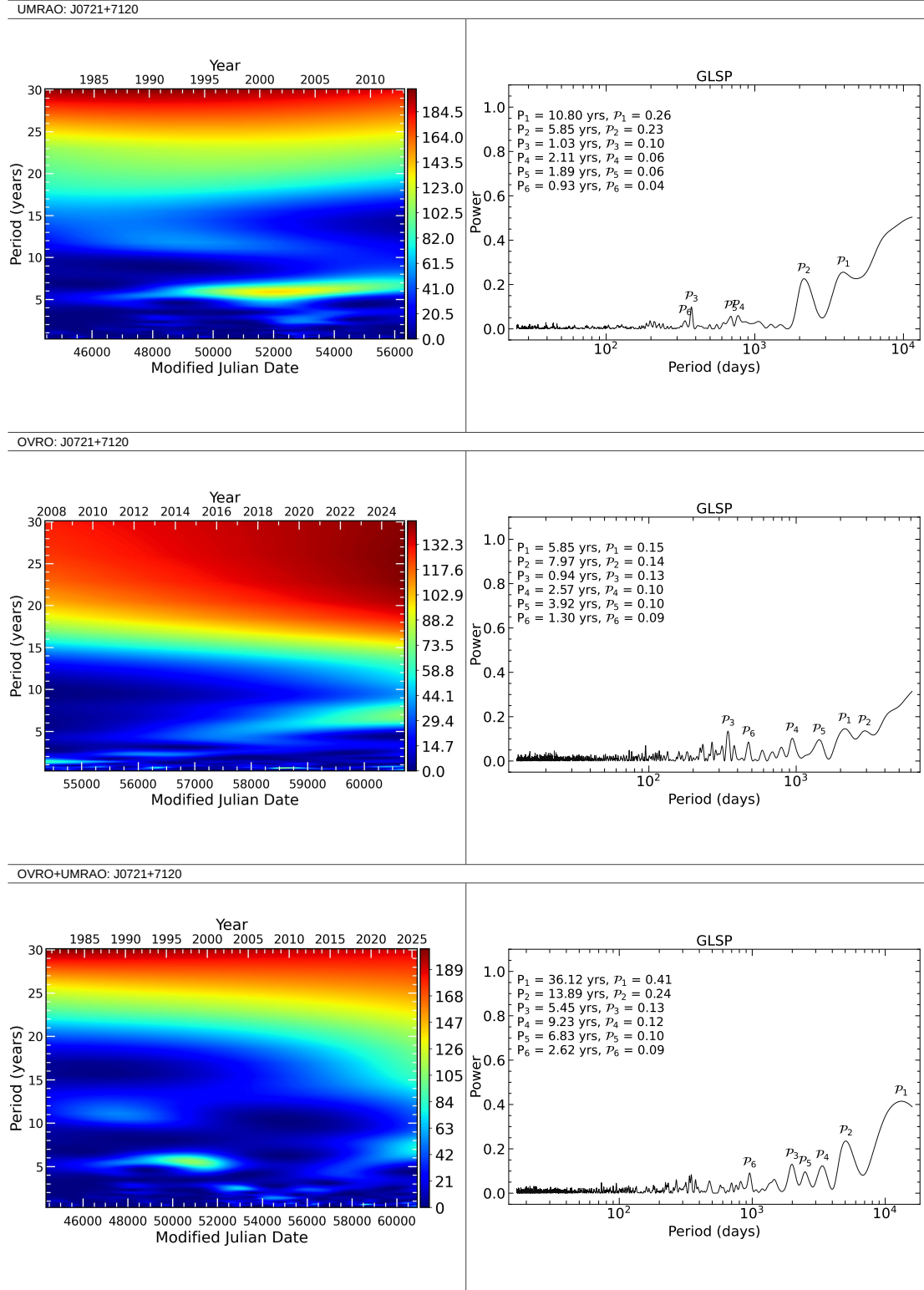
Figure A.24: J0433+0521, (RA = 68.296°, DEC = 5.354°), $z = 0.033$.

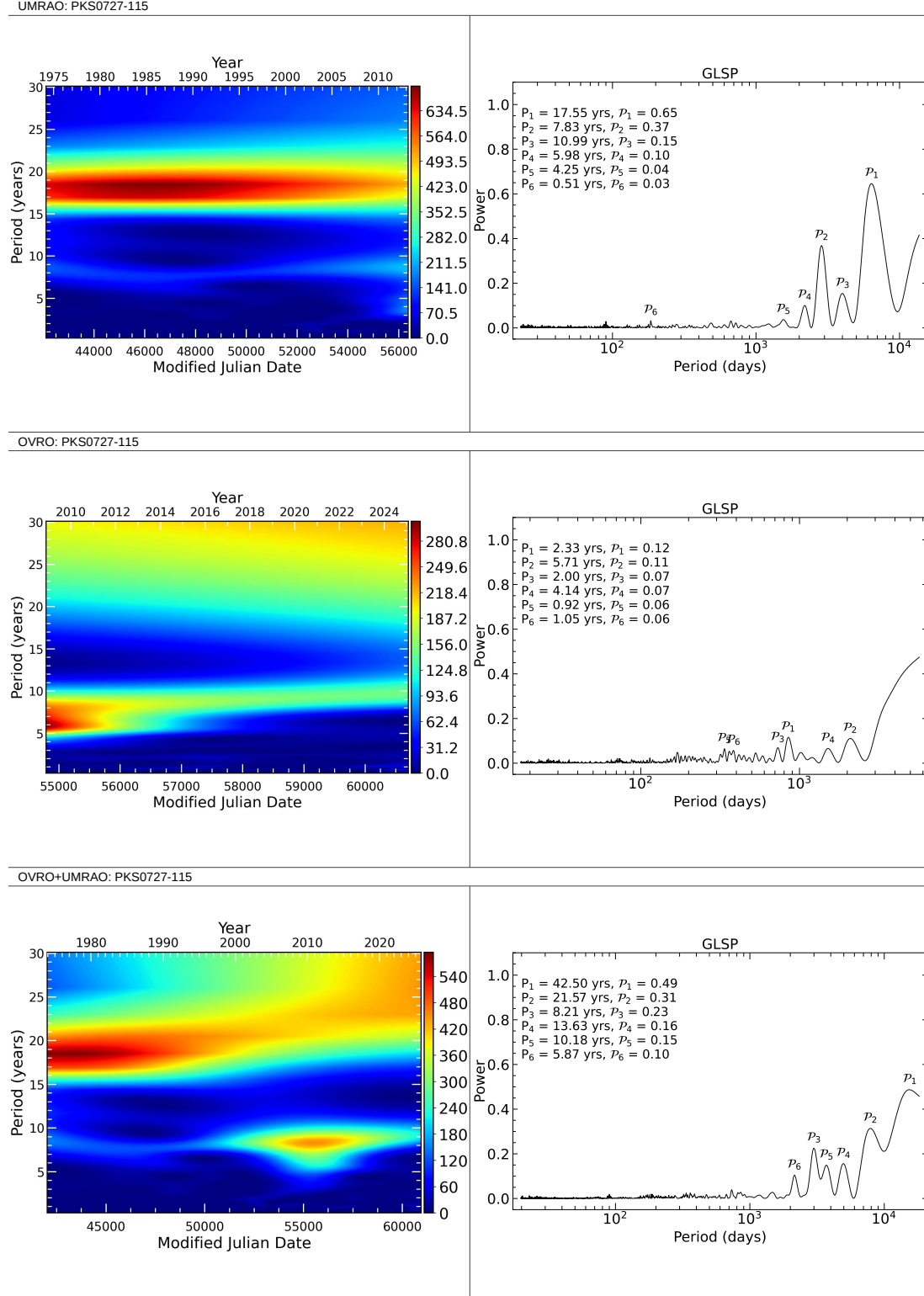
Figure A.25: J0501-0159, (RA = 75.303°, DEC = −1.987°), $z = 2.286$.

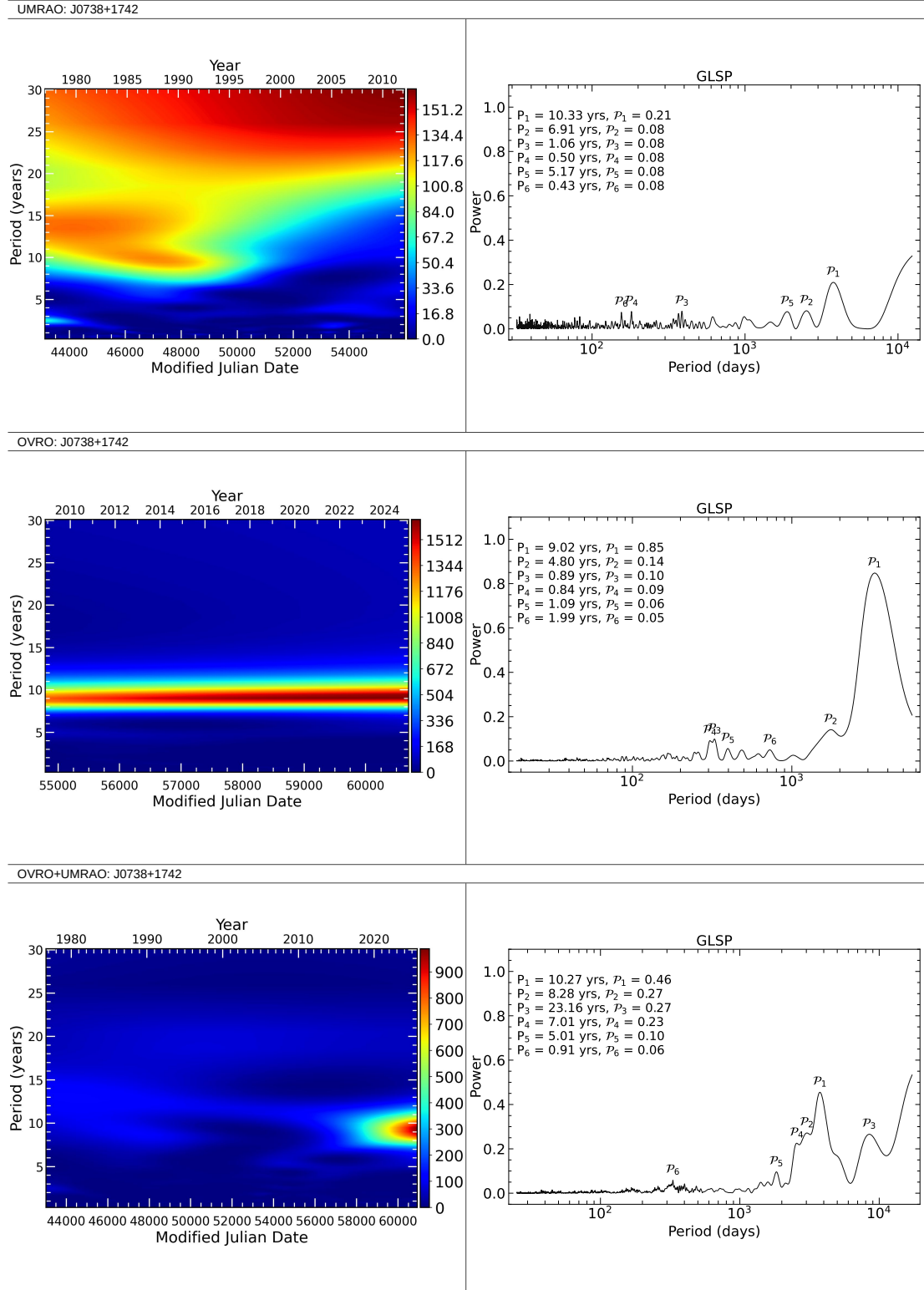
Figure A.26: J0530+1331, (RA = 82.735°, DEC = 13.532°), $z = 2.070$.

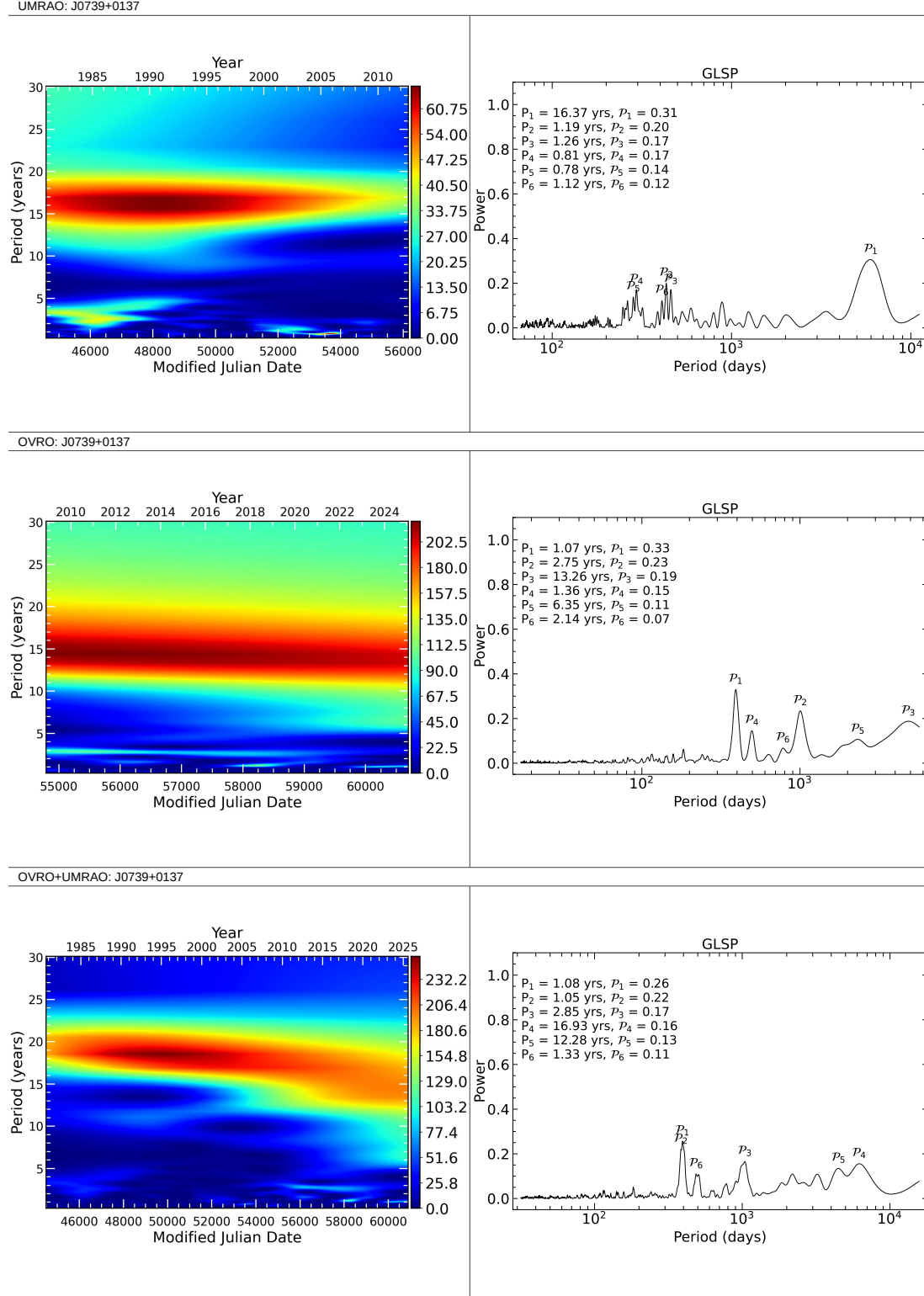
Figure A.27: J0607-0834, (RA = 91.999°, DEC = −8.581°, $z = 0.872$).

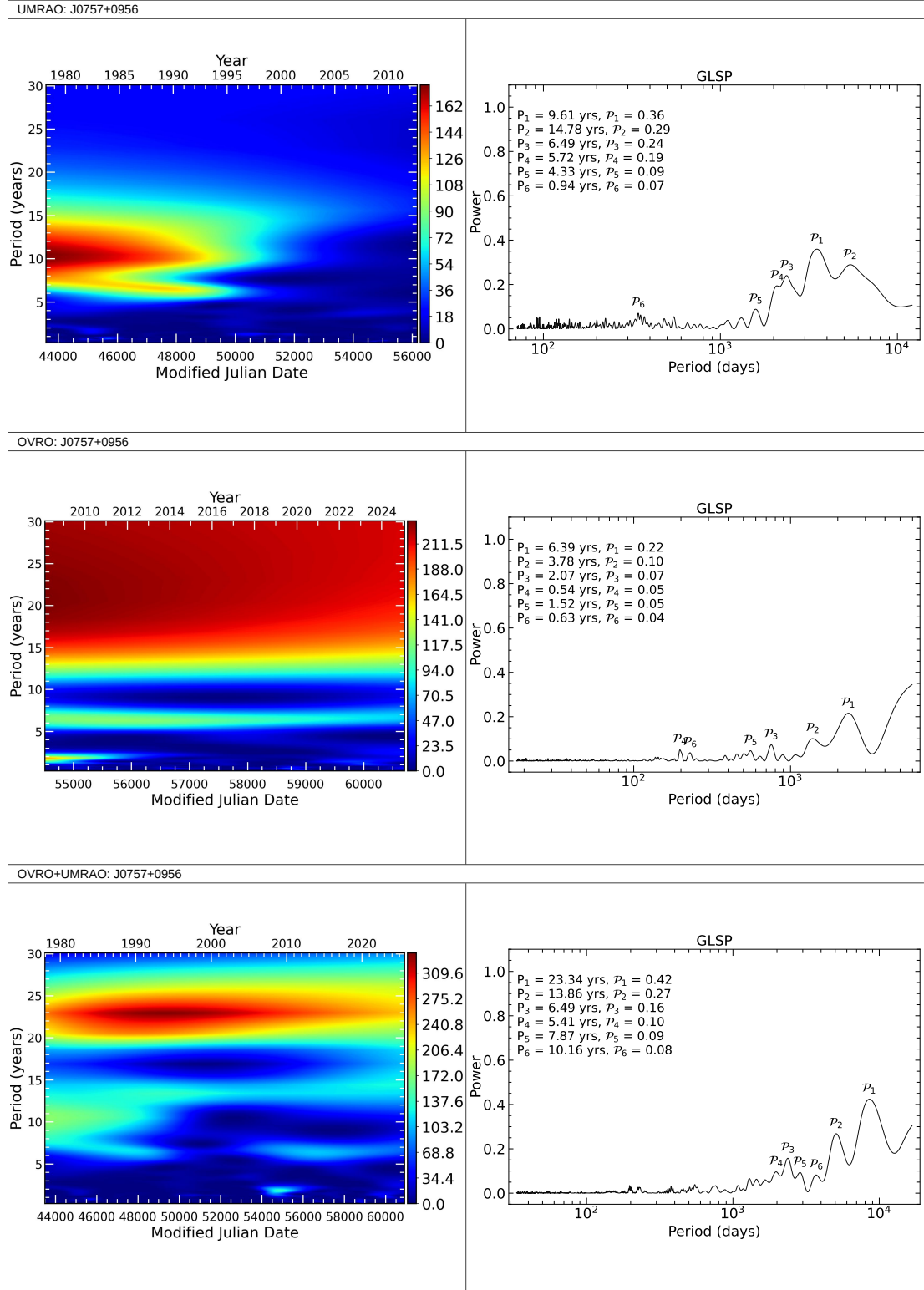
Figure A.28: J0609-1542, (RA = 92.421°, DEC = −15.711°), $z = 0.324$.

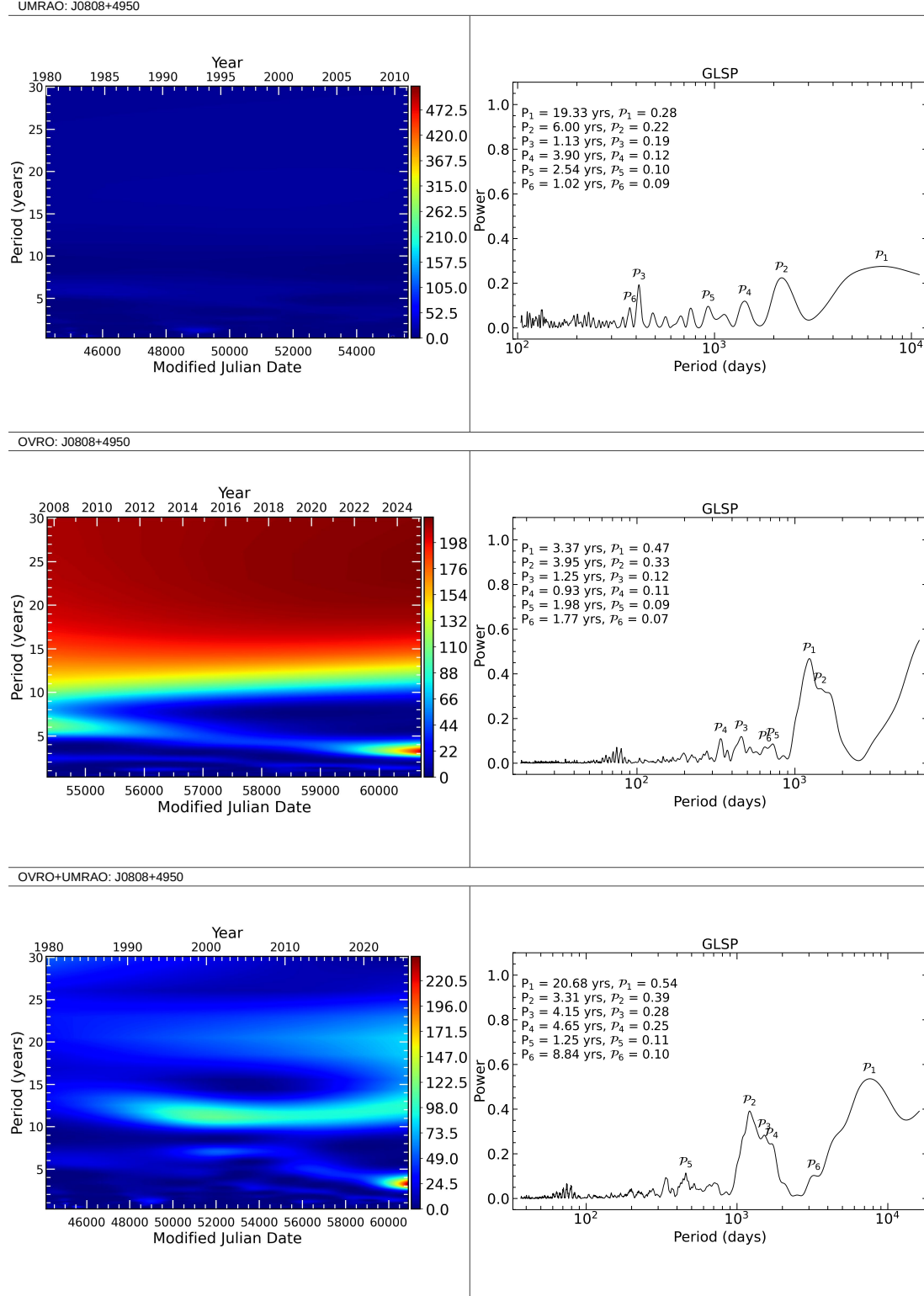
Figure A.29: J0721+7120, (RA = 110.473°, DEC = 71.343°), $z = U$.

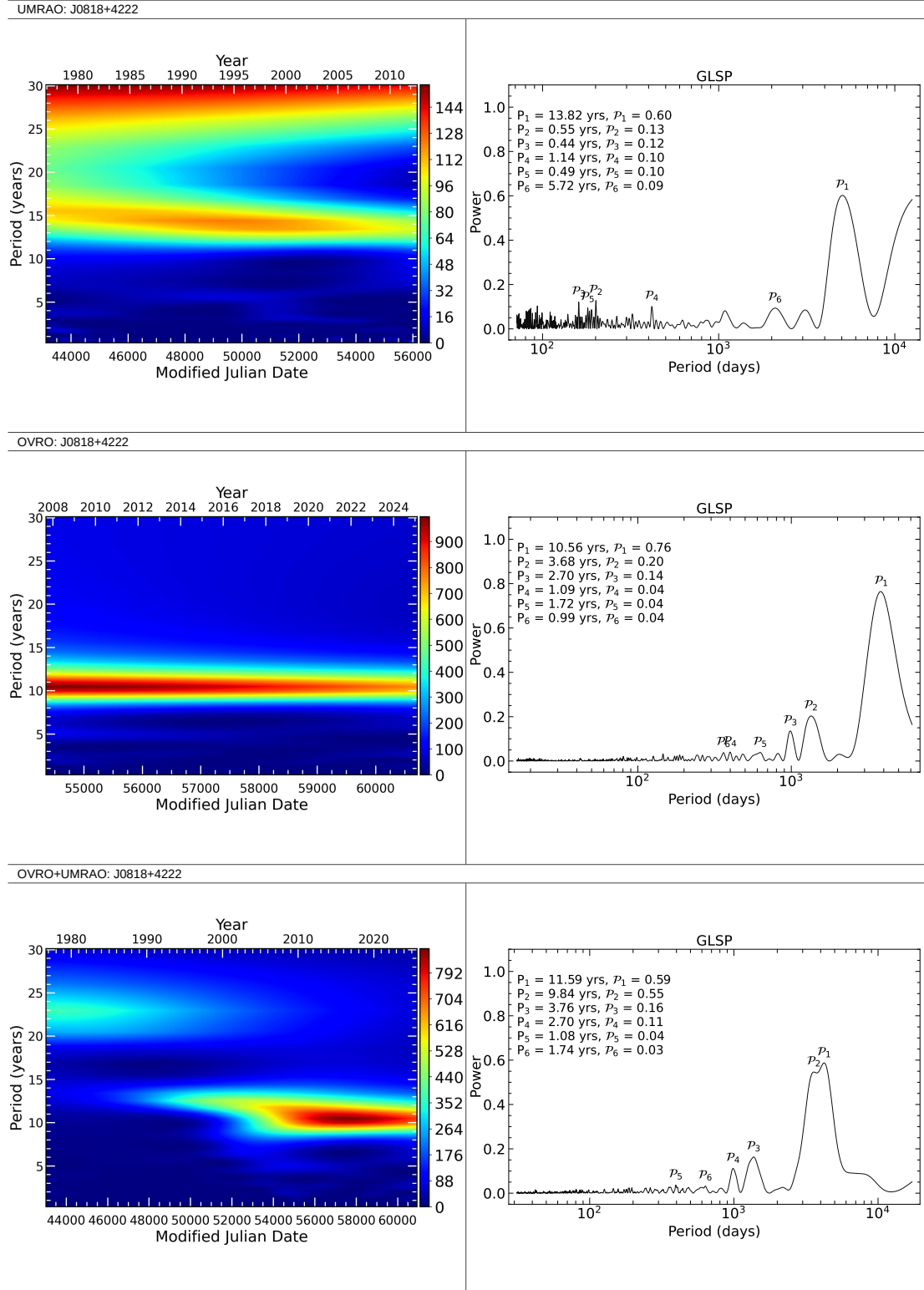
Figure A.30: J0730-1141, (RA = 112.580°, DEC = -11.687°), $z = 1.591$.

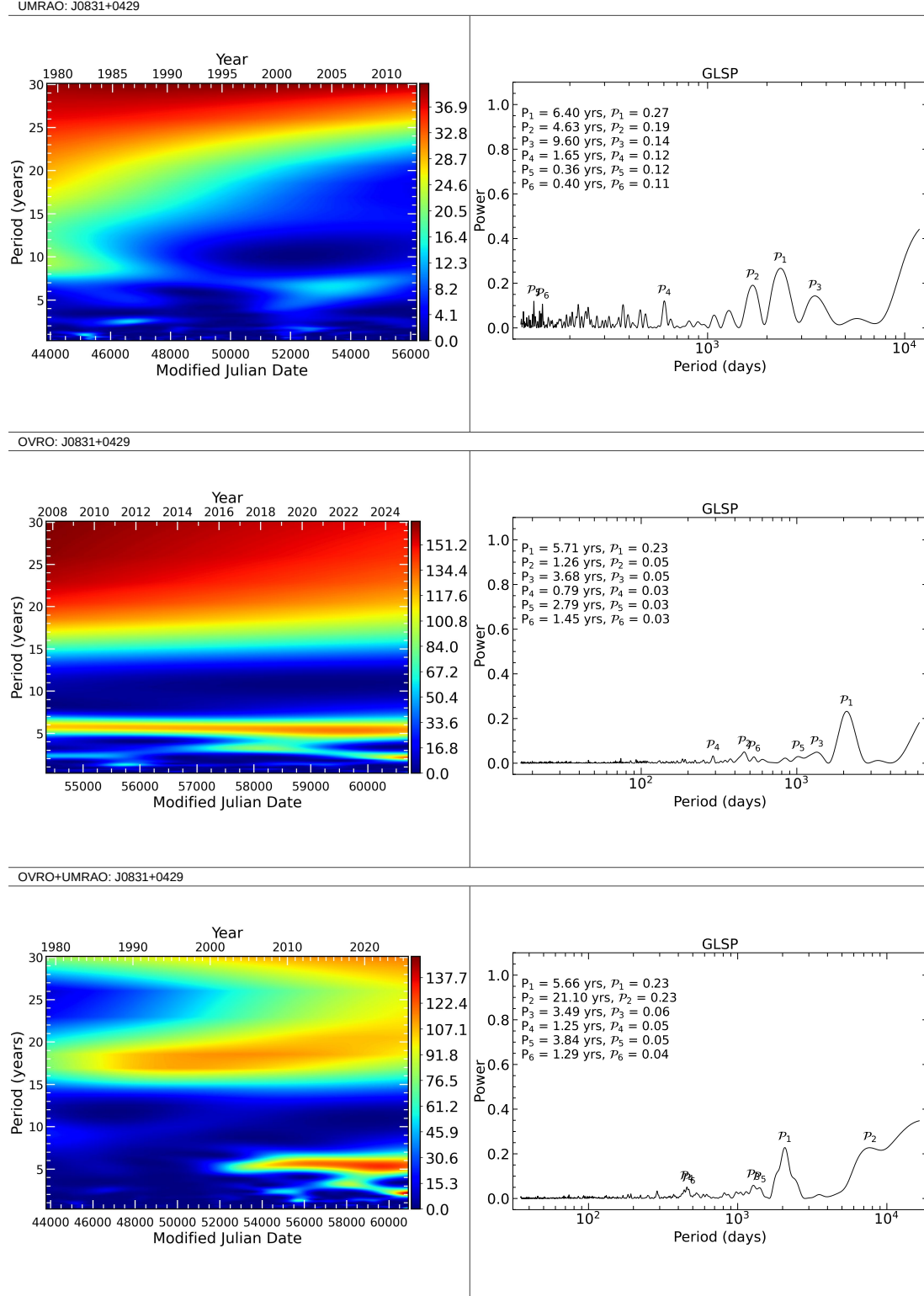
Figure A.31: J0738+1742, (RA = 114.531°, DEC = 17.705°), $z = \text{U}$.

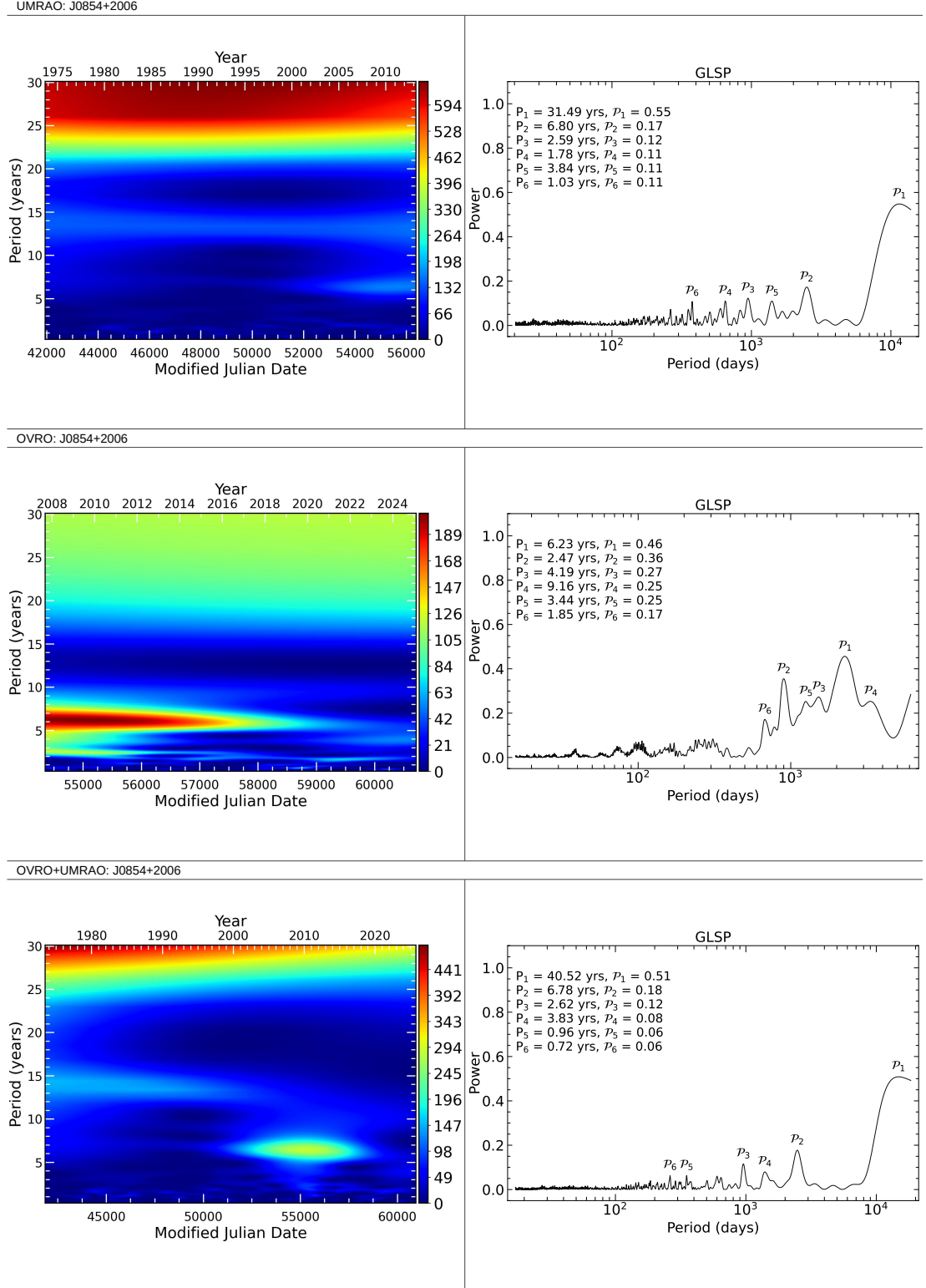
Figure A.32: J0739+0137, (RA = 114.825°, DEC = 1.618°), $z = 0.189$.

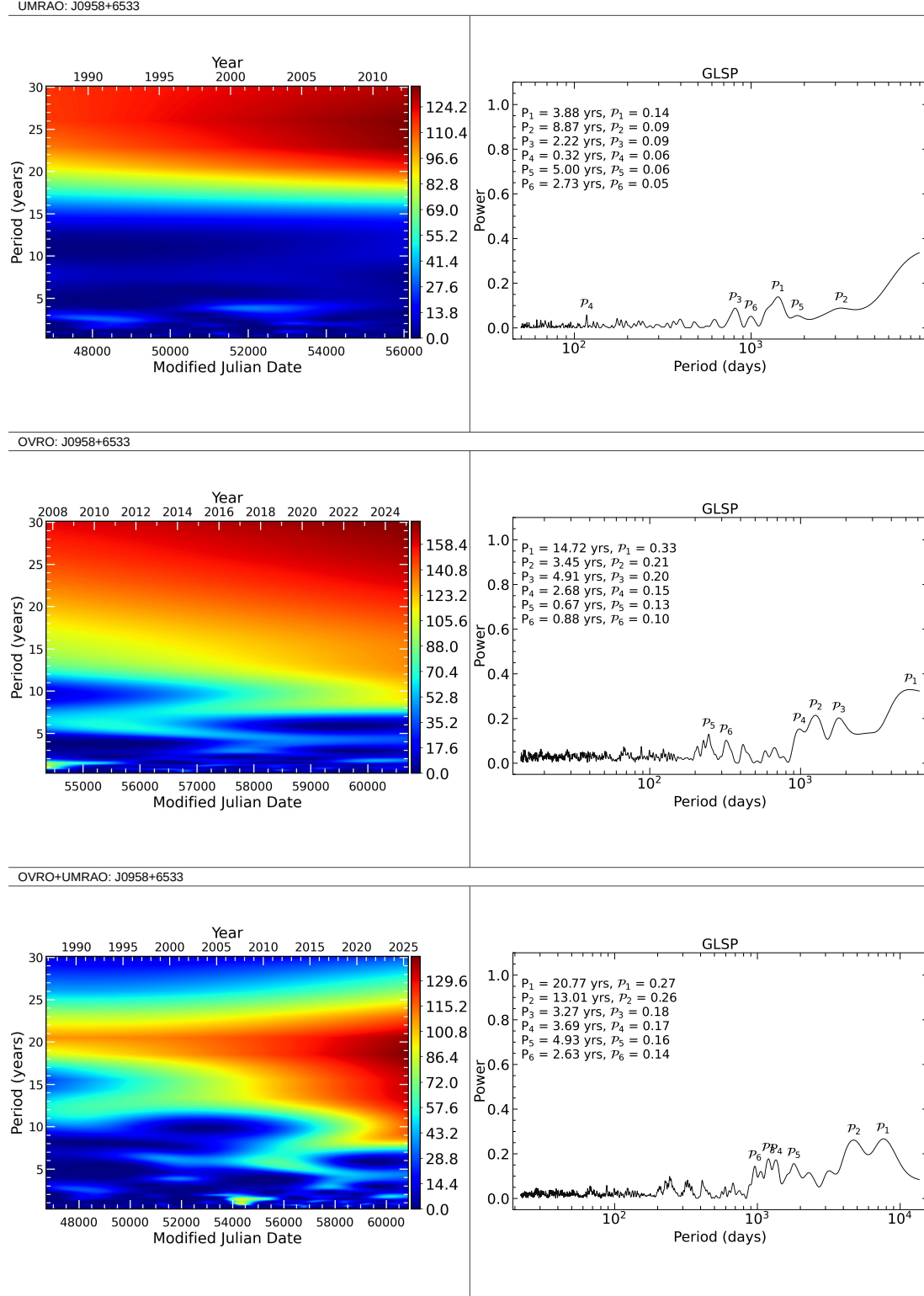
Figure A.33: J0757+0956, (RA = 119.278°, DEC = 9.943°), $z = 0.266$.

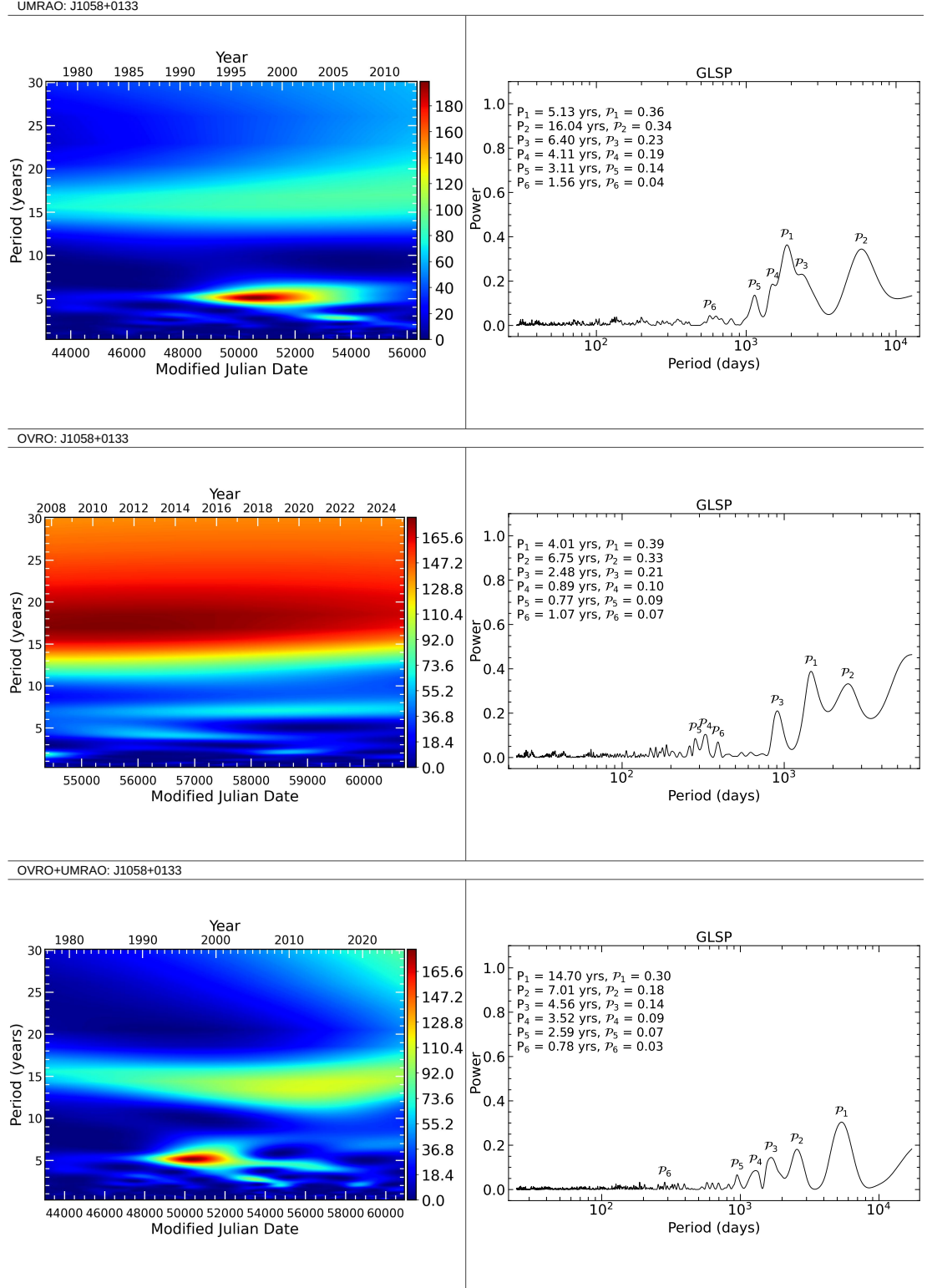
Figure A.34: J0808+4950, (RA = 122.165°, DEC = 49.843°), $z = 1.435$.

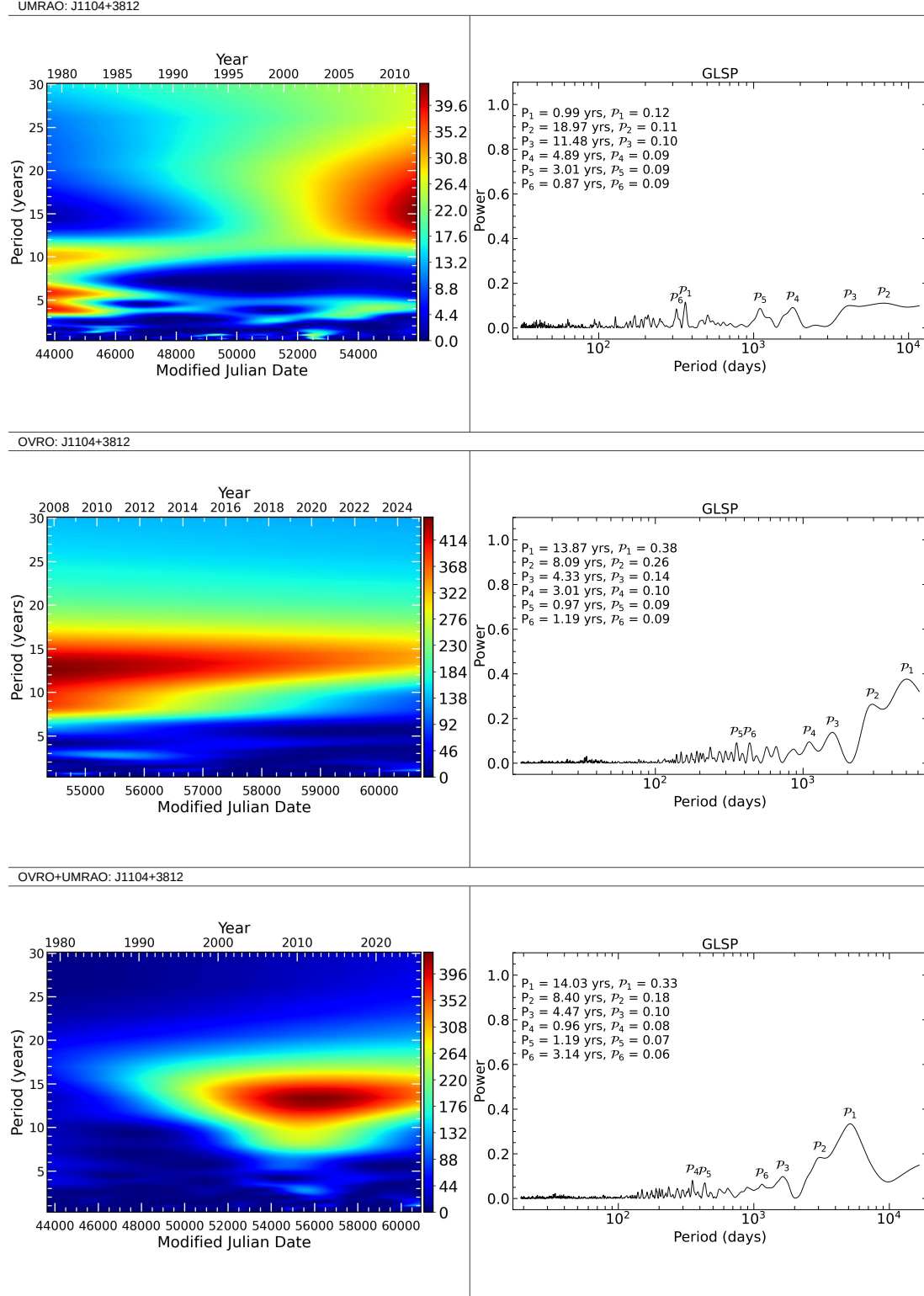
Figure A.35: J0818+4222, (RA = 124.567°, DEC = 42.379°), $z = \text{U}$.

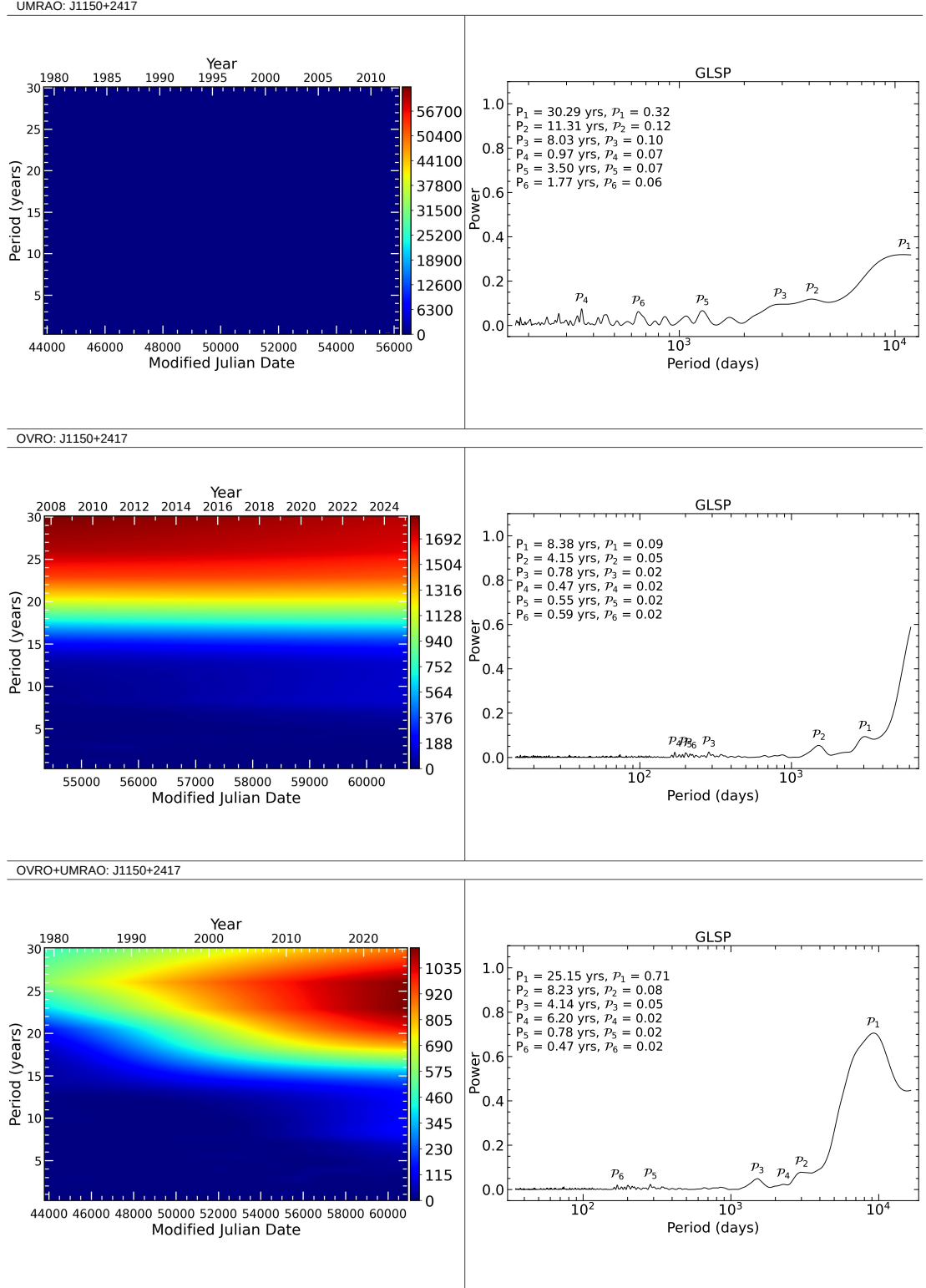
Figure A.36: J0831+0429, (RA = 127.954°, DEC = 4.494°), $z = 0.175$.

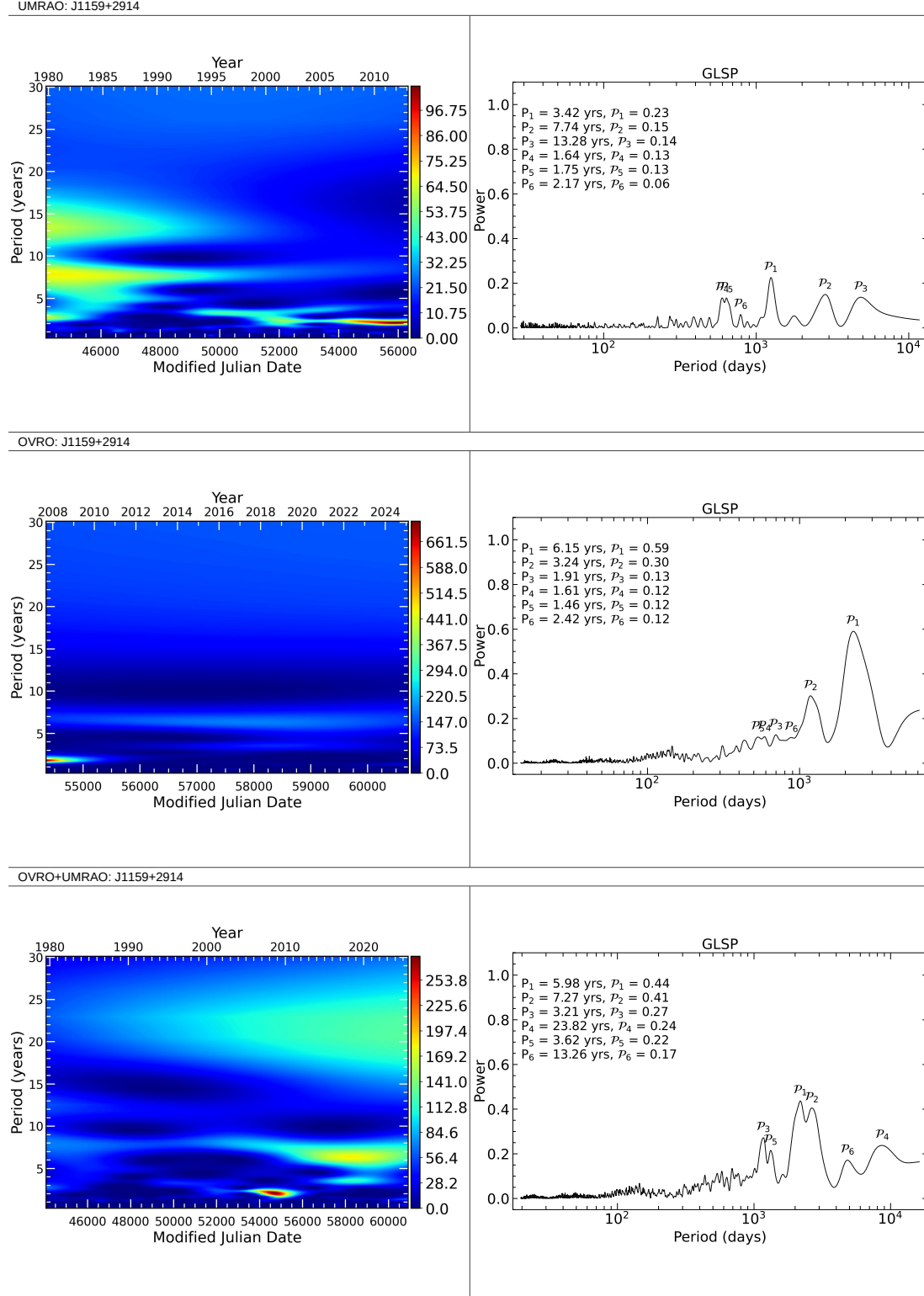
Figure A.37: J0854+2006, (RA = 133.704°, DEC = 20.108°), $z = 0.306$.

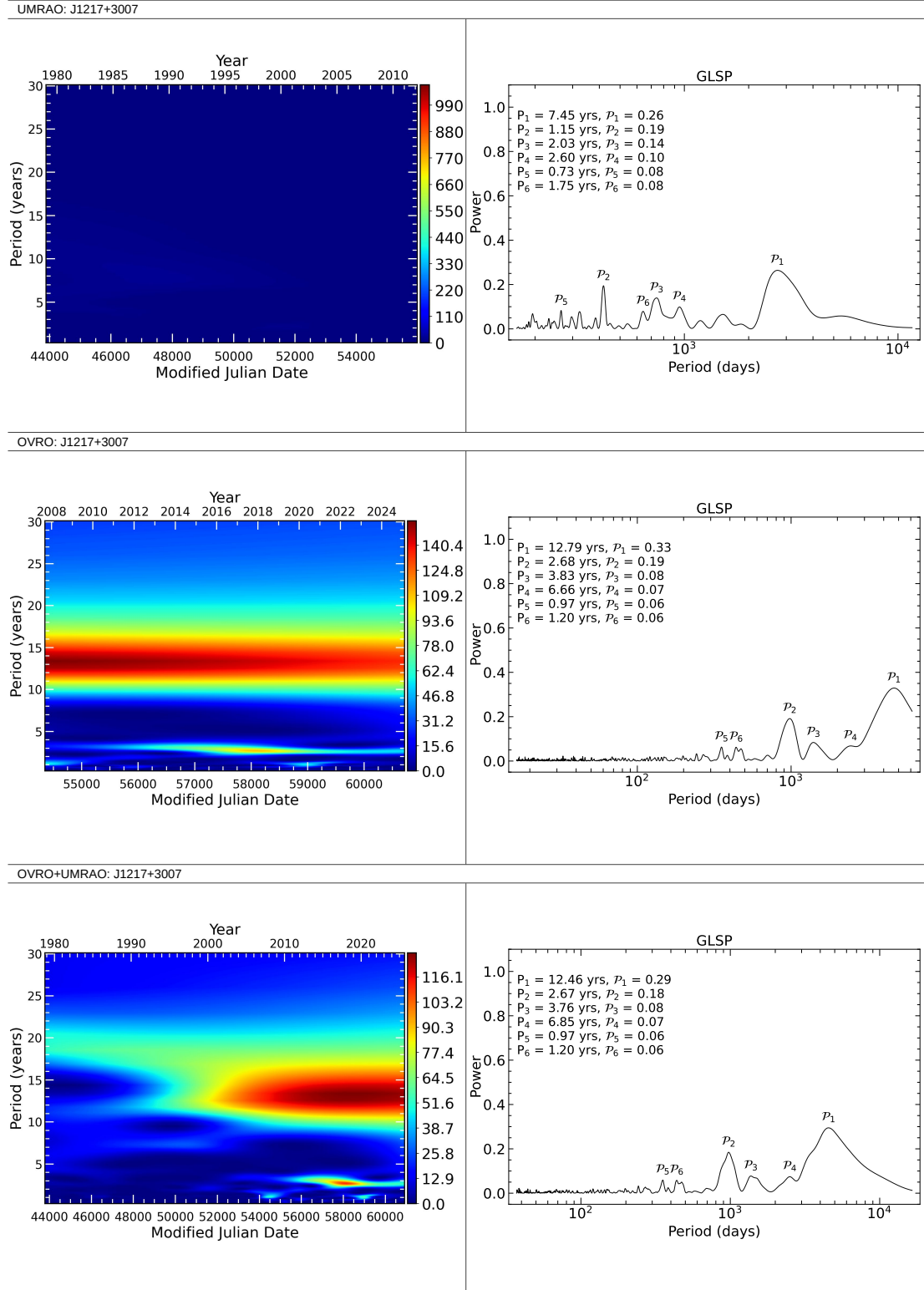
Figure A.38: J0958+6533, (RA = 149.697°, DEC = 65.565°), $z = 0.367$.

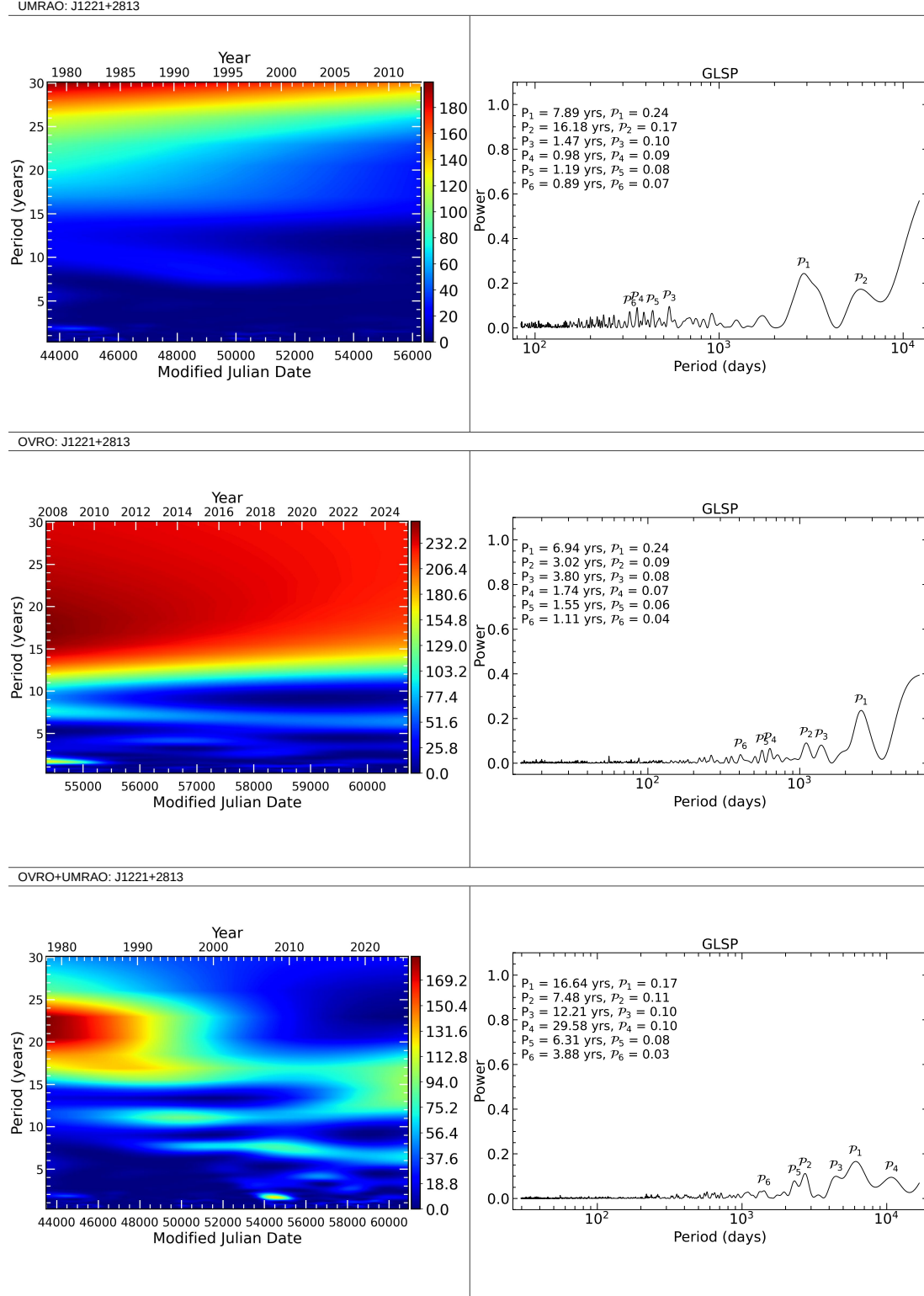
Figure A.39: J1058+0133, (RA = 164.623°, DEC = 1.566°), $z = 0.891$.

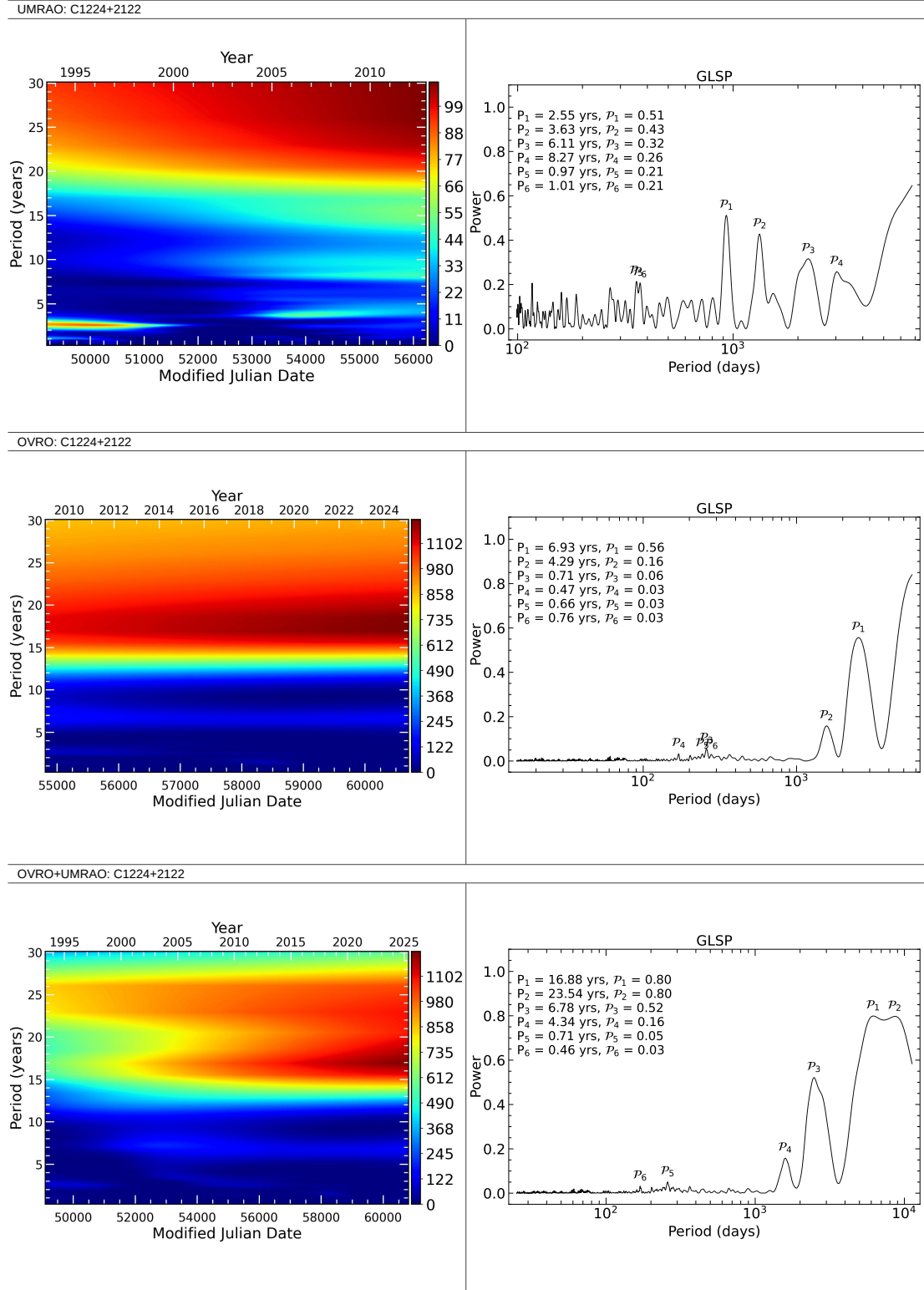
Figure A.40: J1104+3812, (RA = 166.114°, DEC = 38.209°), $z = 0.030$.

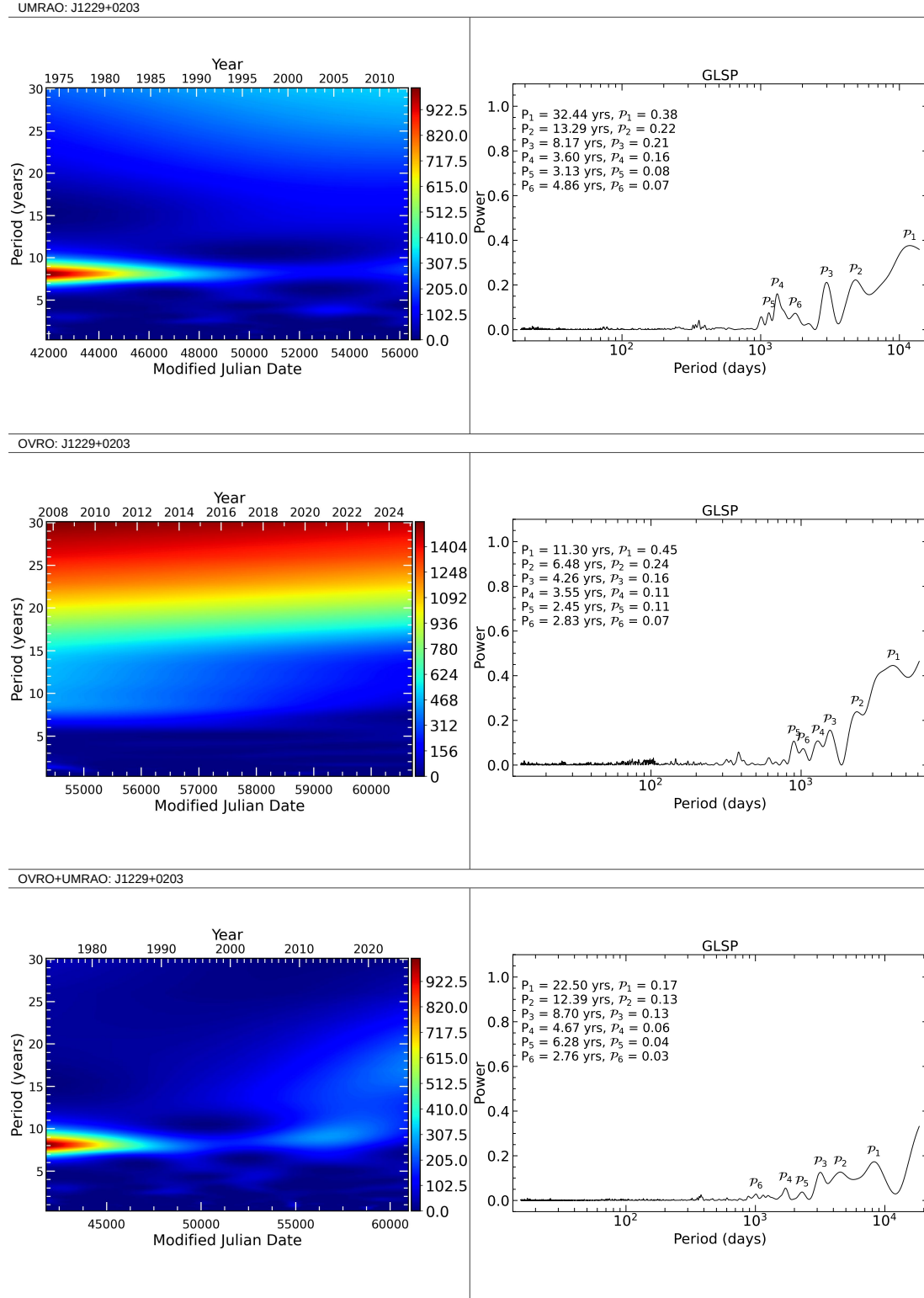
Figure A.41: J1150+2417, (RA = 177.580°, DEC = 24.298°), $z = 0.209$.

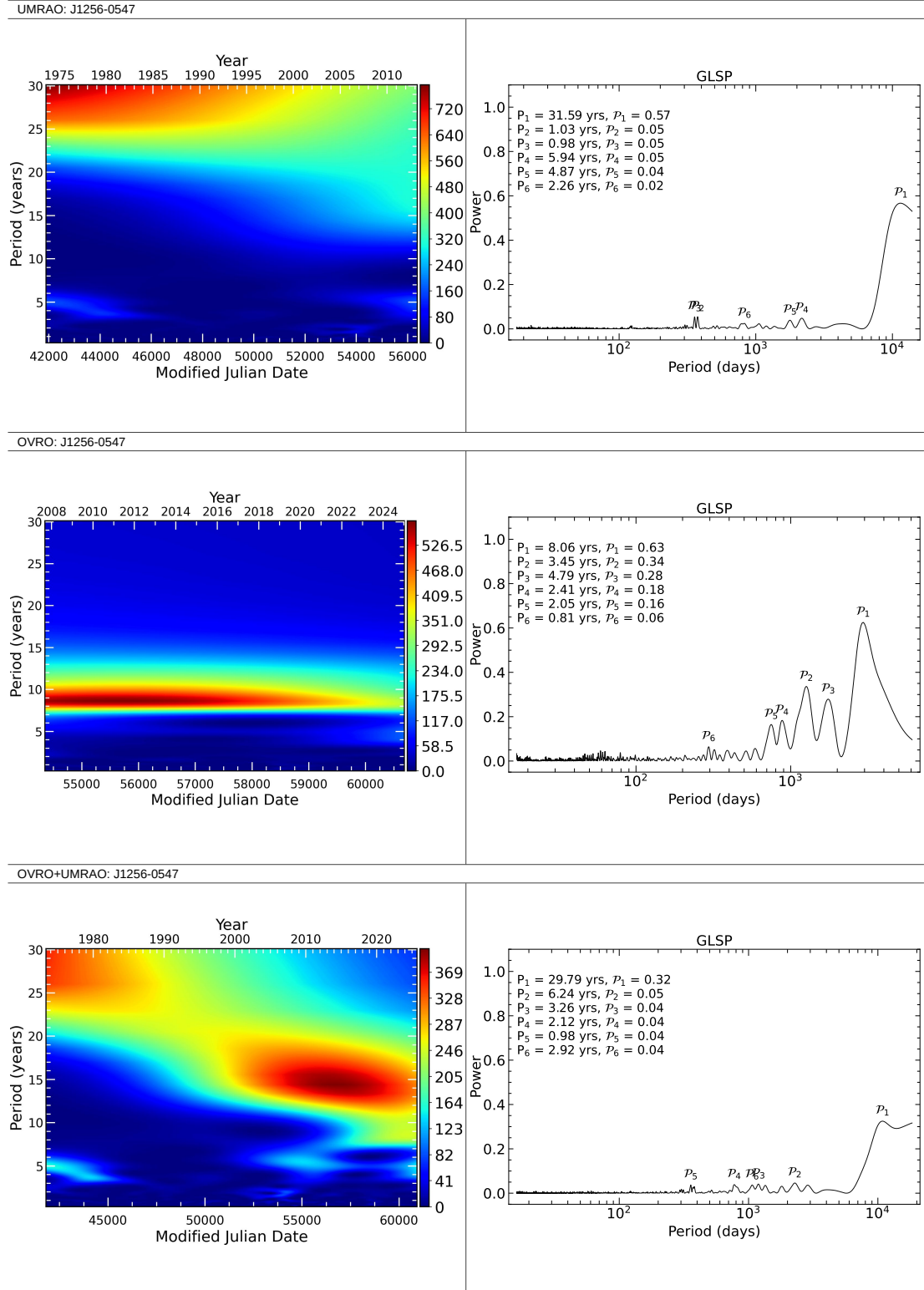
Figure A.42: J1159+2914, (RA = 179.883°, DEC = 29.245°), $z = 0.725$.

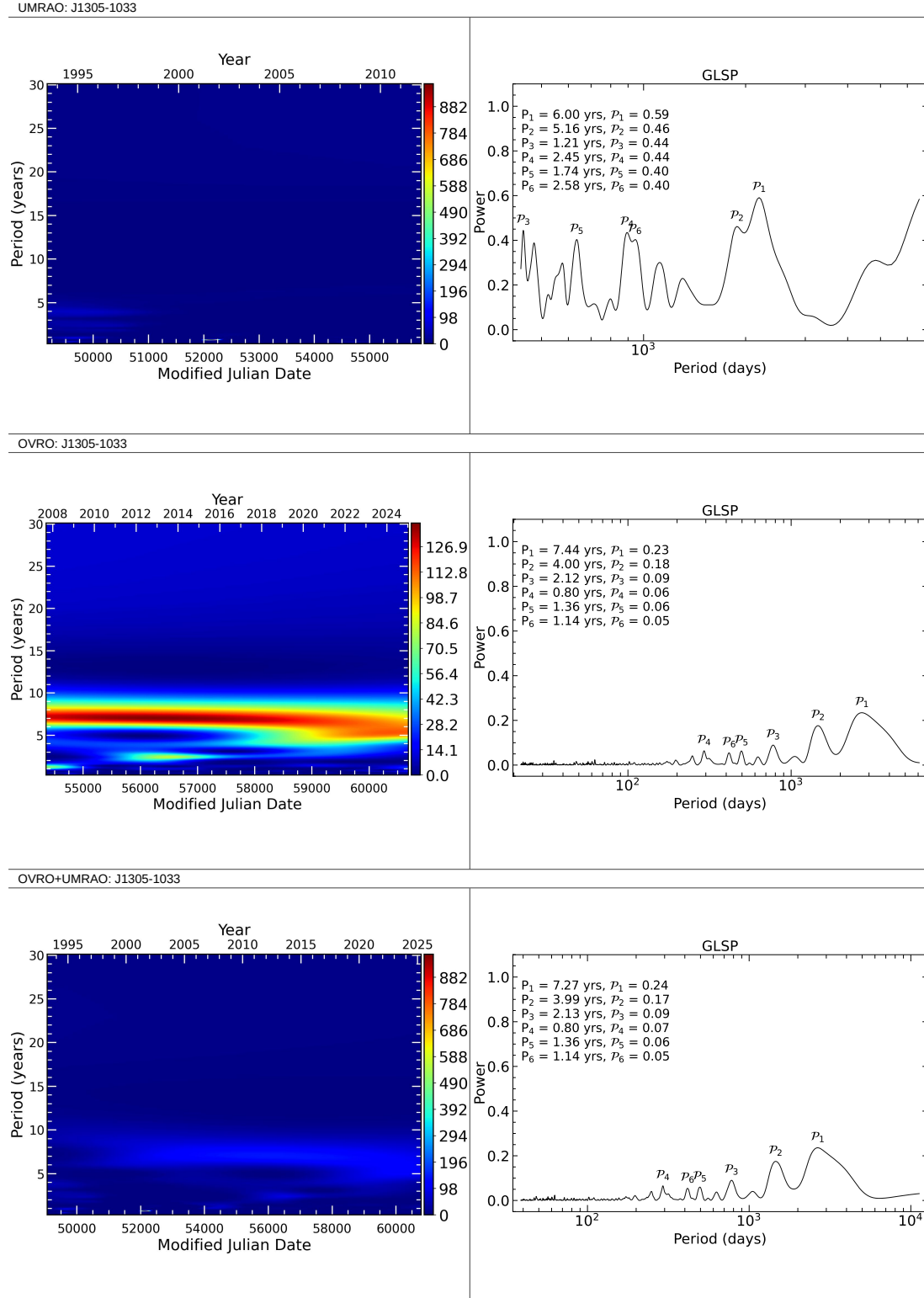
Figure A.43: J1217+3007, (RA = 184.467°, DEC = 30.117°), $z = 0.130$.

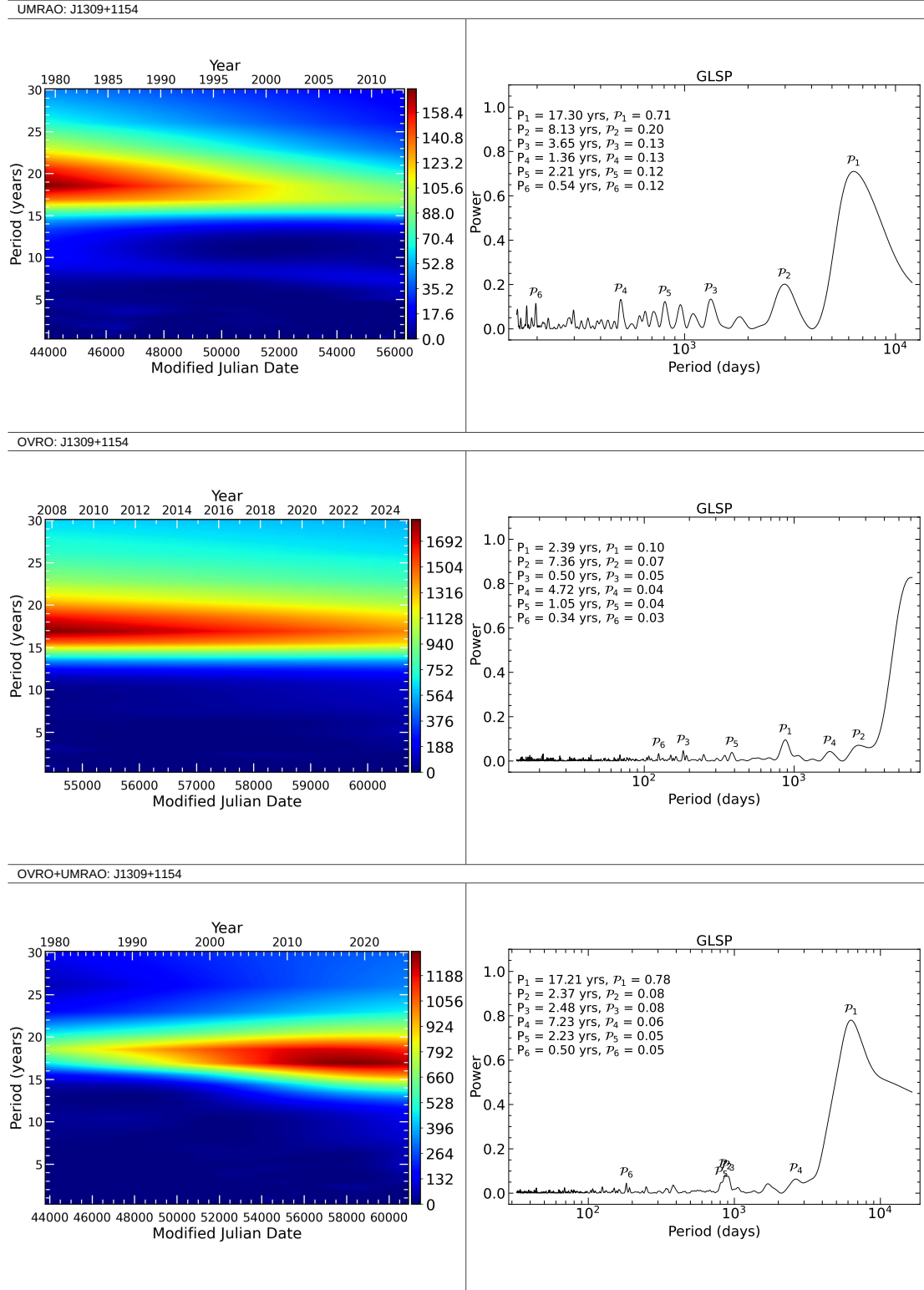
Figure A.44: J1221+2813, (RA = 185.382°, DEC = 28.233°), $z = 0.102$.

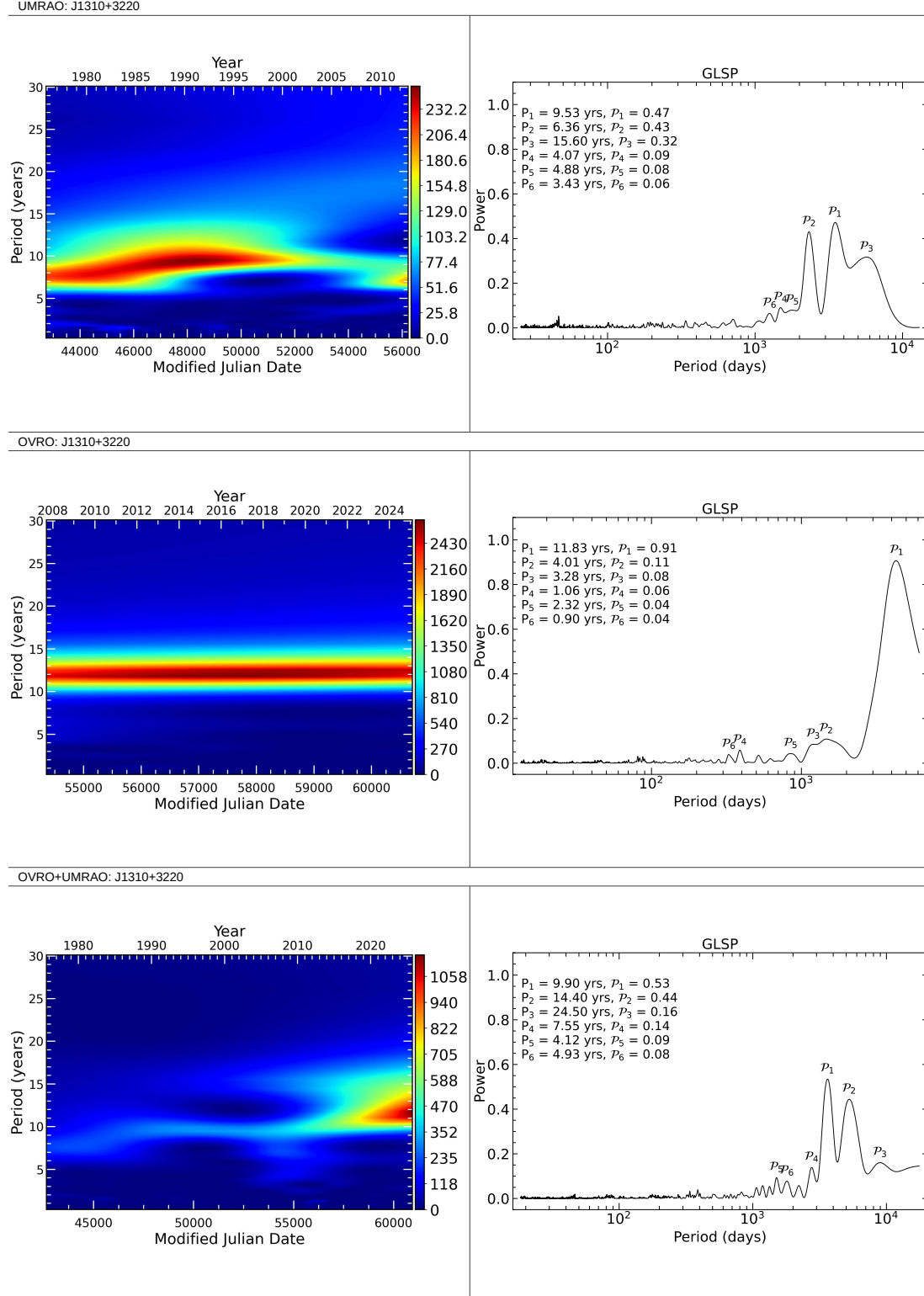
Figure A.45: J1224+2122, (RA = 186.227°, DEC = 21.380°), $z = 0.433$.

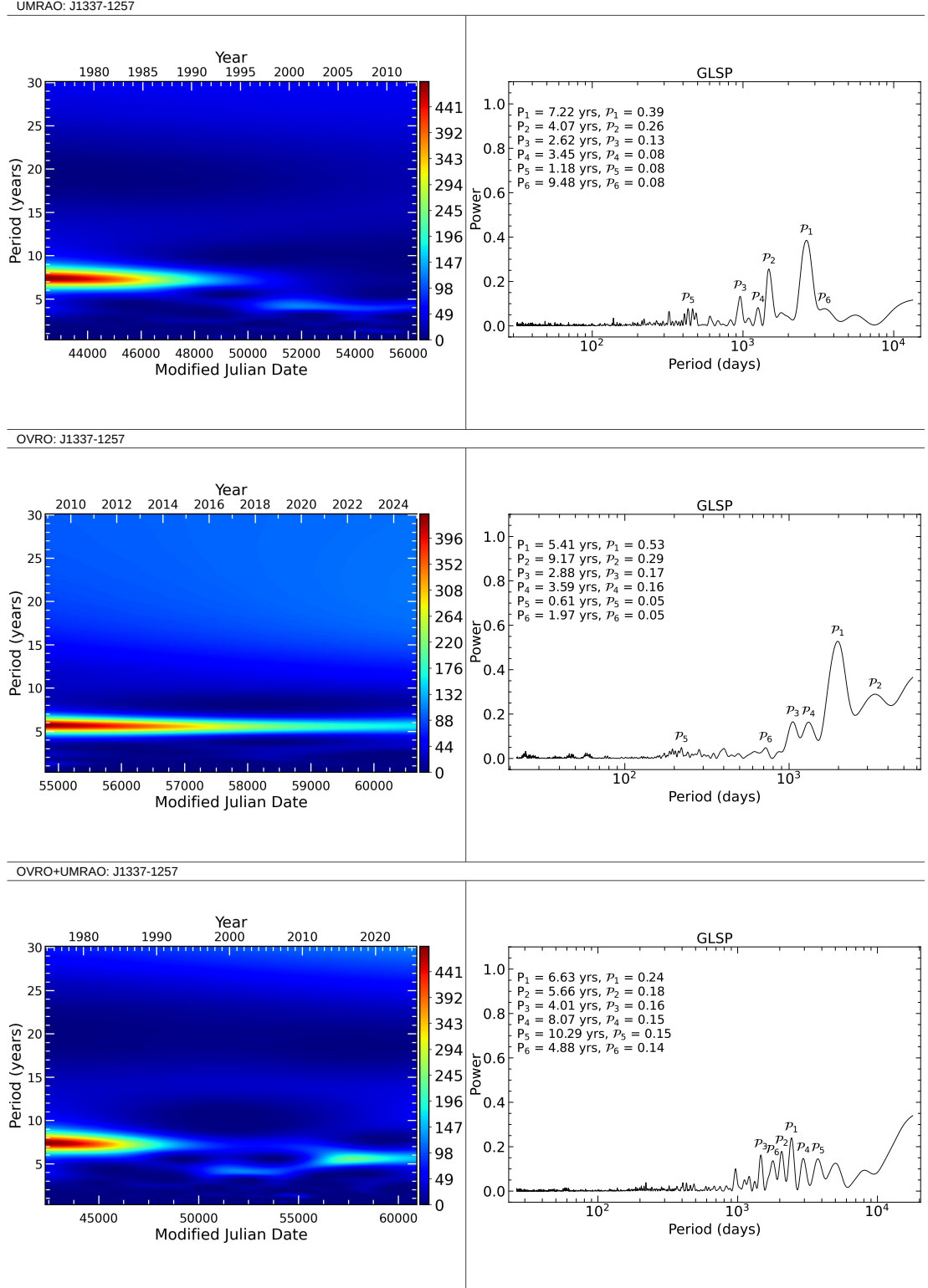
Figure A.46: J1229+0203, (RA = 187.278°, DEC = 2.052°), $z = 0.158$.

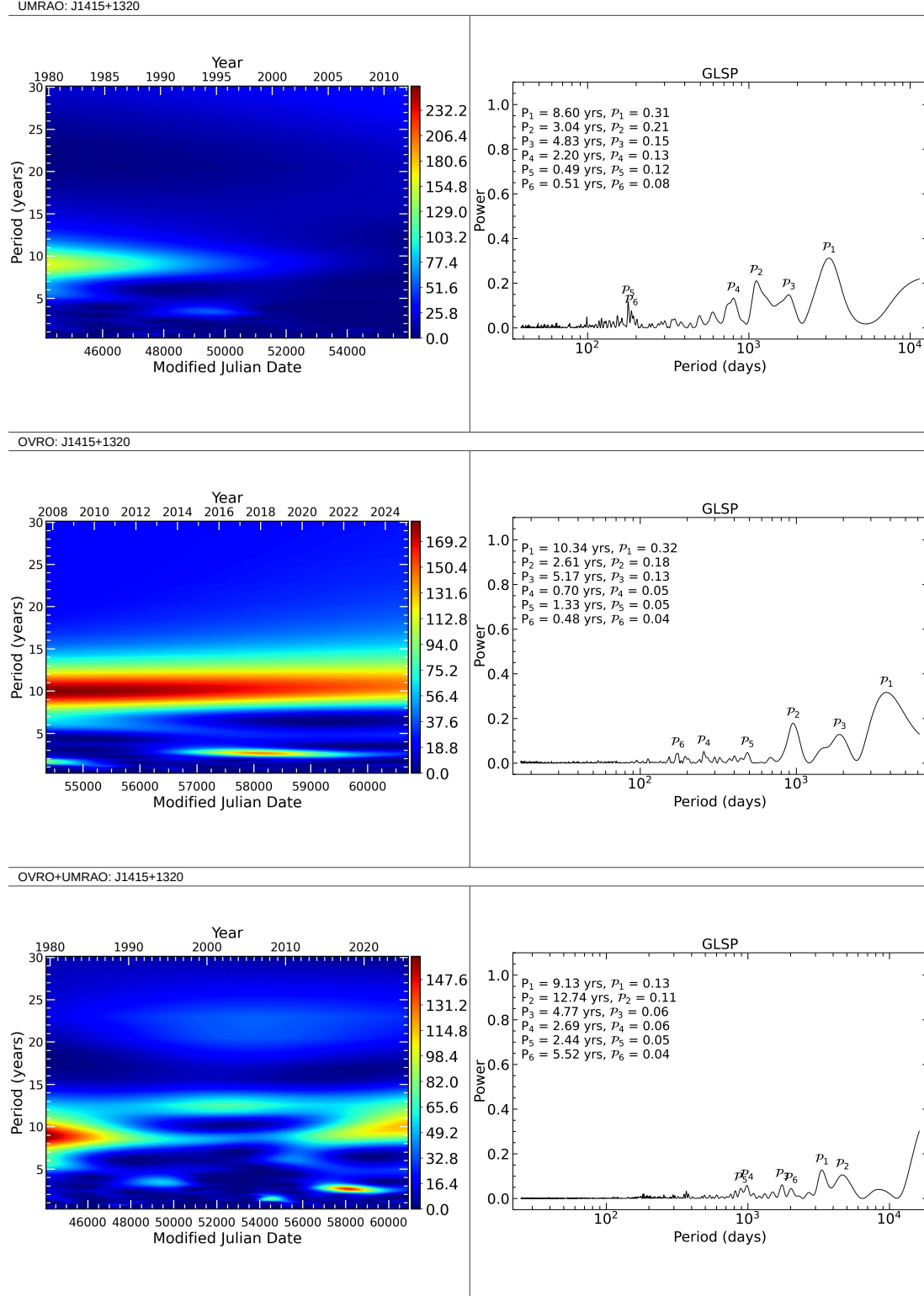
Figure A.47: J1256-0547, (RA = 194.047°, DEC = -5.789°), $z = 0.536$.

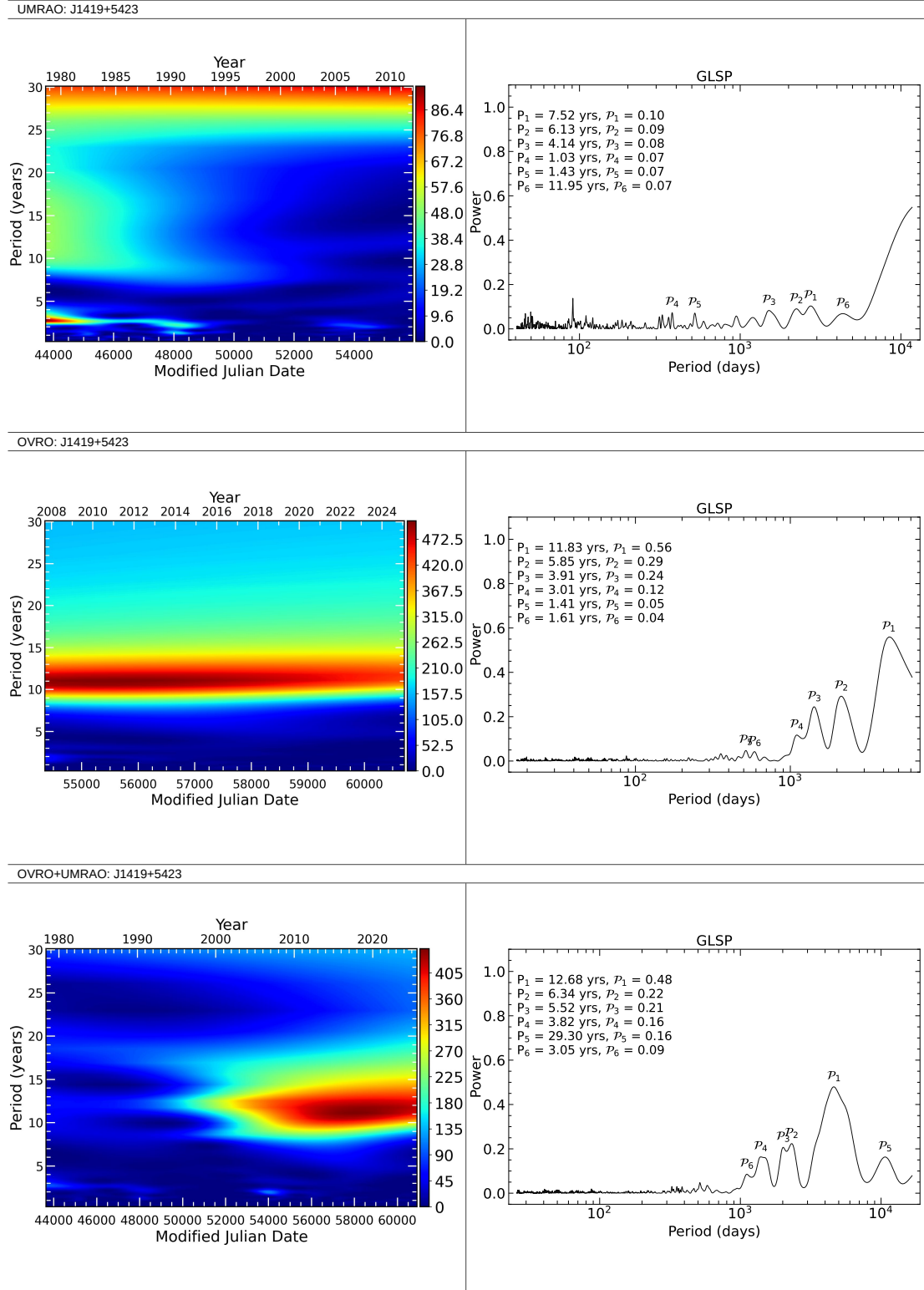
Figure A.48: J1305-1033, (RA = 196.388°, DEC = −10.555°), $z = 0.278$.

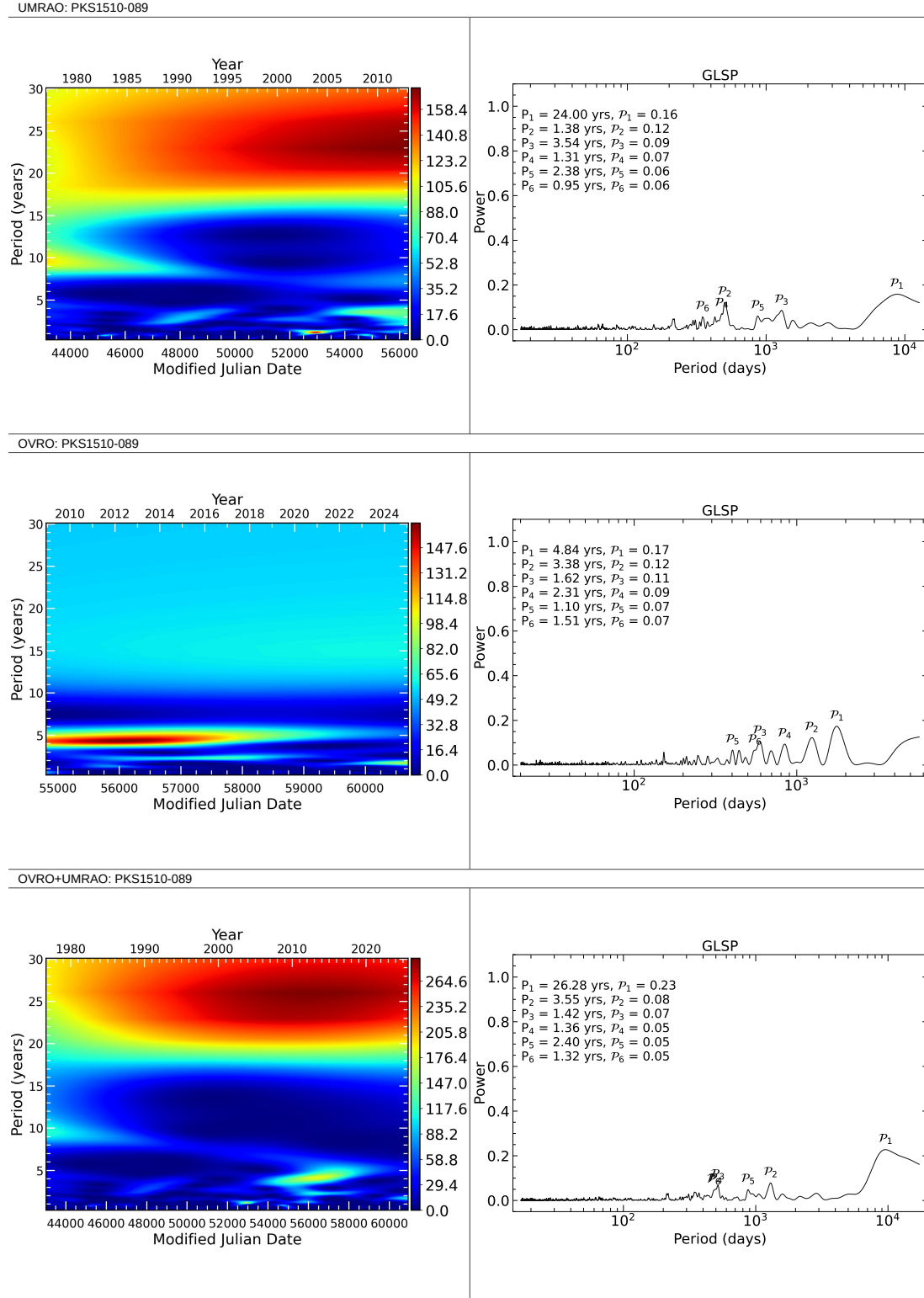
Figure A.49: J1309+1154, (RA = 197.391°, DEC = 11.907°), $z = \text{U}$.

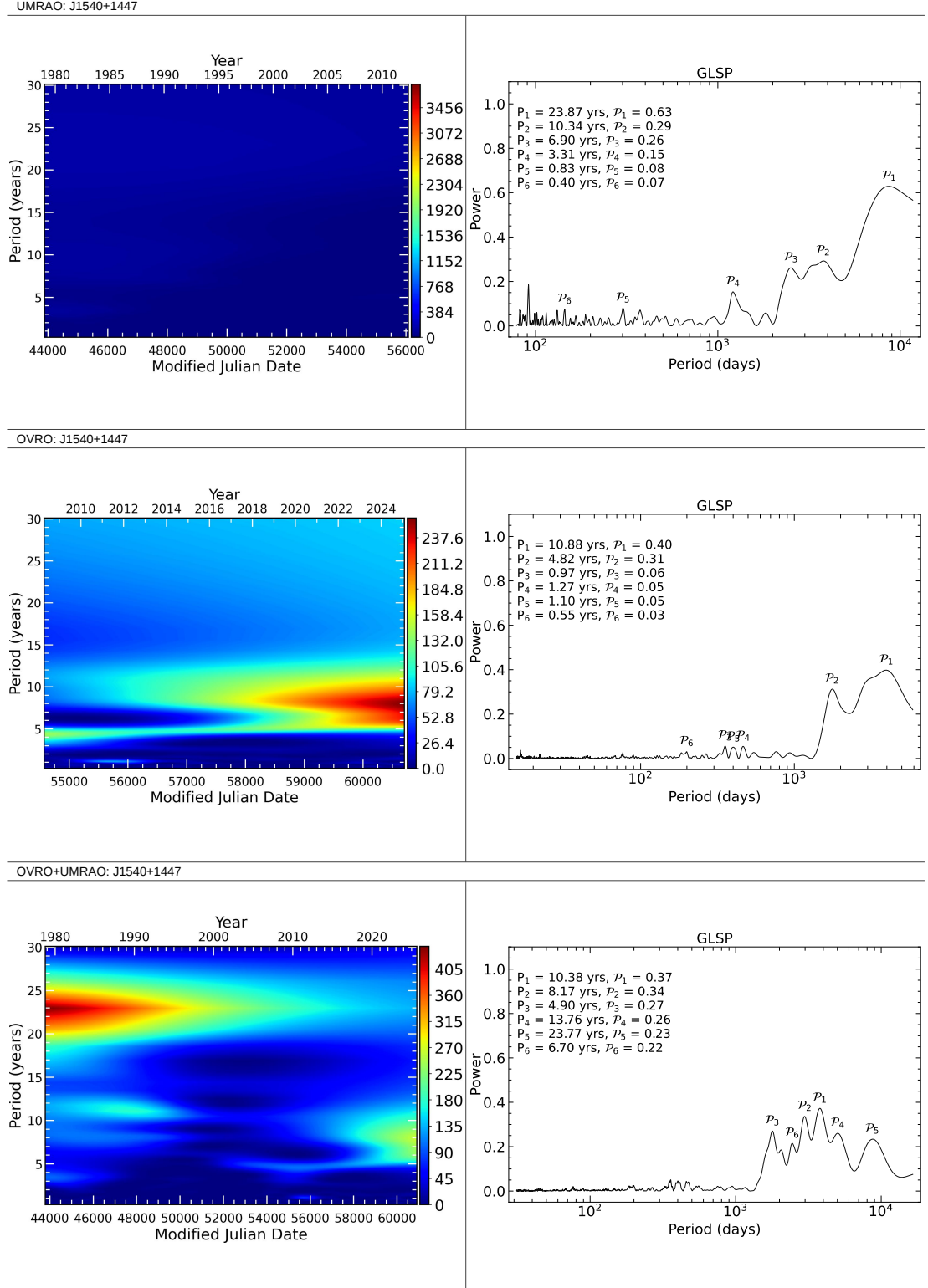
Figure A.50: J1310+3220, (RA = 197.619°, DEC = 32.345°), $z = 0.995$.

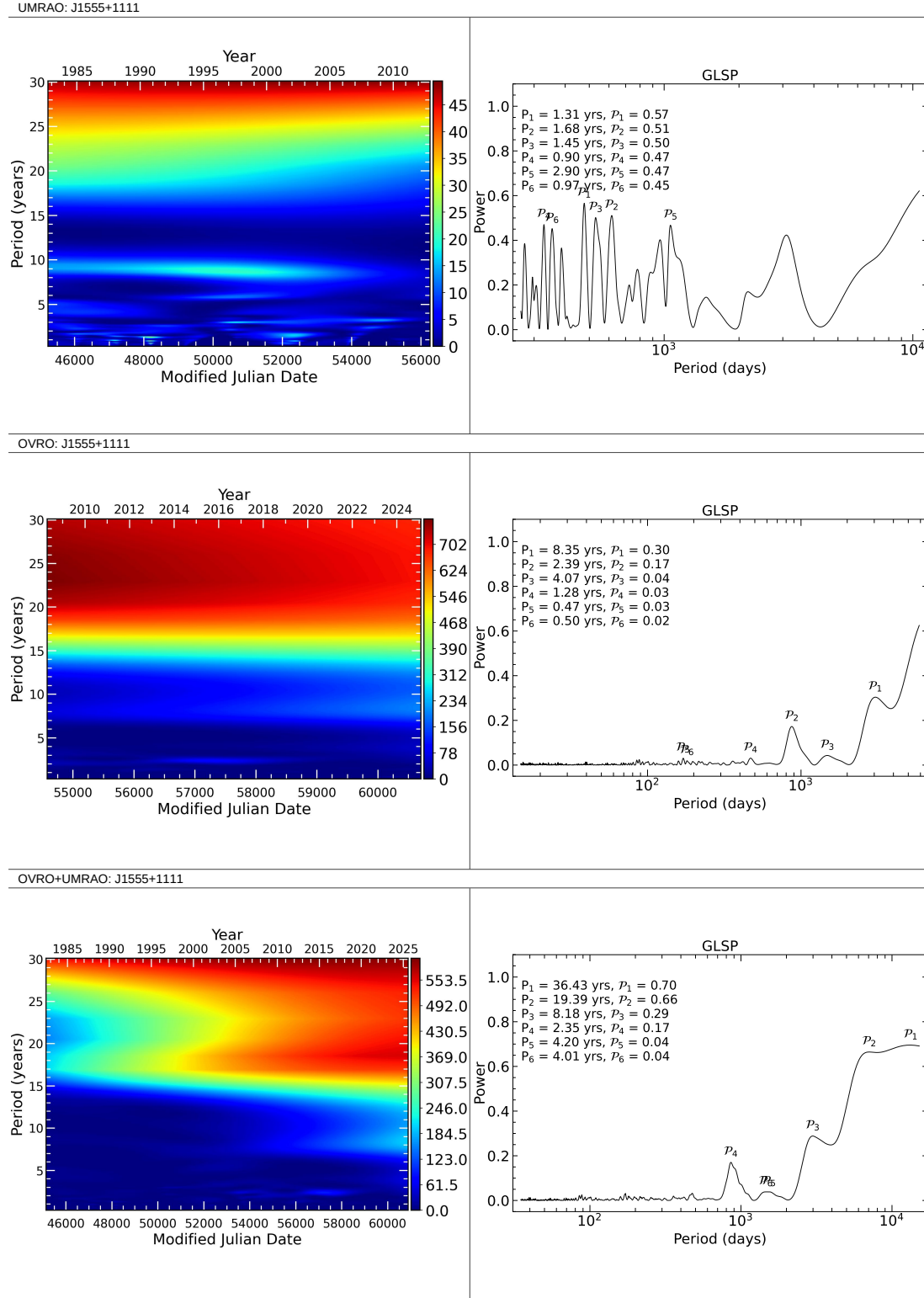
Figure A.51: J1337-1257, (RA = 204.416°, DEC = -12.957°), $z = 0.539$.

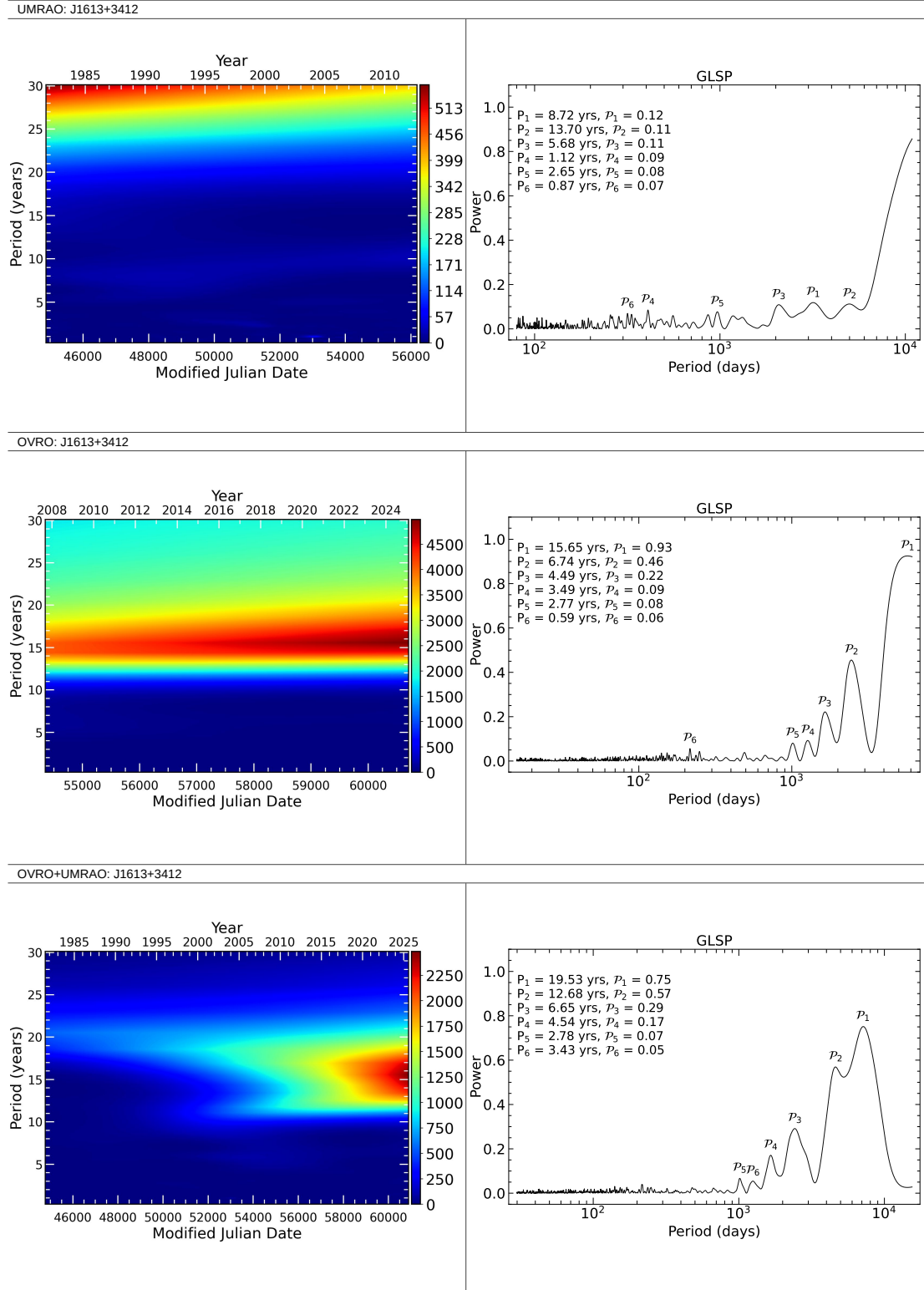
Figure A.52: J1415+1320, (RA = 213.995°, DEC = 13.340°), $z = 0.247$.

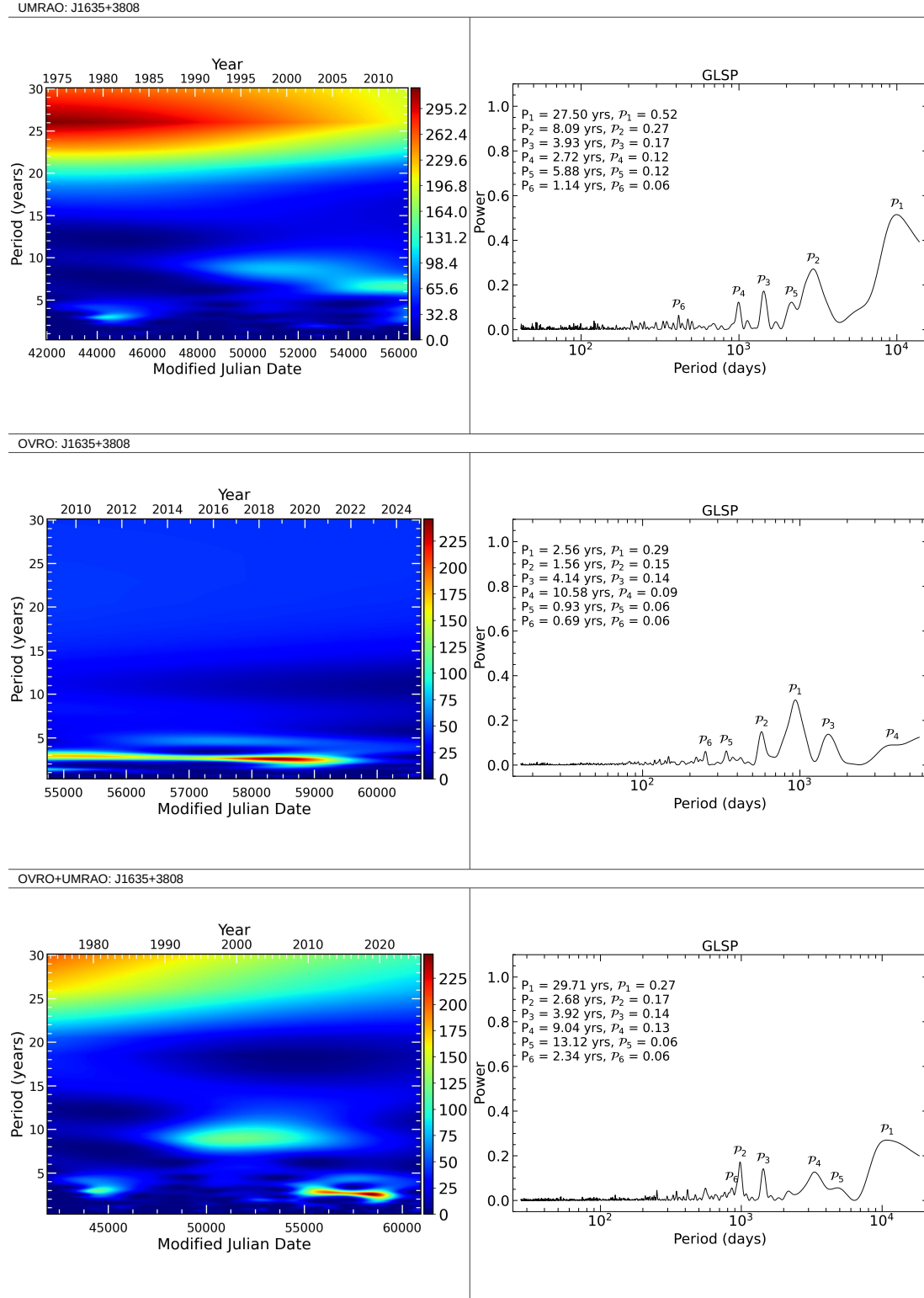
Figure A.53: J1419+5423, (RA = 214.944°, DEC = 54.387°), $z = 0.152$.

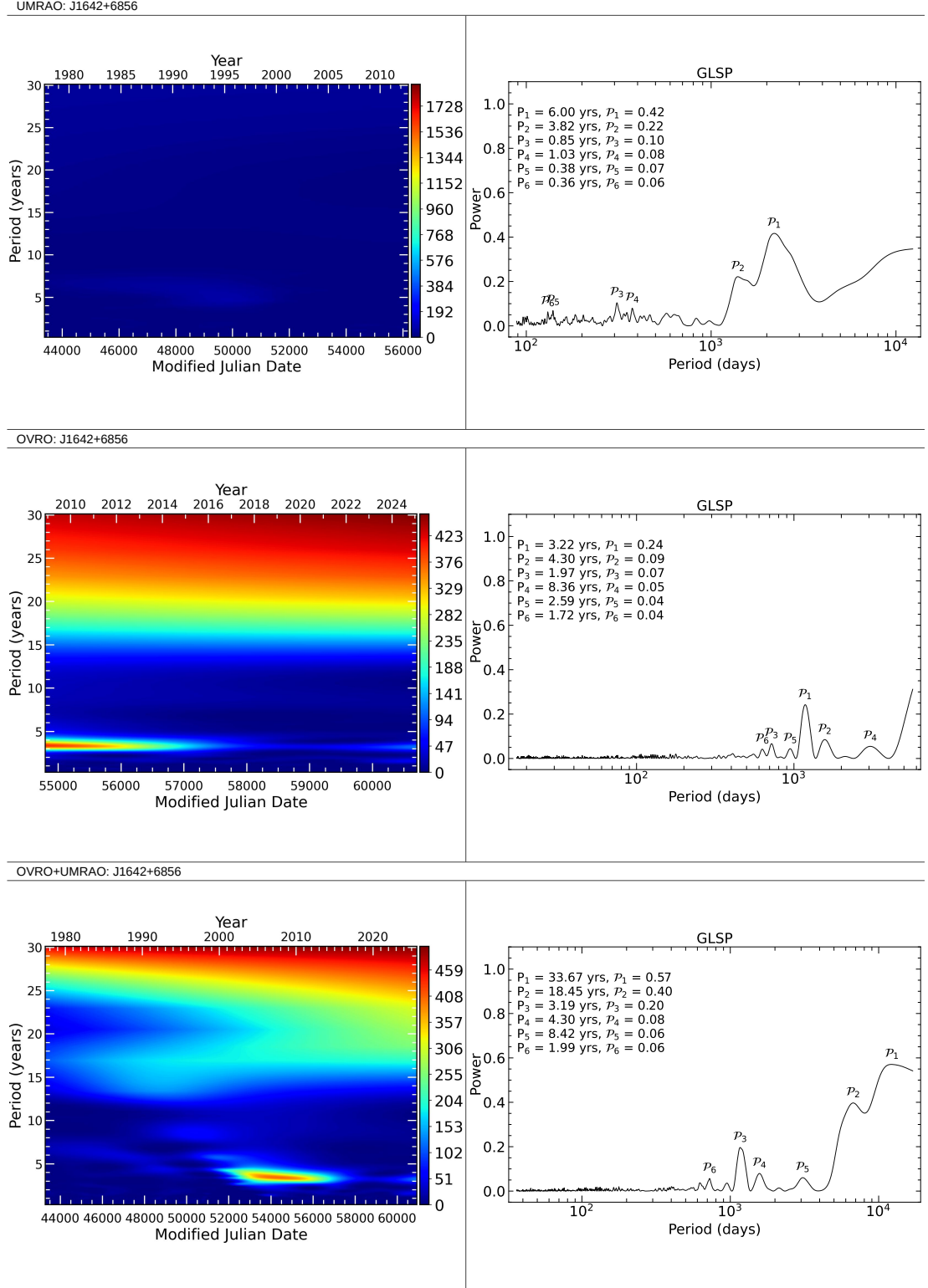
Figure A.54: J1512-0905, (RA = 228.211°, DEC = -9.100°), $z = 0.360$.

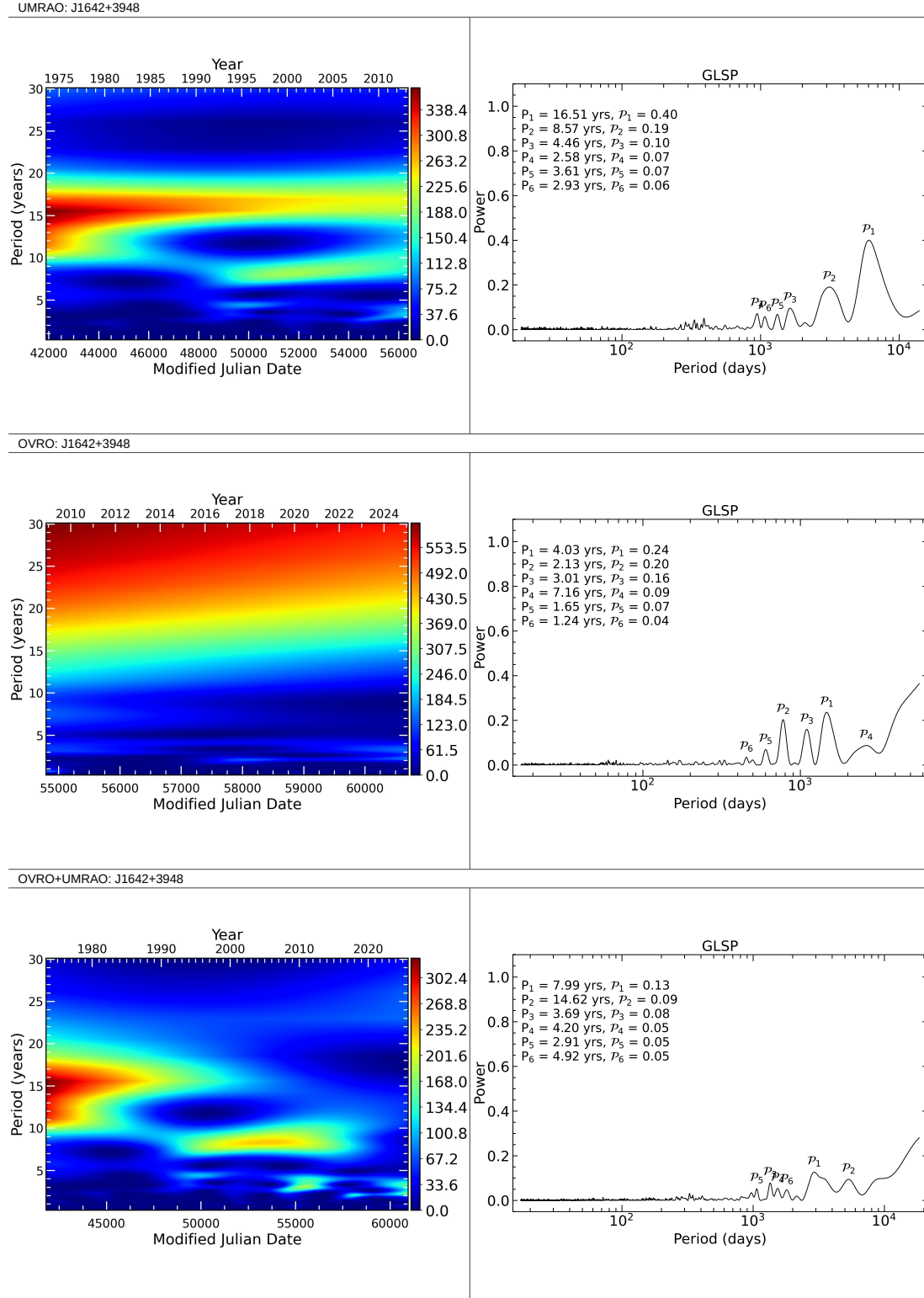
Figure A.55: J1540+1447, (RA = 235.206°, DEC = 14.796°), $z = 0.605$.

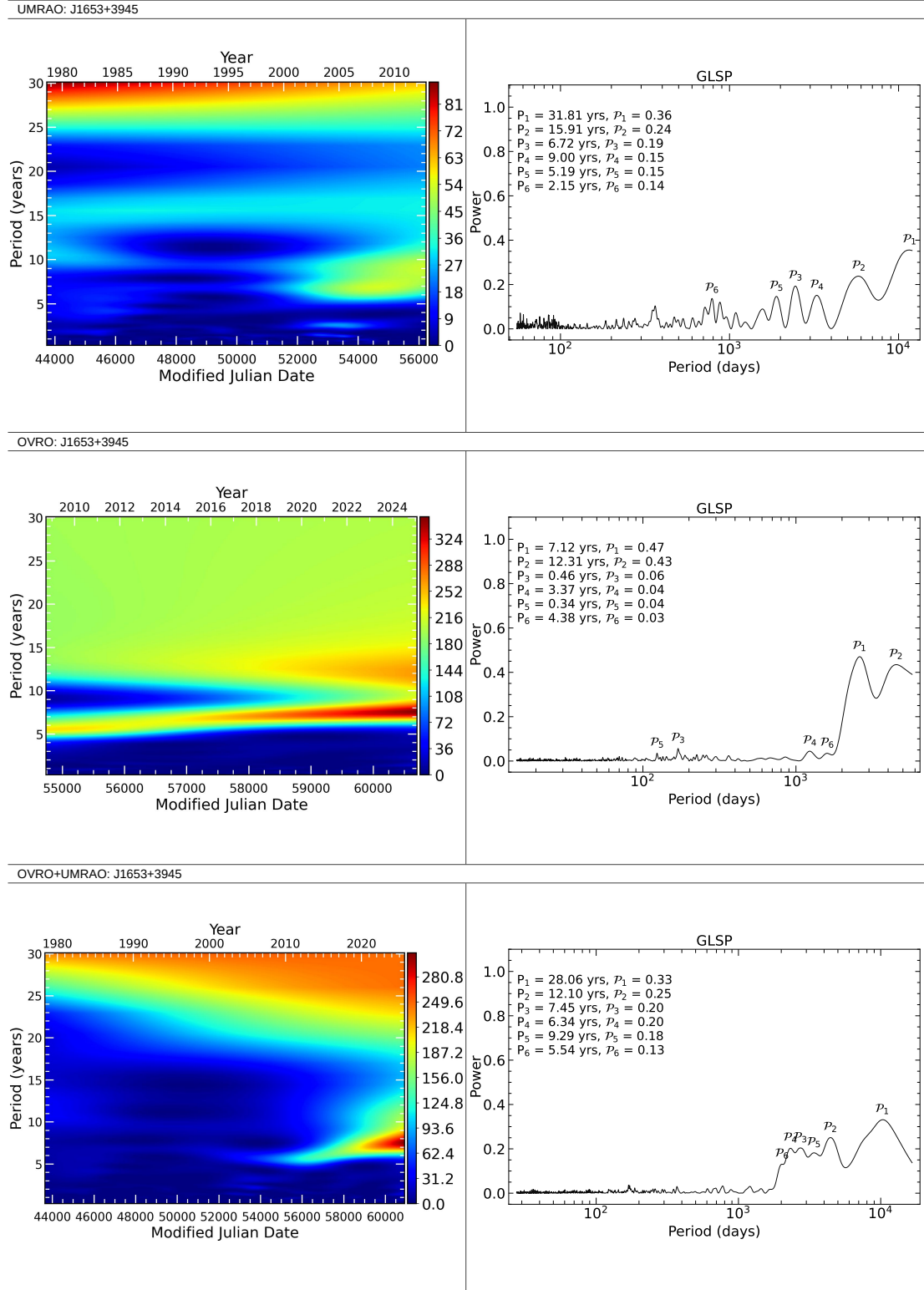
Figure A.56: J1555+1111, (RA = 238.929°, DEC = 11.190°), $z = 0.433$.

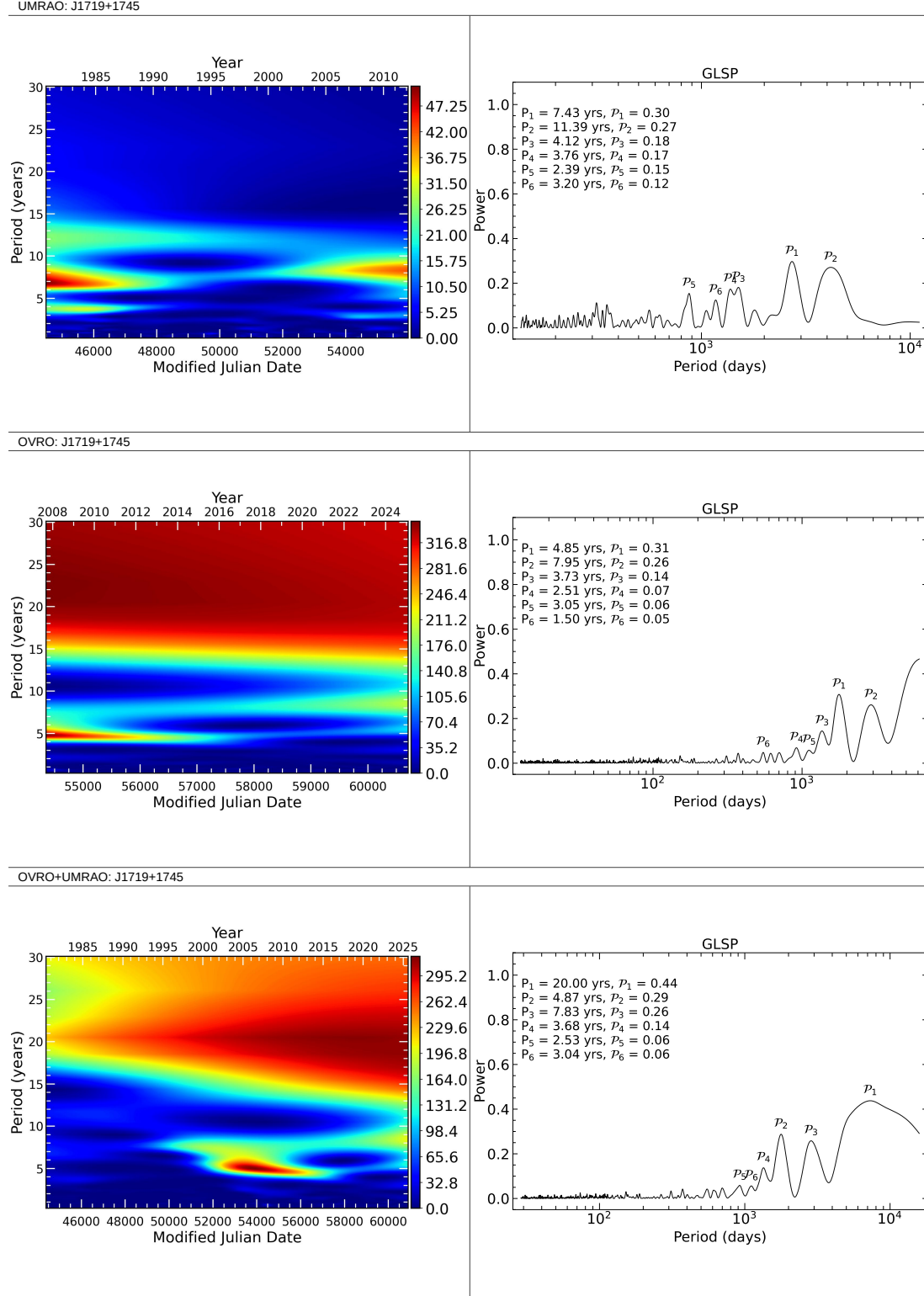
Figure A.57: J1613+3412, (RA = 243.421°, DEC = 34.213°), $z = 1.399$.

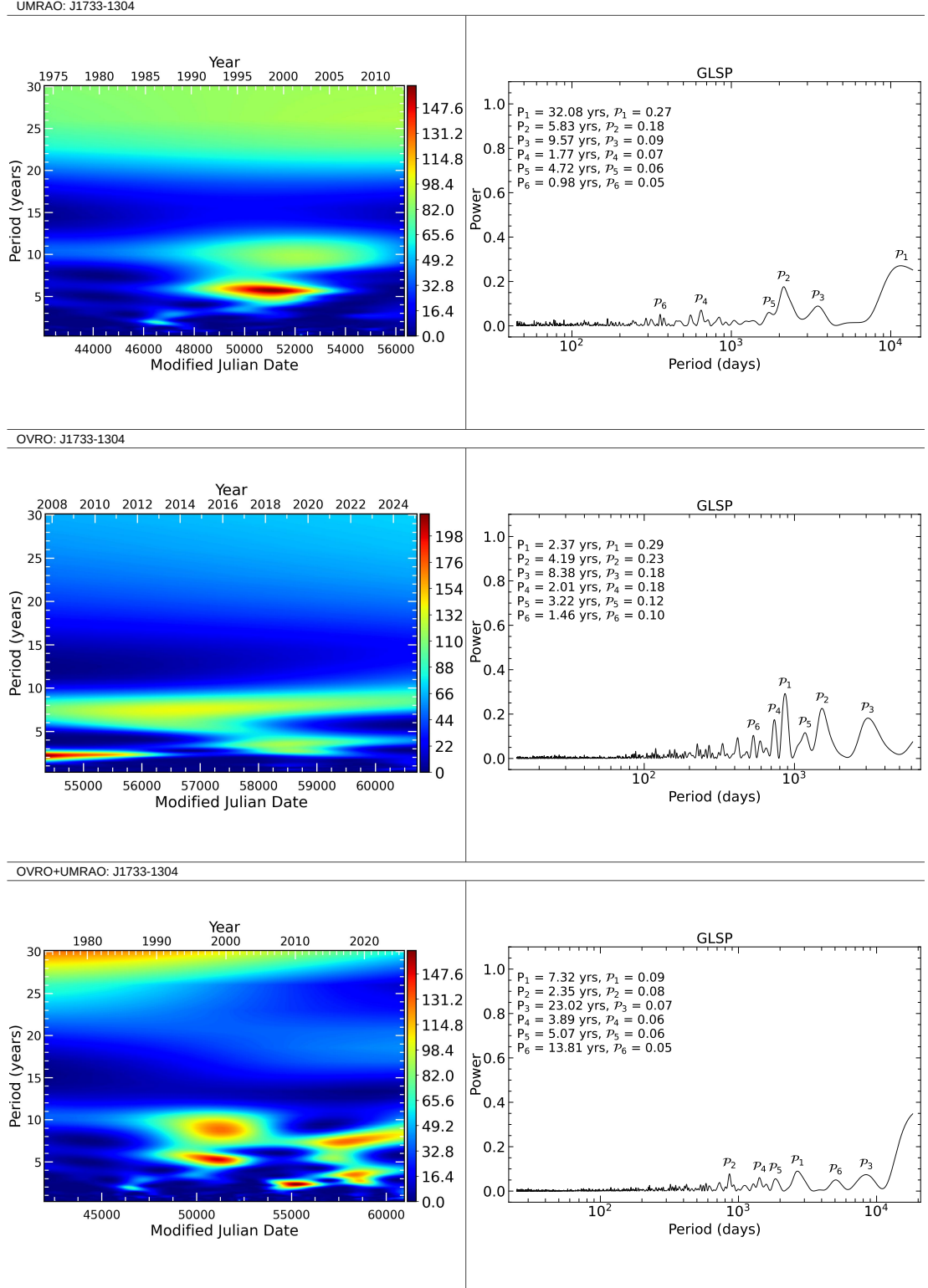
Figure A.58: J1635+3808, (RA = 248.815°, DEC = 38.134°), $z = 1.815$.

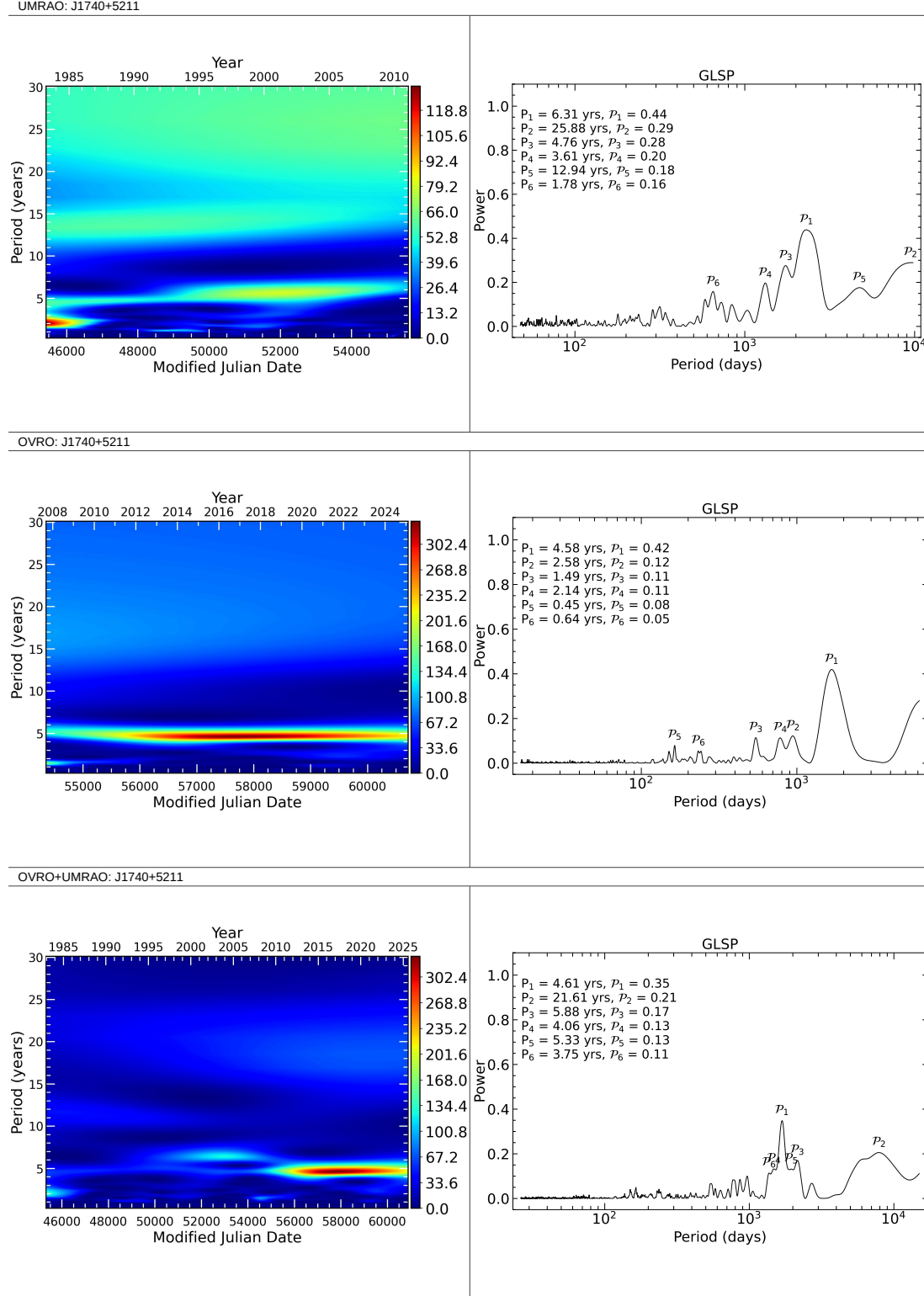
Figure A.59: J1642+6856, (RA = 250.533°, DEC = 68.944°), $z = 0.751$.

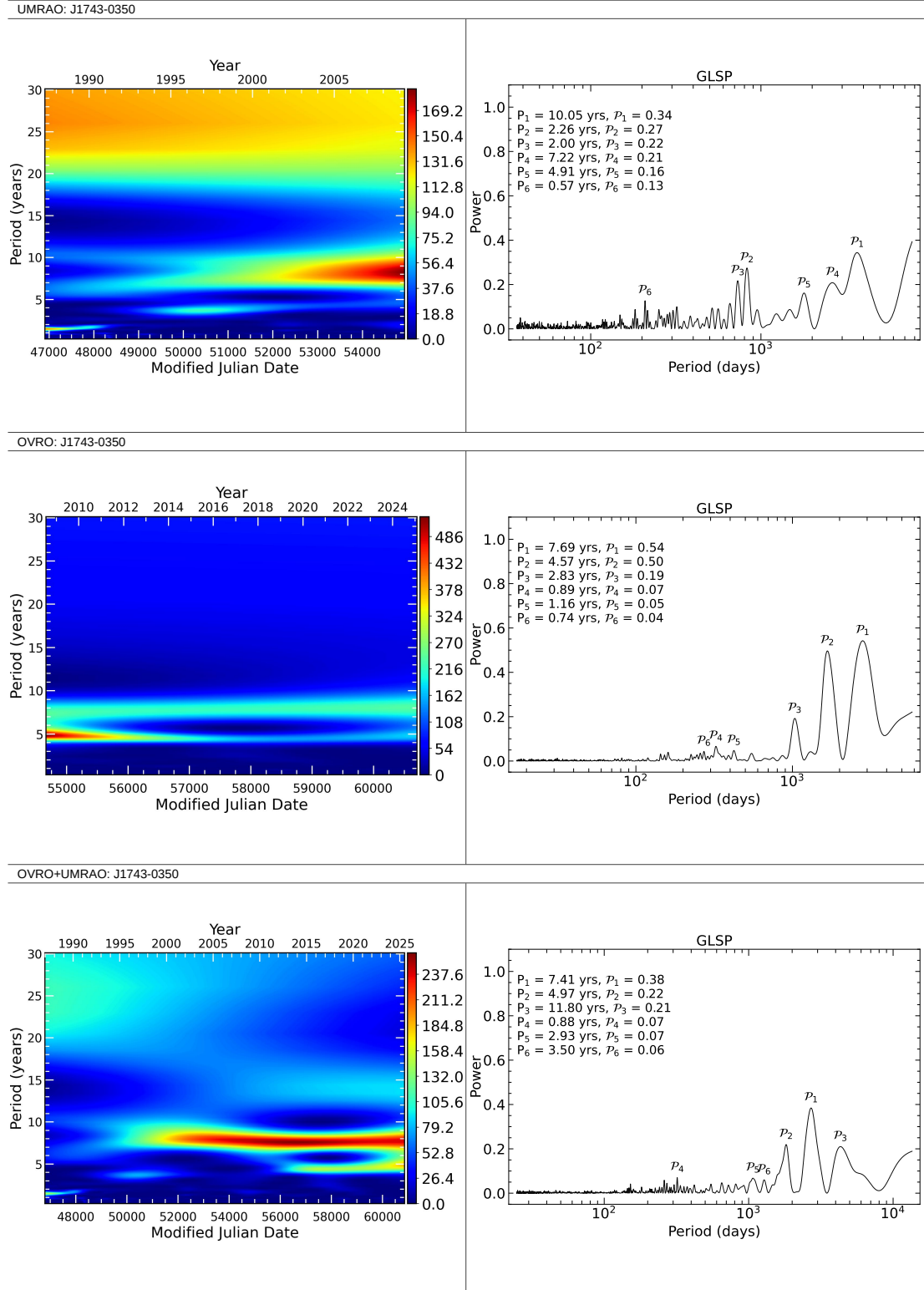
Figure A.60: J1642+3948, (RA = 250.745°, DEC = 39.810°), $z = 0.593$.

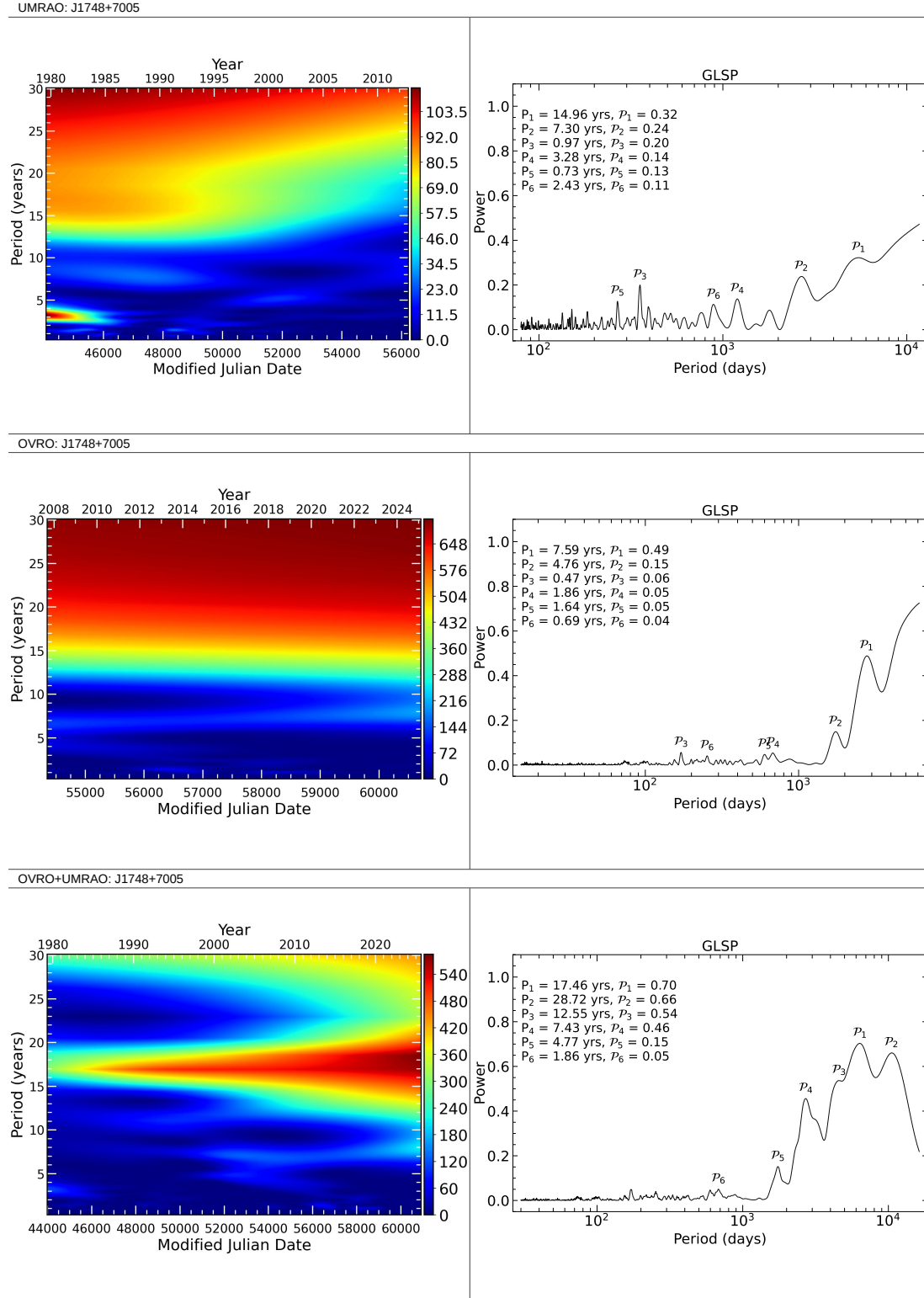
Figure A.61: J1653+3945, (RA = 253.468°, DEC = 39.760°), $z = 0.034$.

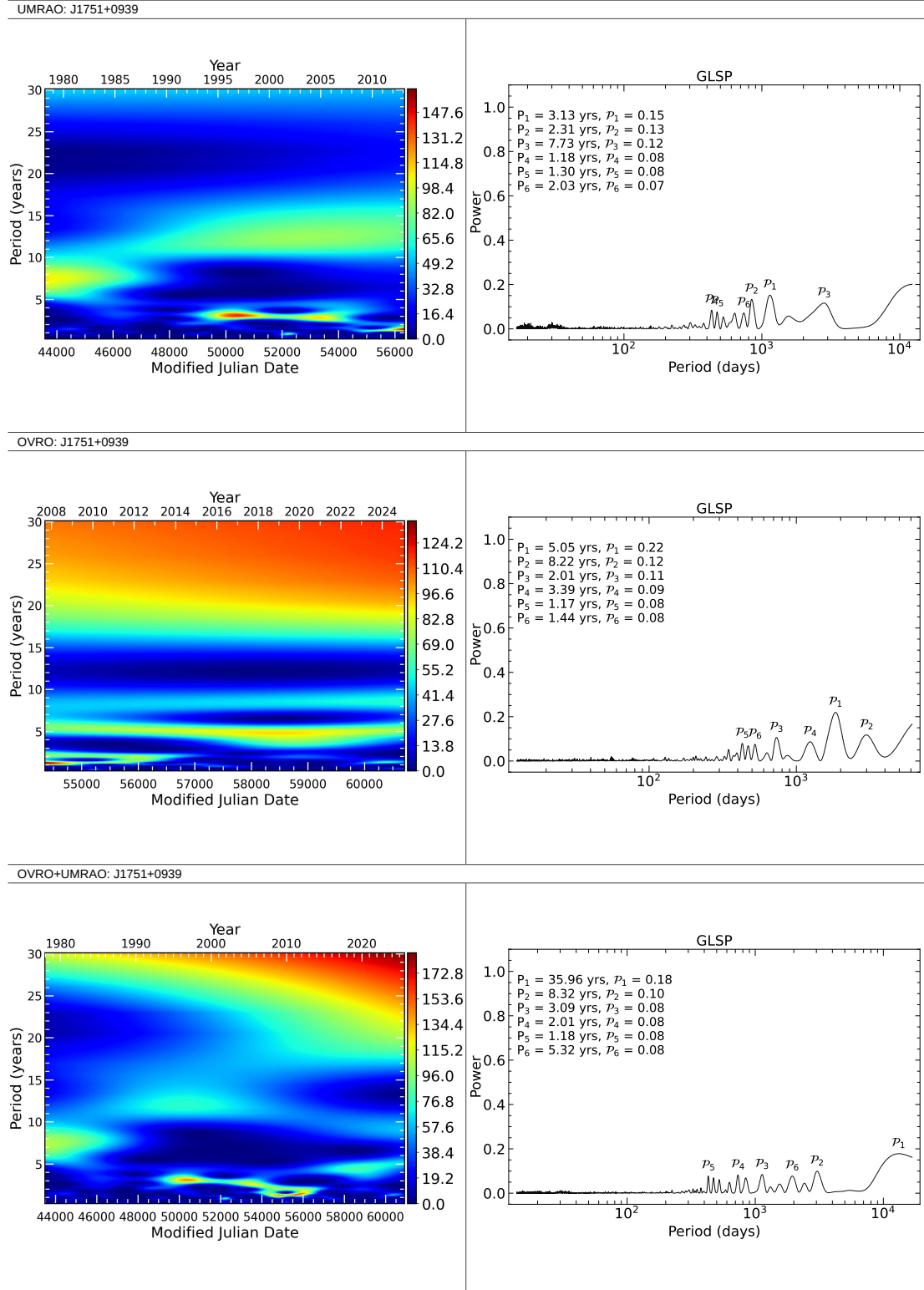
Figure A.62: J1719+1745, (RA = 259.804°, DEC = 17.752°), $z = 0.137$.

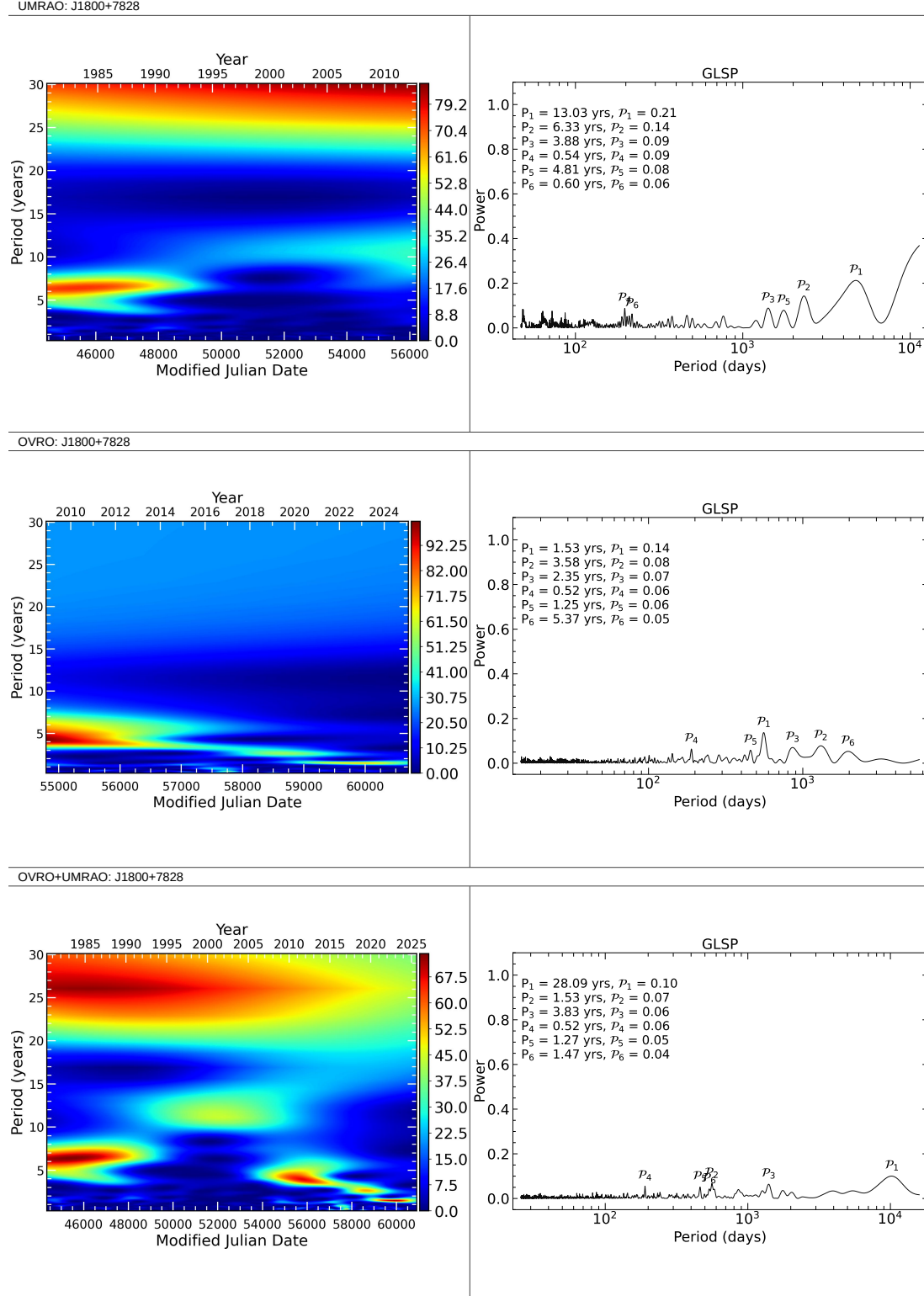
Figure A.63: J1733-1304, (RA = 263.261°, DEC = −13.080°), $z = 0.902$.

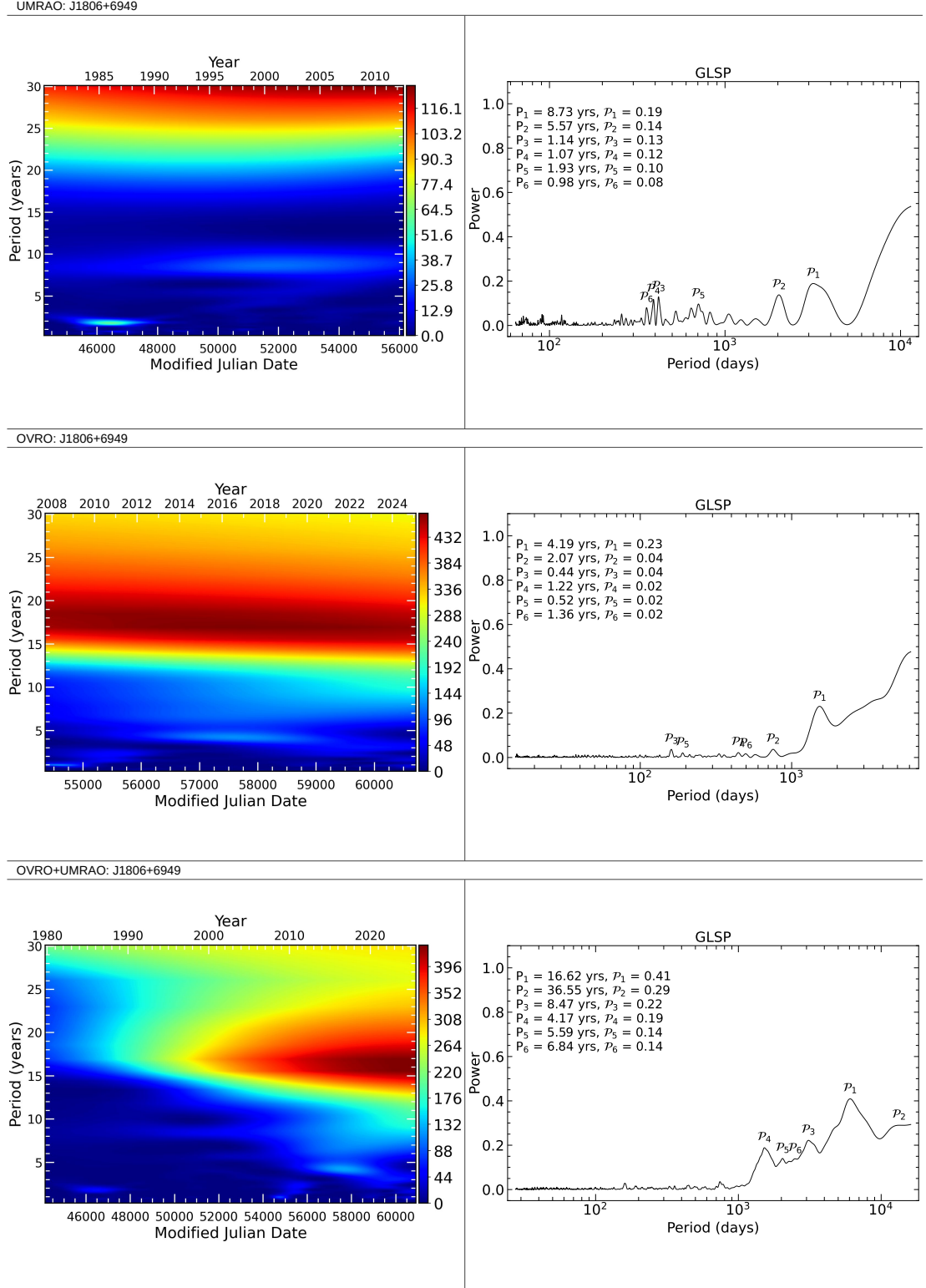
Figure A.64: J1740+5211, (RA = 265.154°, DEC = 52.195°), $z = 1.379$.

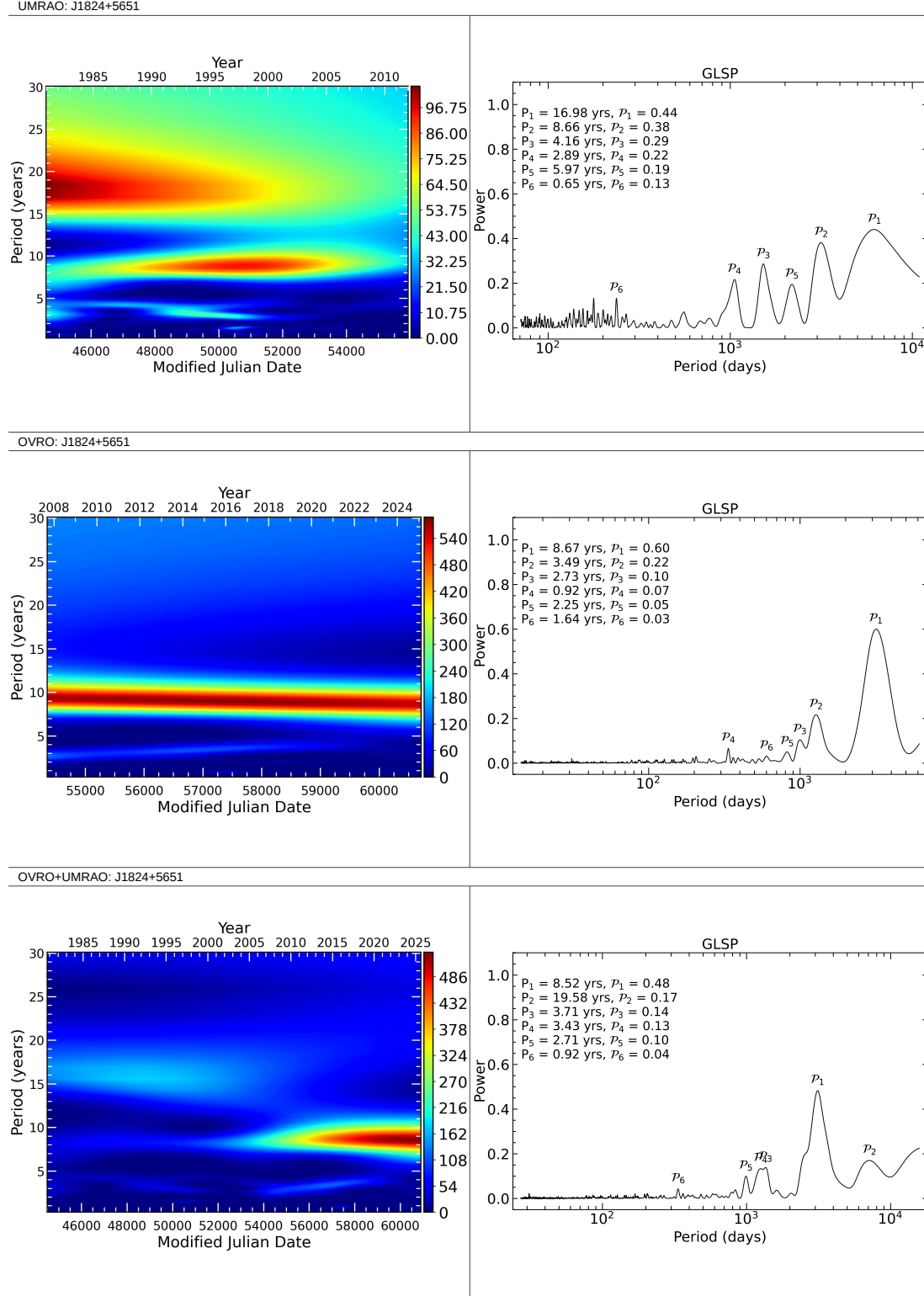
Figure A.65: J1743-0350, (RA = 265.995°, DEC = -3.834°), $z = 1.054$.

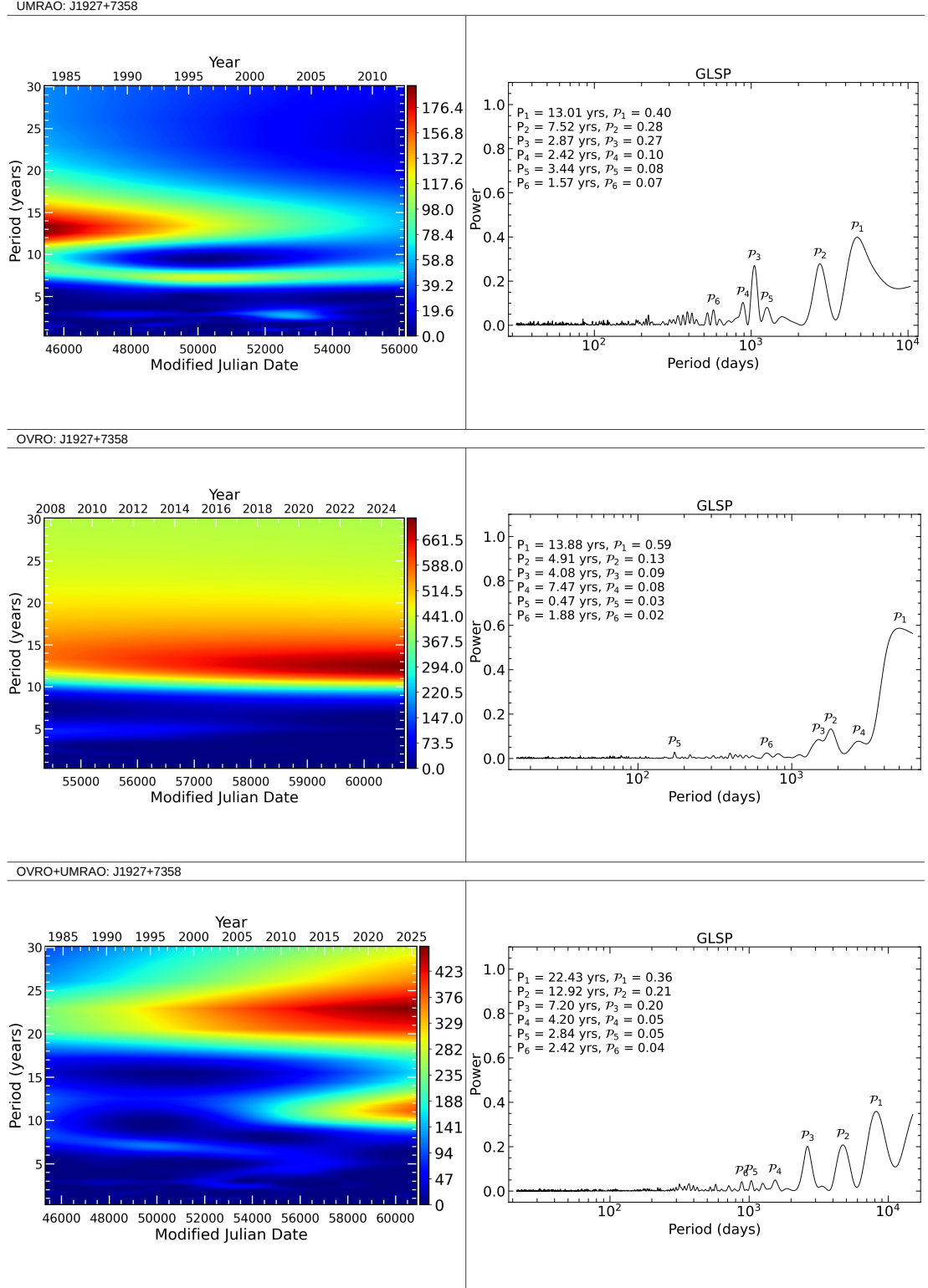
Figure A.66: J1748+7005, (RA = 267.137°, DEC = 70.097°), $z = 0.770$.

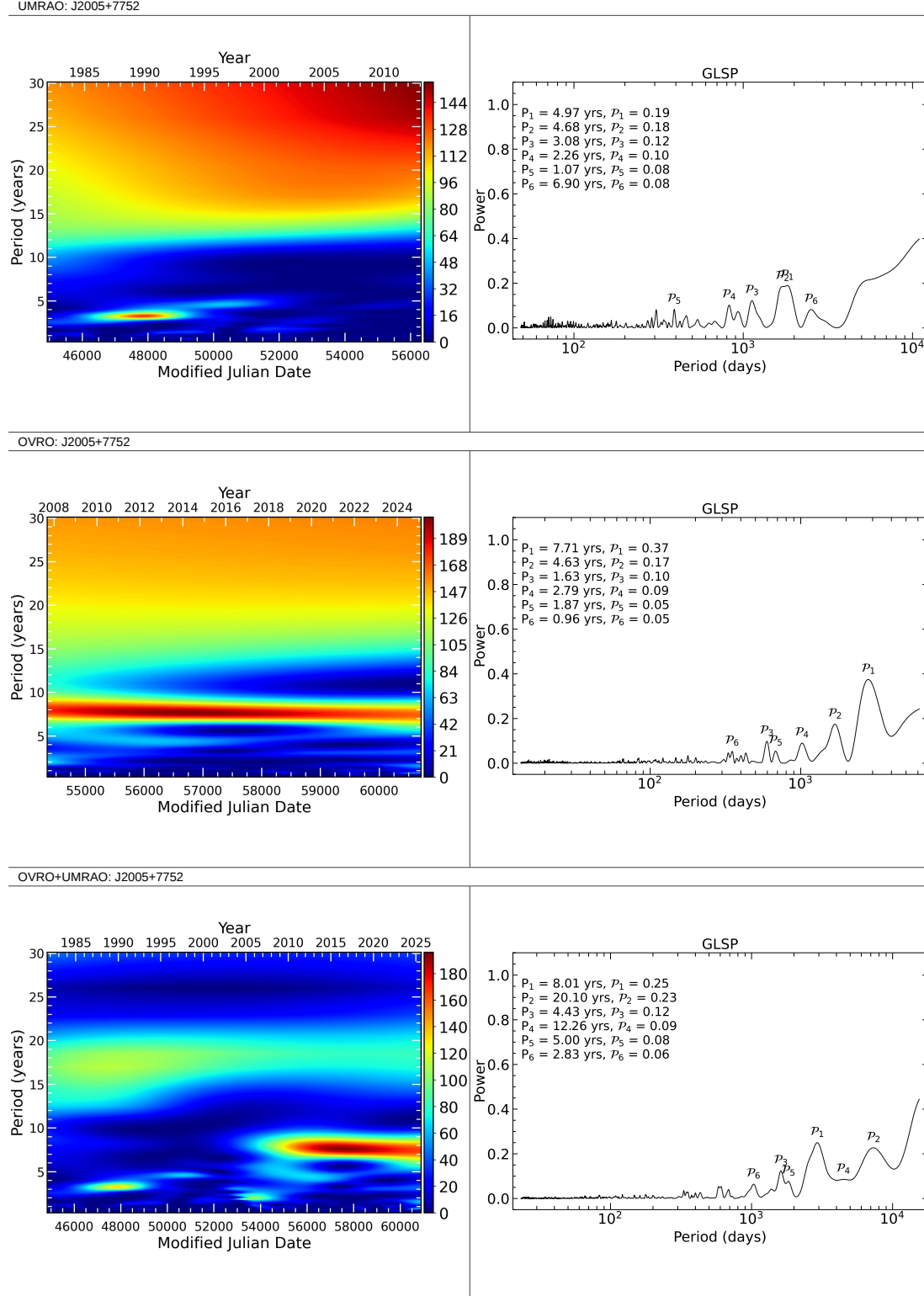
Figure A.67: J1751+0939, (RA = 267.887°, DEC = 9.650°), $z = 0.320$.

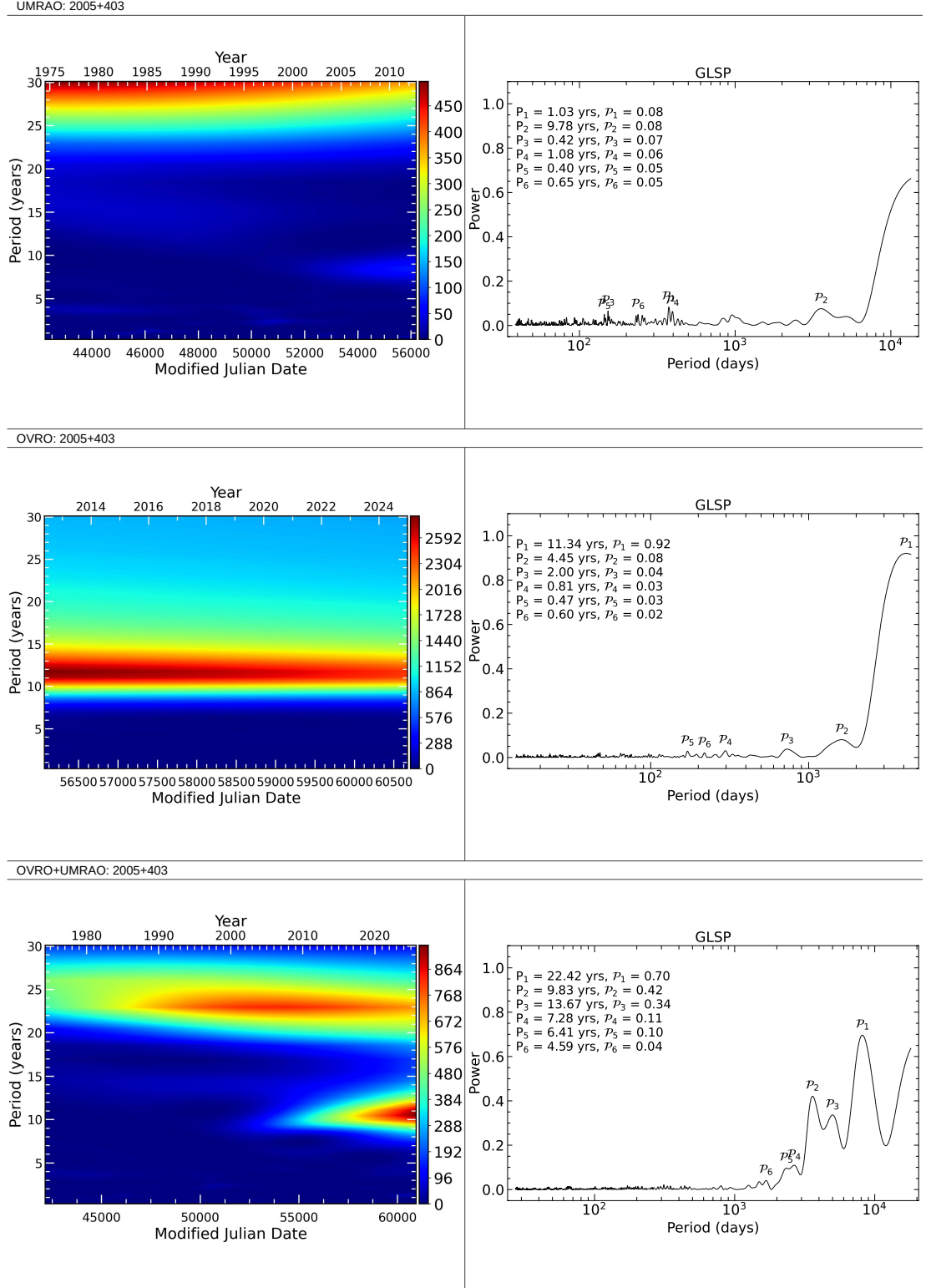
Figure A.68: J1800+7828, (RA = 270.190°, DEC = 78.468°), $z = 0.680$.

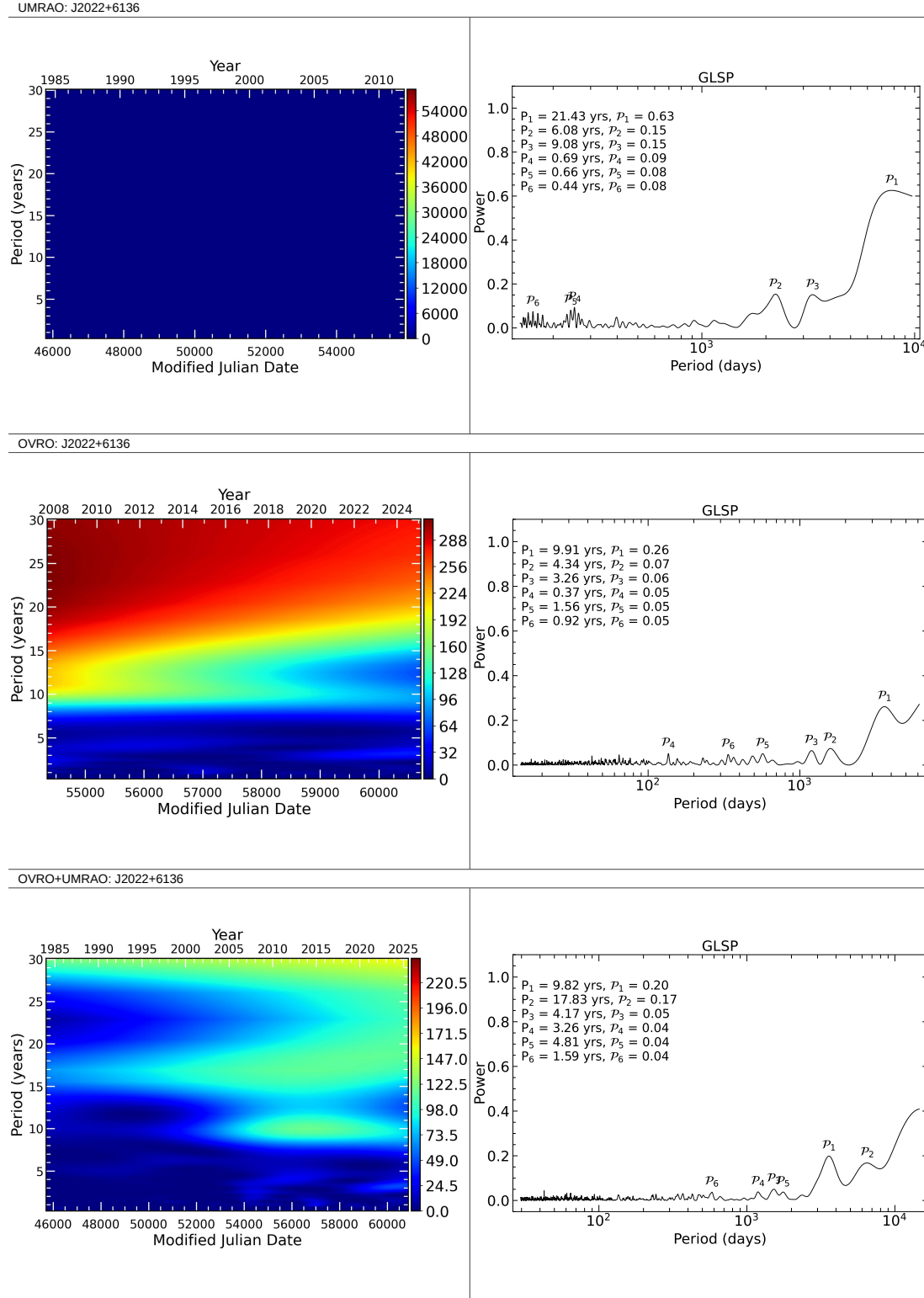
Figure A.69: J1806+6949, (RA = 271.711°, DEC = 69.824°), $z = 0.051$.

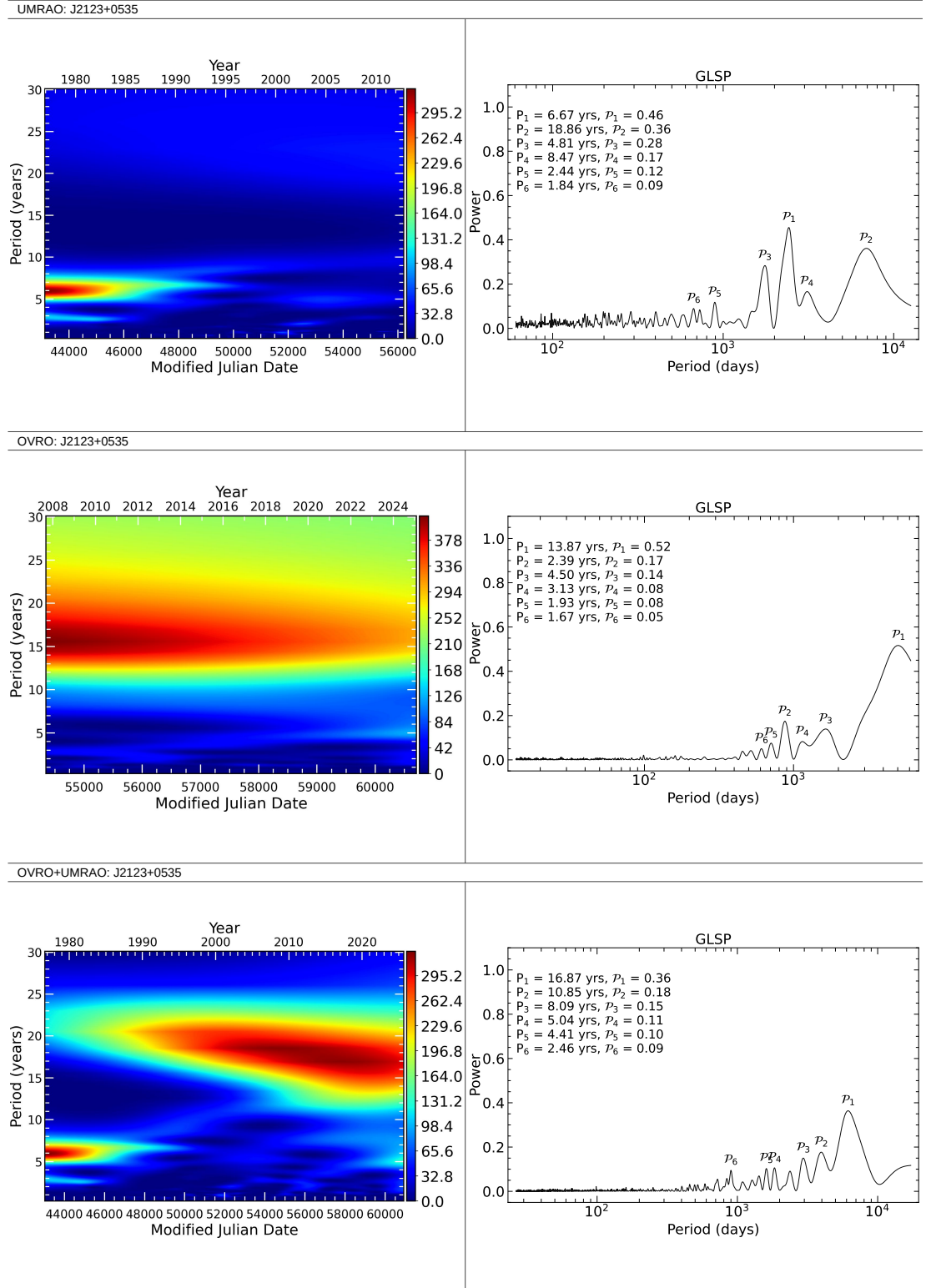
Figure A.70: J1824+5651, (RA = 276.029°, DEC = 56.850°), $z = 0.663$.

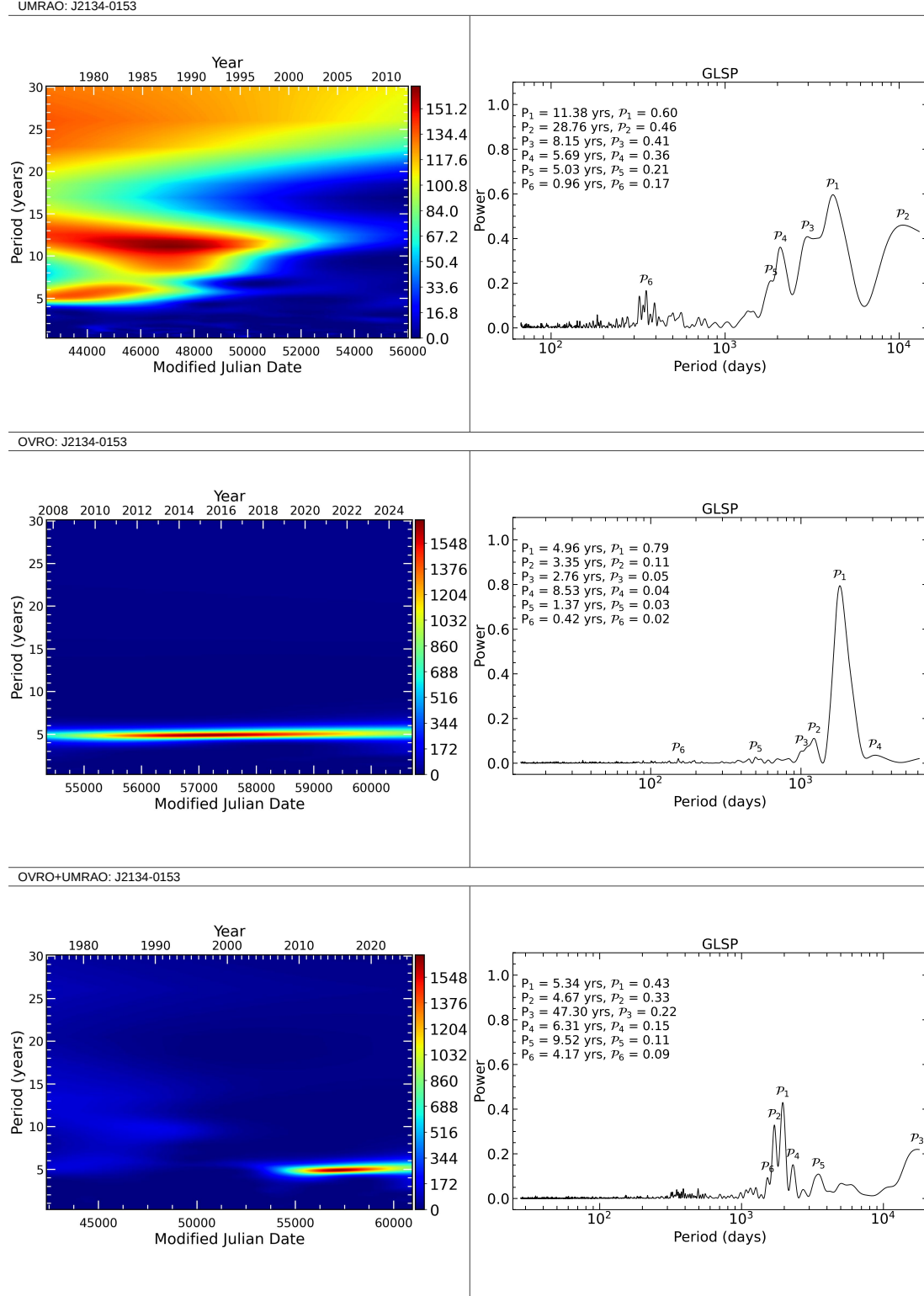
Figure A.71: J1927+7358, (RA = 291.952°, DEC = 73.967°), $z = 0.302$.

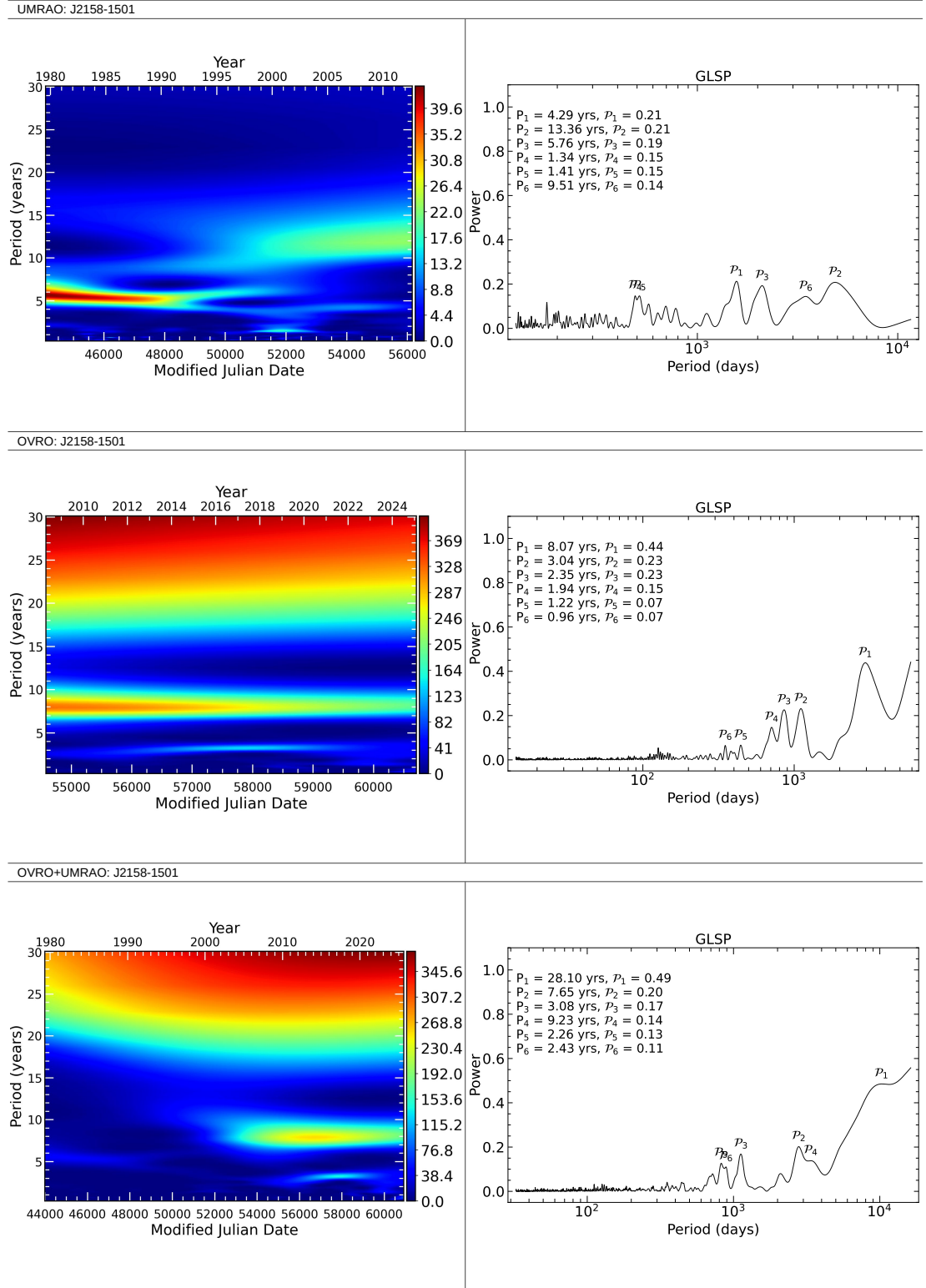
Figure A.72: J2005+7752, (RA = 301.379°, DEC = 77.879°), $z = 0.342$.

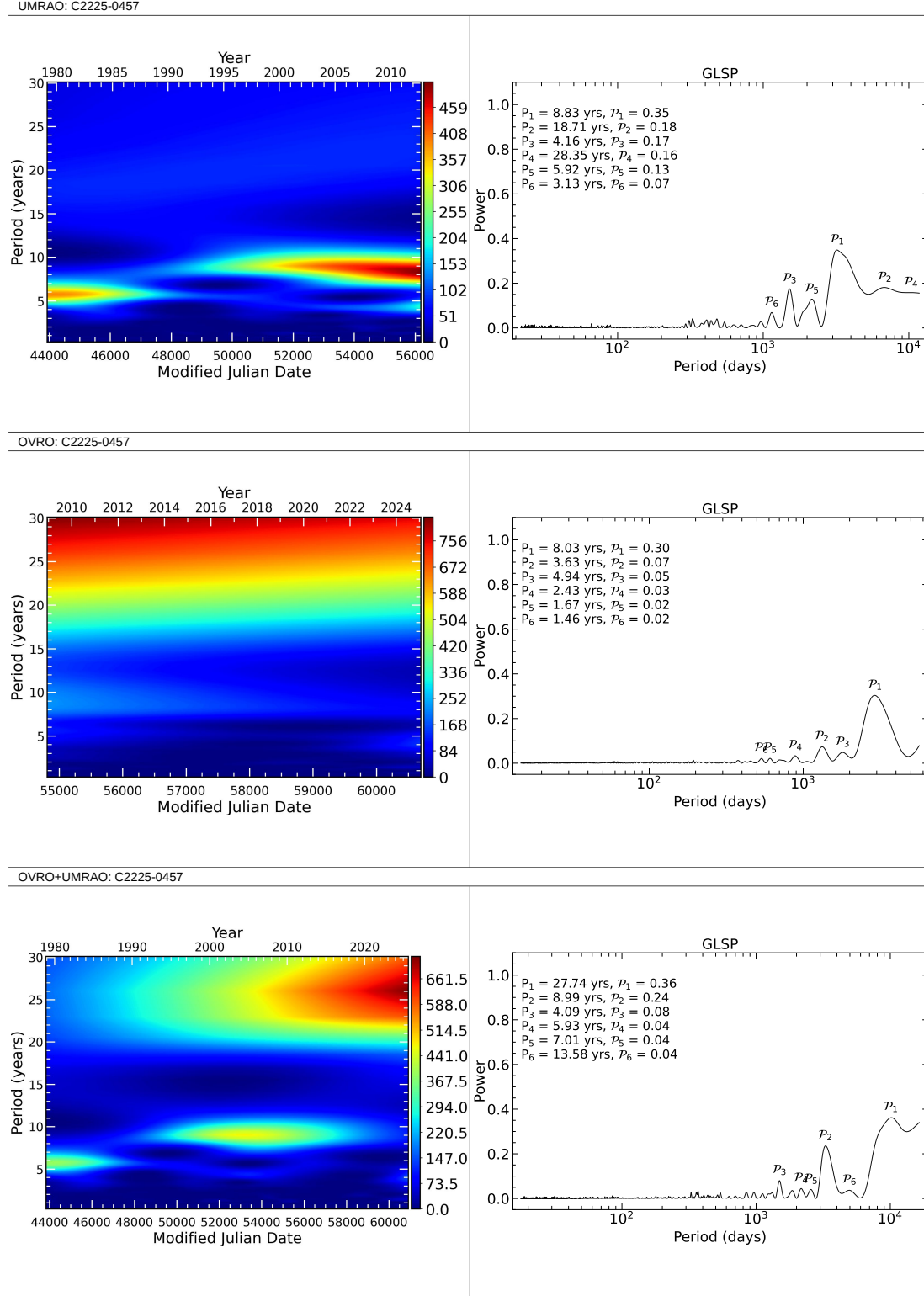
Figure A.73: J2007+4029, (RA = 301.937°, DEC = 40.497°), $z = 1.736$.

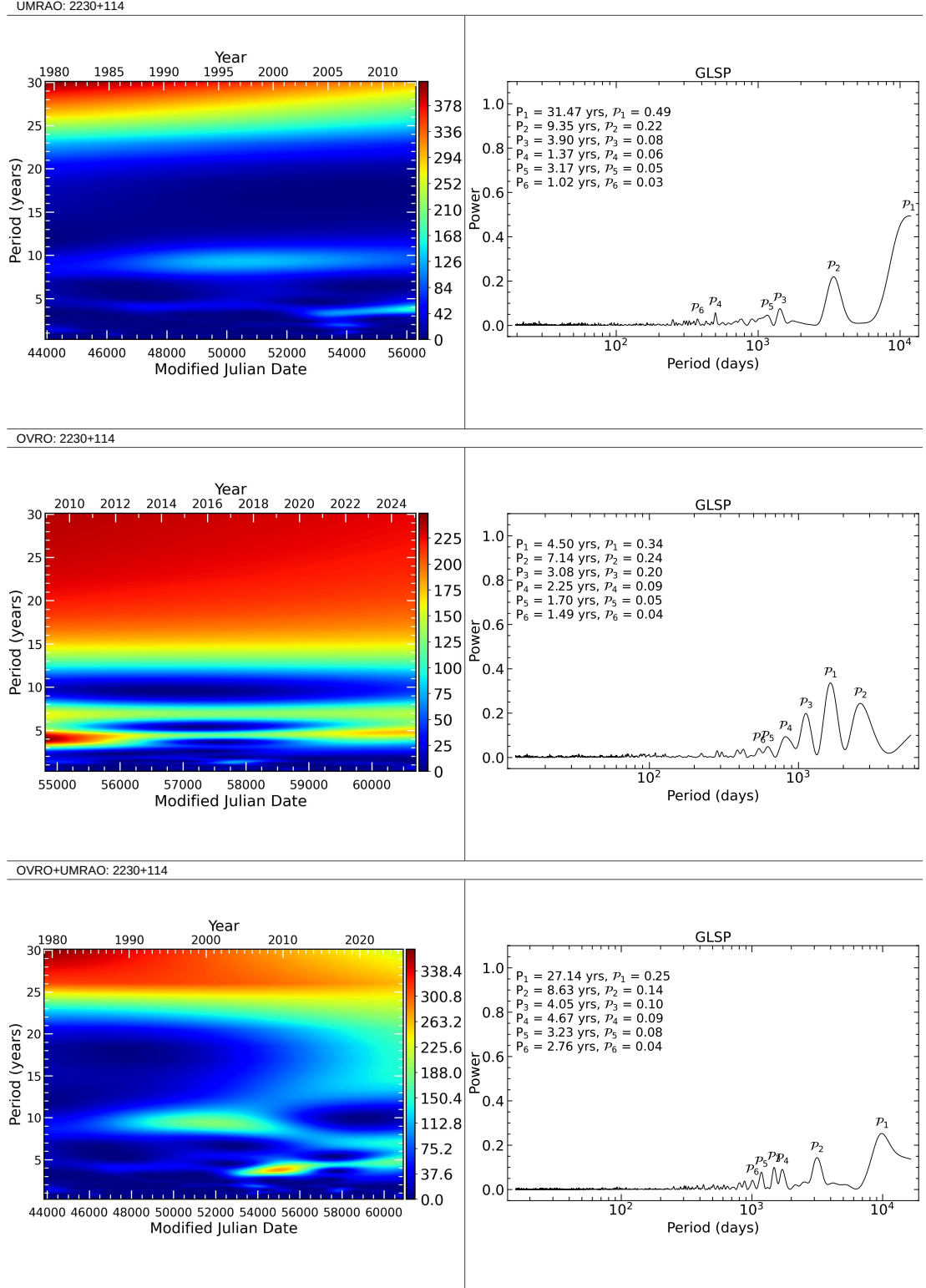
Figure A.74: J2022+6136, (RA = 305.528°, DEC = 61.616°), $z = 0.227$.

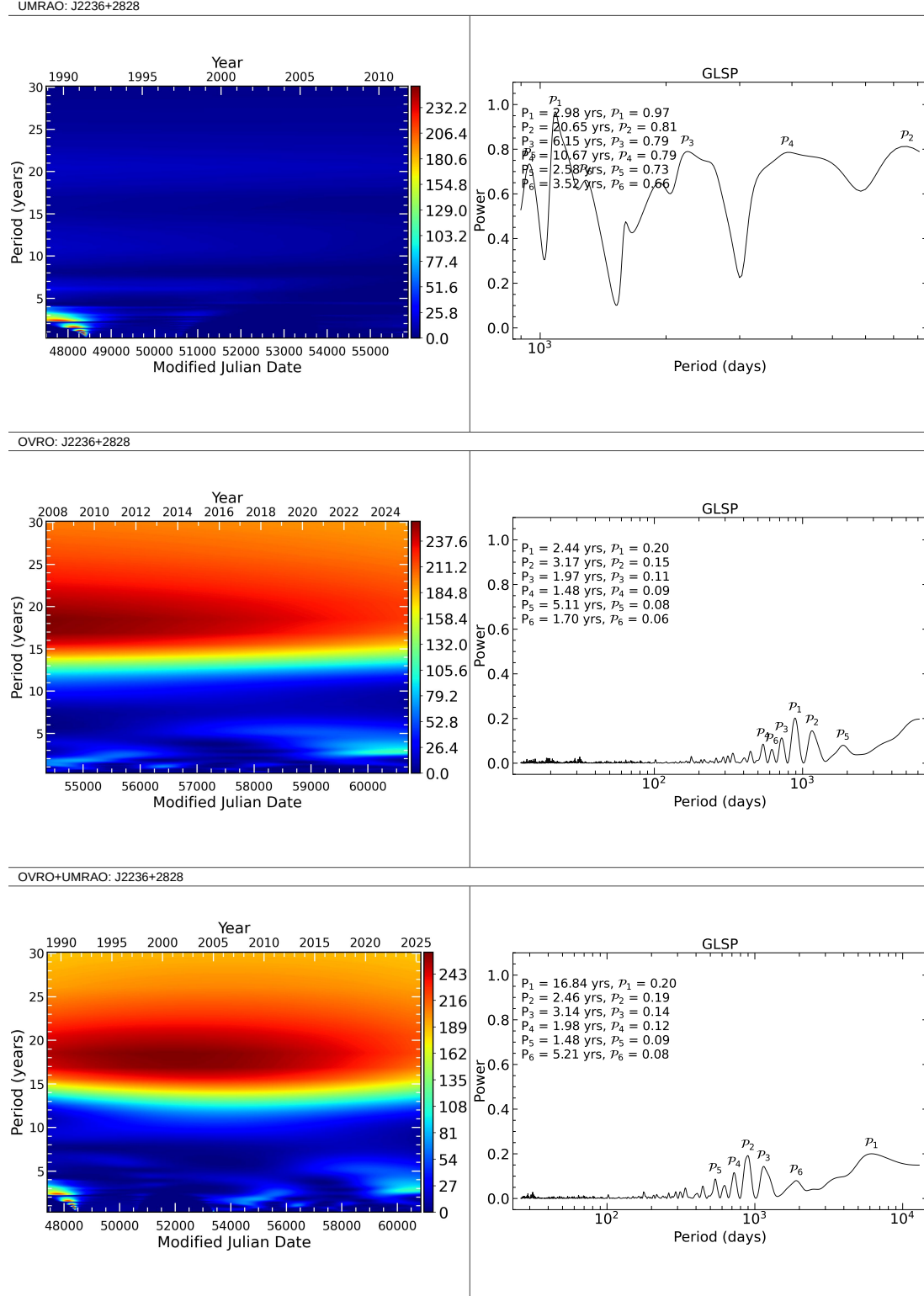
Figure A.75: J2123+0535, (RA = 320.935°, DEC = 5.589°), $z = 1.941$.

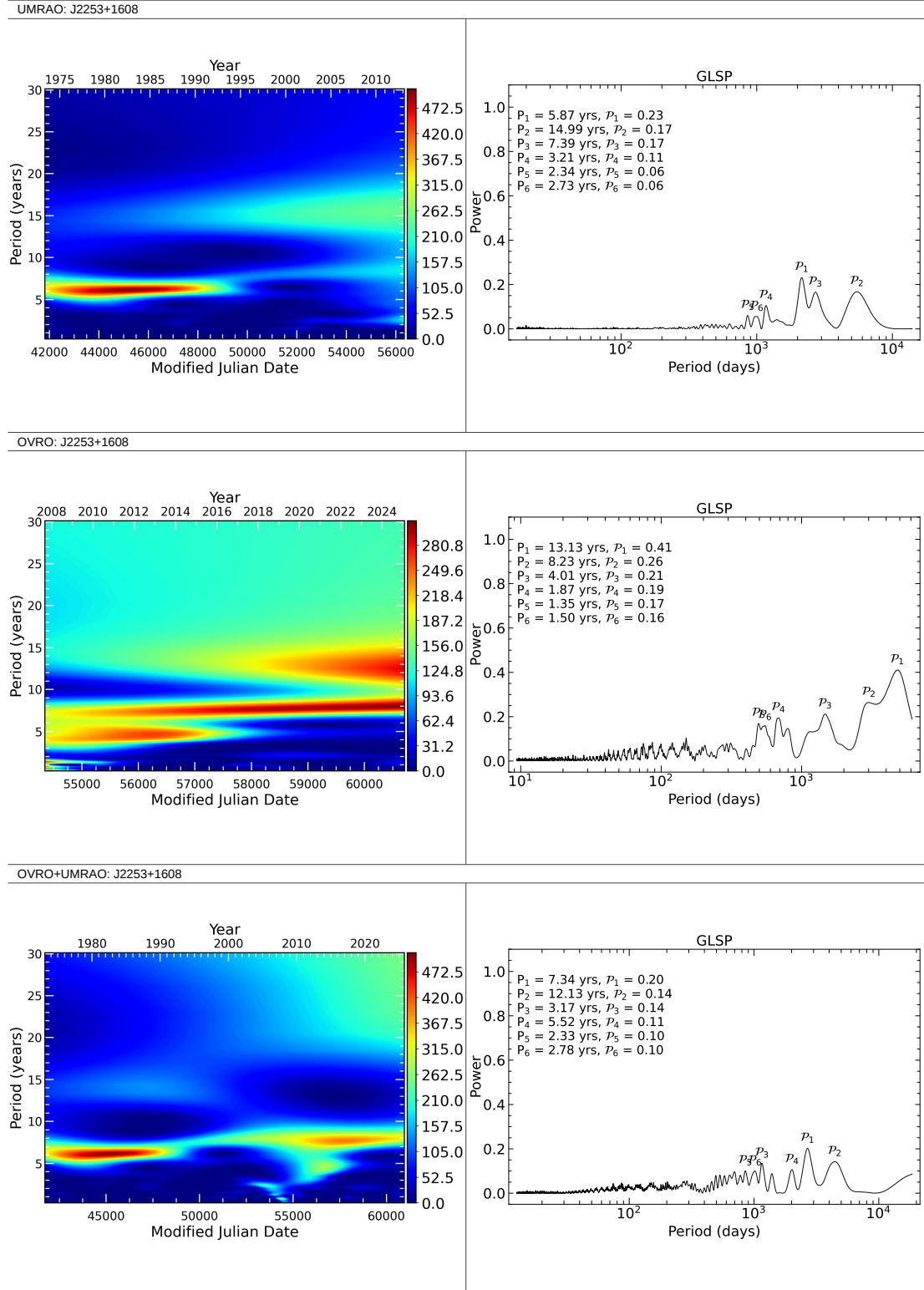
Figure A.76: J2134-0153, (RA = 323.543°, DEC = -1.888°), $z = 1.283$.

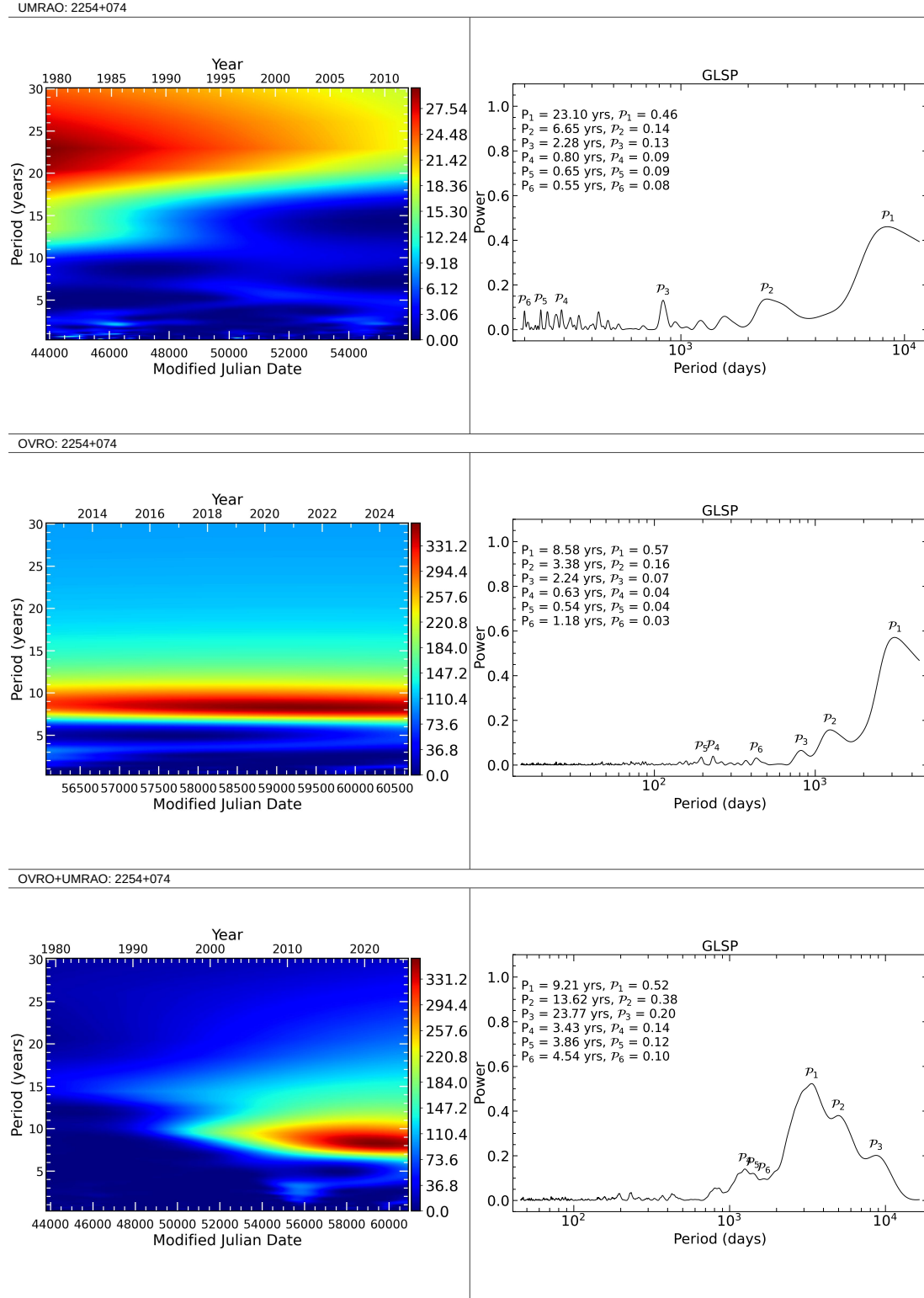
Figure A.77: J2158-1501, (RA = 329.526°, DEC = −15.019°), $z = 0.672$.

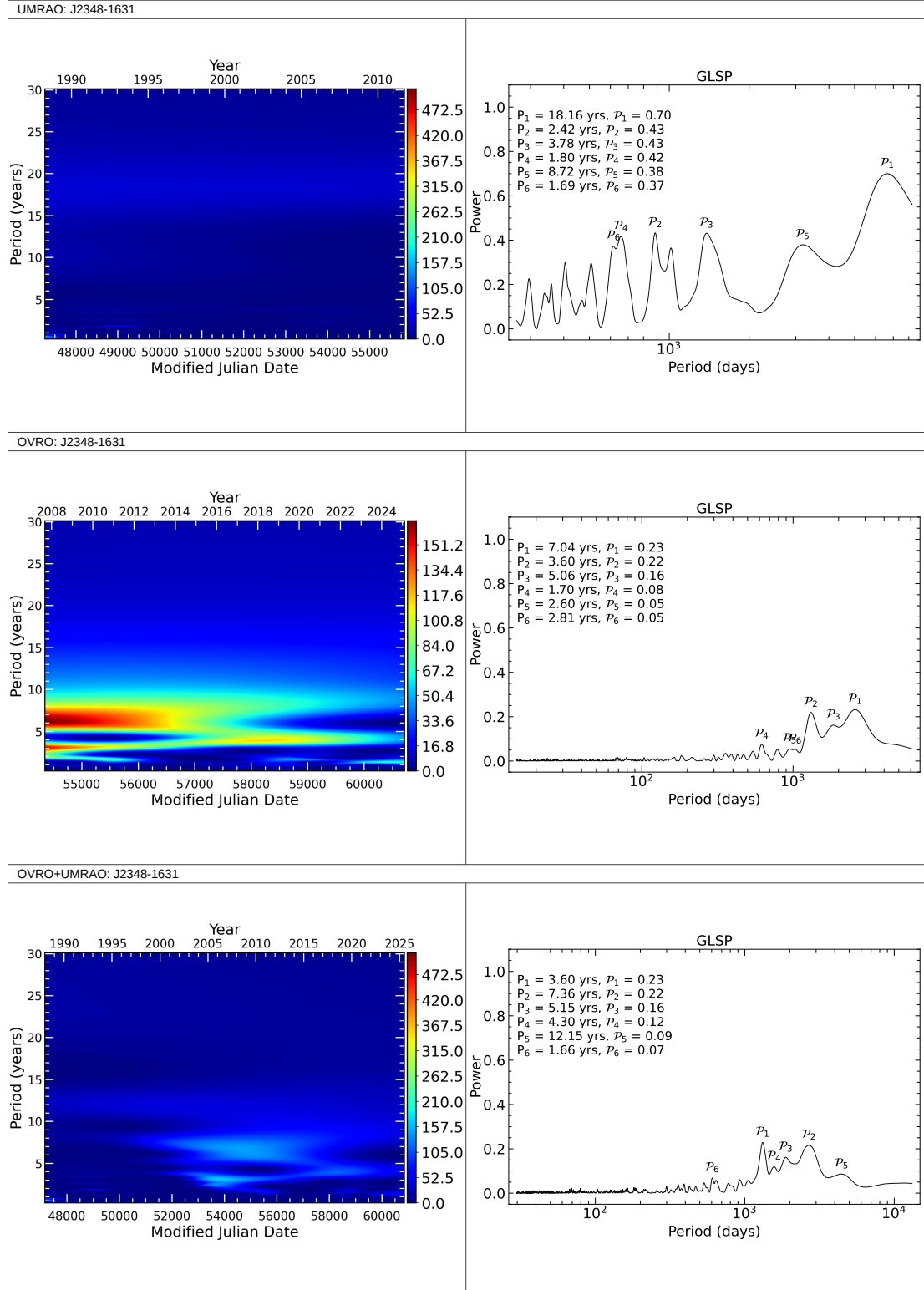
Figure A.78: J2225-0457, (RA = 336.447° , DEC = -4.950°), $z = 1.404$.

Figure A.79: J2232+1143, (RA = 338.152°, DEC = 11.731°), $z = 1.037$.



Figure A.81: J2253+1608, (RA = 343.491°, DEC = 16.148°), $z = 0.859$.

Figure A.82: J2257+0743, (RA = 344.322°, DEC = 7.720°), $z = 0.190$.

Figure A.83: J2348-1631, (RA = 357.011°, DEC = −16.520°), $z = 0.576$.

Appendix B

GLSP Comparison for OVRO, UMRAO, and Combined Data

This appendix presents the overlaid GLSP plots by telescope for the 83 sources in the sample. UMRAO, OVRO, and OVRO+UMRAO are shown in red, blue, and green, respectively. For each periodogram, the six strongest peaks are listed with their periods, frequencies, and GLSP powers. Periods are given in years and frequencies in d^{-1} .

The GLSP frequency grid was defined as

$$T = t_{\max} - t_{\min}, \quad (\text{B.1})$$

$$f_{\text{low}} = \frac{1}{1.8 T}, \quad (\text{B.2})$$

$$f_{\text{high}} = \min \left[\frac{1.1 N}{2T}, \frac{1}{2\Delta t_{\text{med}}} \right], \quad (\text{B.3})$$

where T is the observed temporal baseline, N is the number of measurements, and Δt_{med} is the median sampling interval. The factor 1.8 avoids truncating peaks near $P = T$, while the high frequency limit prevents the GLSP from probing periods shorter than approximately $2\Delta t_{\text{med}}$.

Only peaks with $P > 100$ days were considered when selecting the six strongest peaks. An oversampling factor of $\zeta = 29$ was used to improve the period resolution.

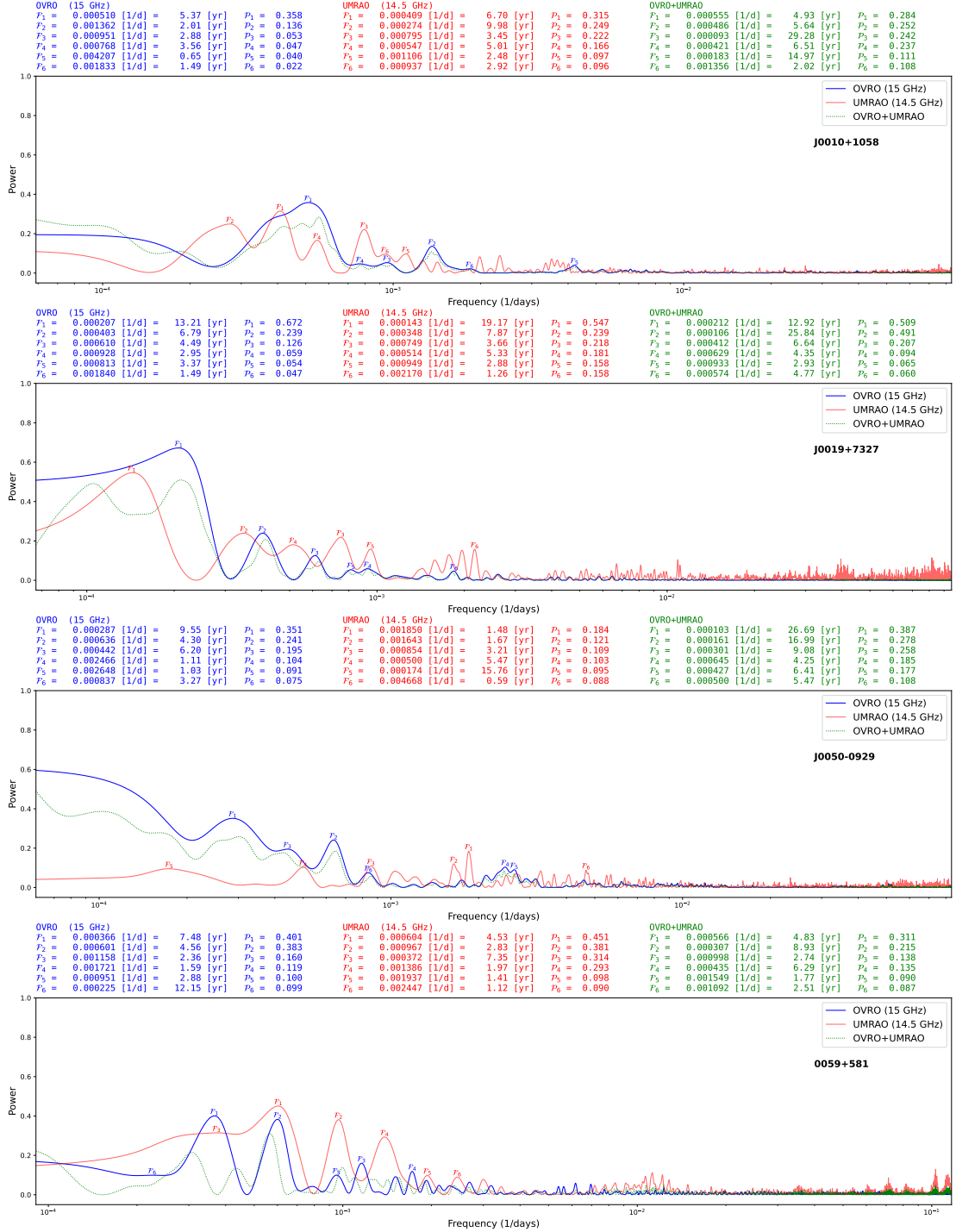


Figure B.1: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRao light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRao data. The sources shown, from top to bottom, are: J0010+1058, J0019+7327, J0050-0929, and 0059+581.

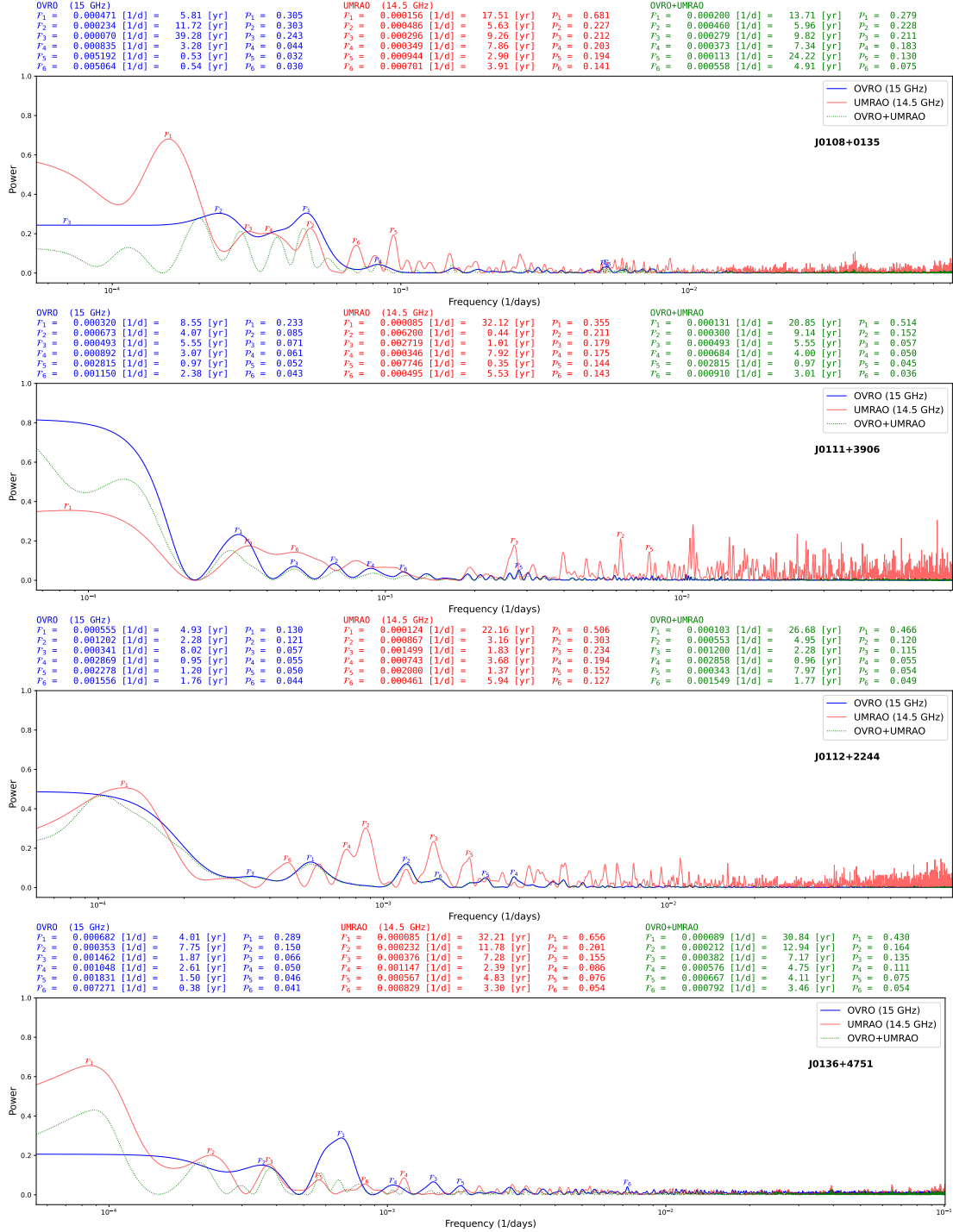


Figure B.2: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0108+0135, J0111+3906, J0112+2244, and J0136+4751.

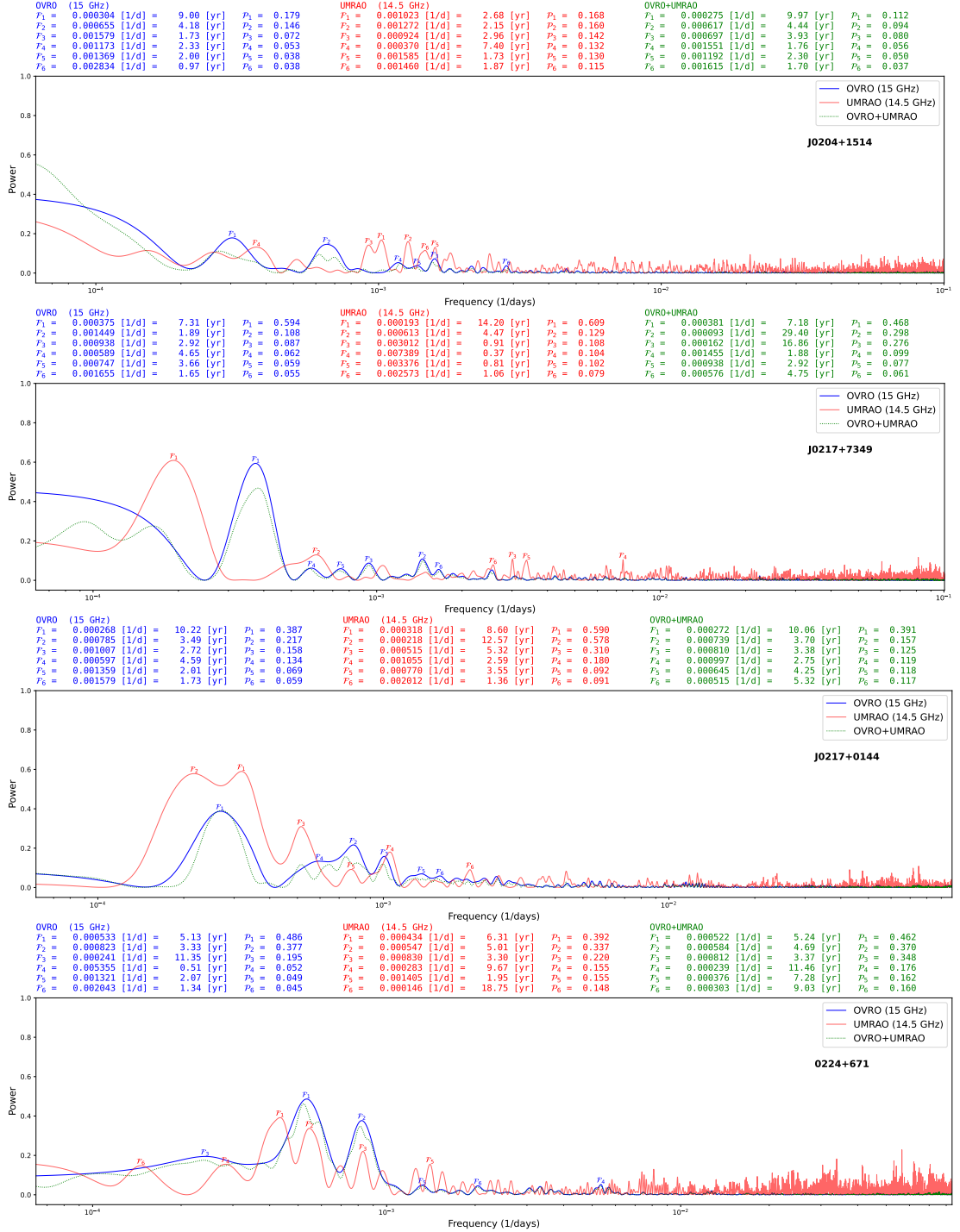


Figure B.3: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0204+1514, J0217+7349, J0217+0144, and 0224+671.

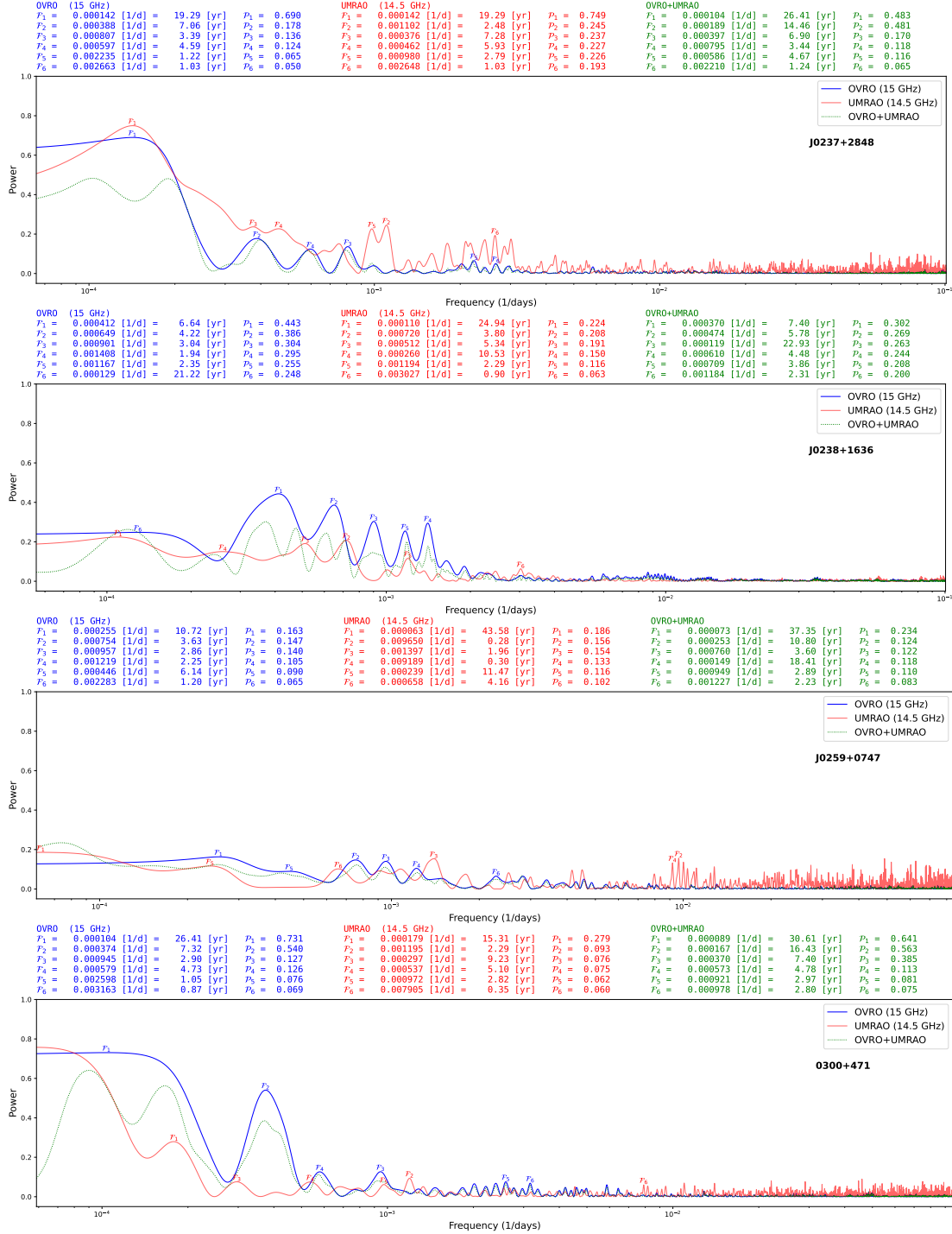


Figure B.4: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0237+2848, J0238+1636, J0259+0747, and 0300+471.

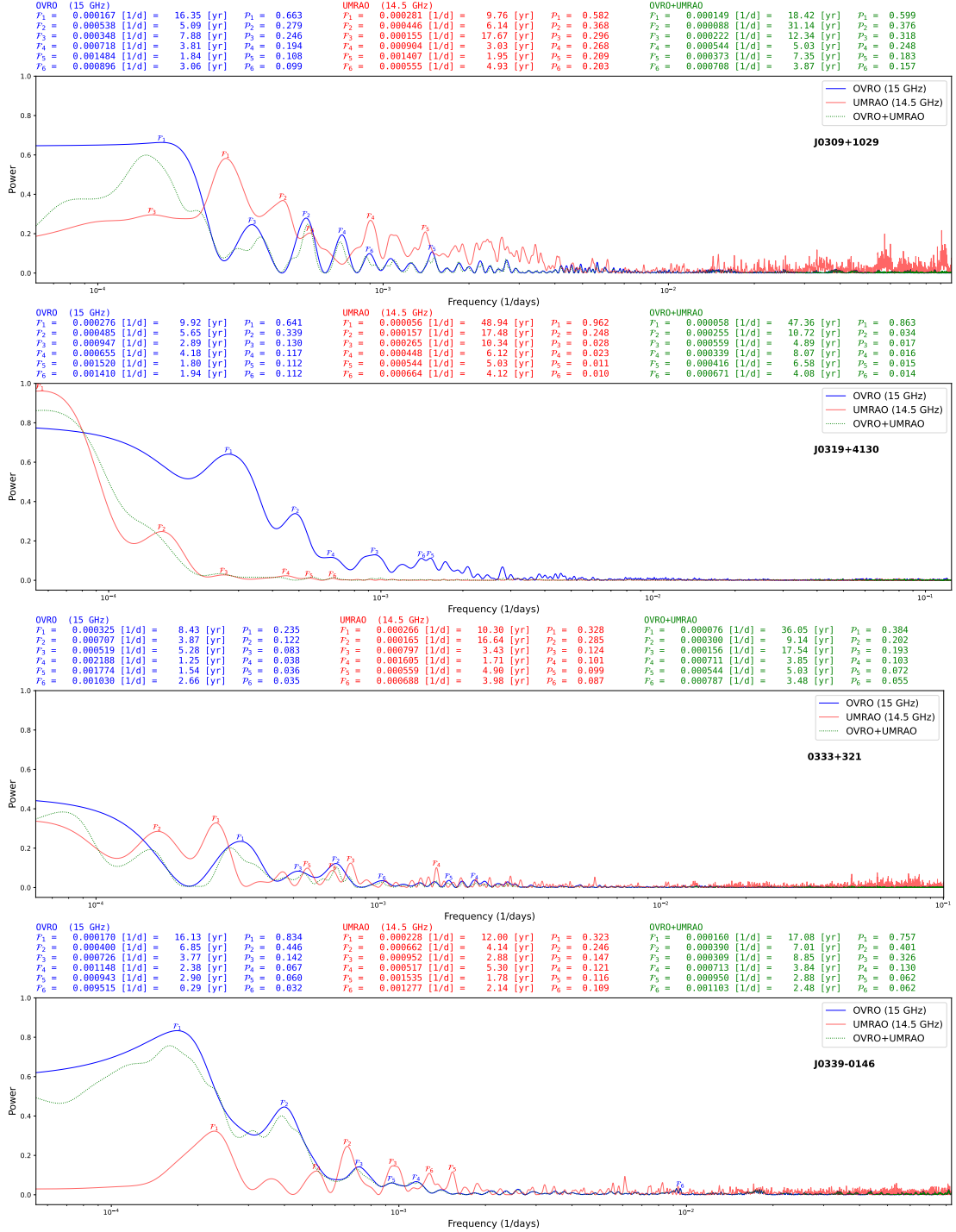


Figure B.5: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0309+1029, J0319+4130, 0333+321, and J0339-0146.

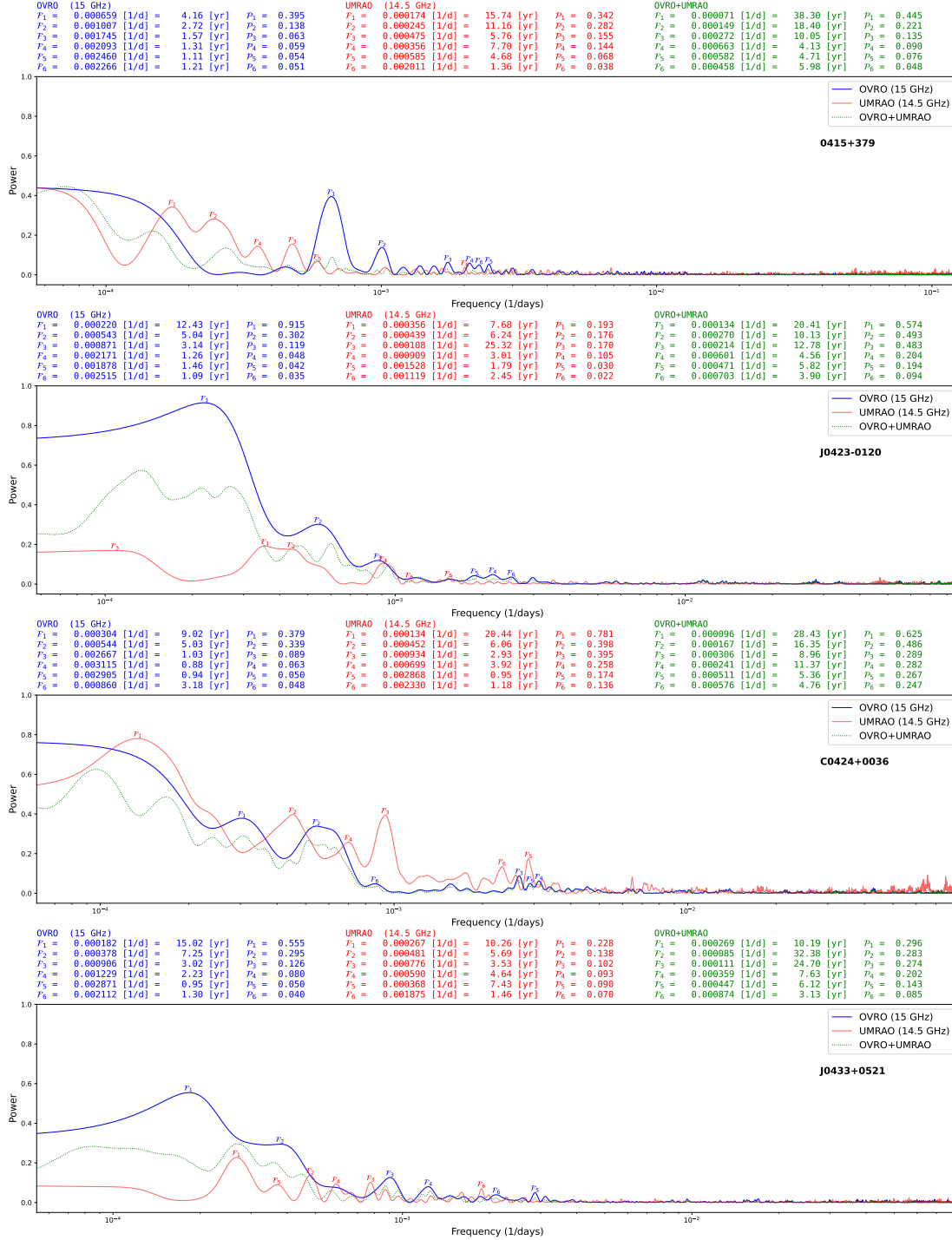


Figure B.6: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: 0415+379, J0423-0120, C0424+0036, and J0433+0521.

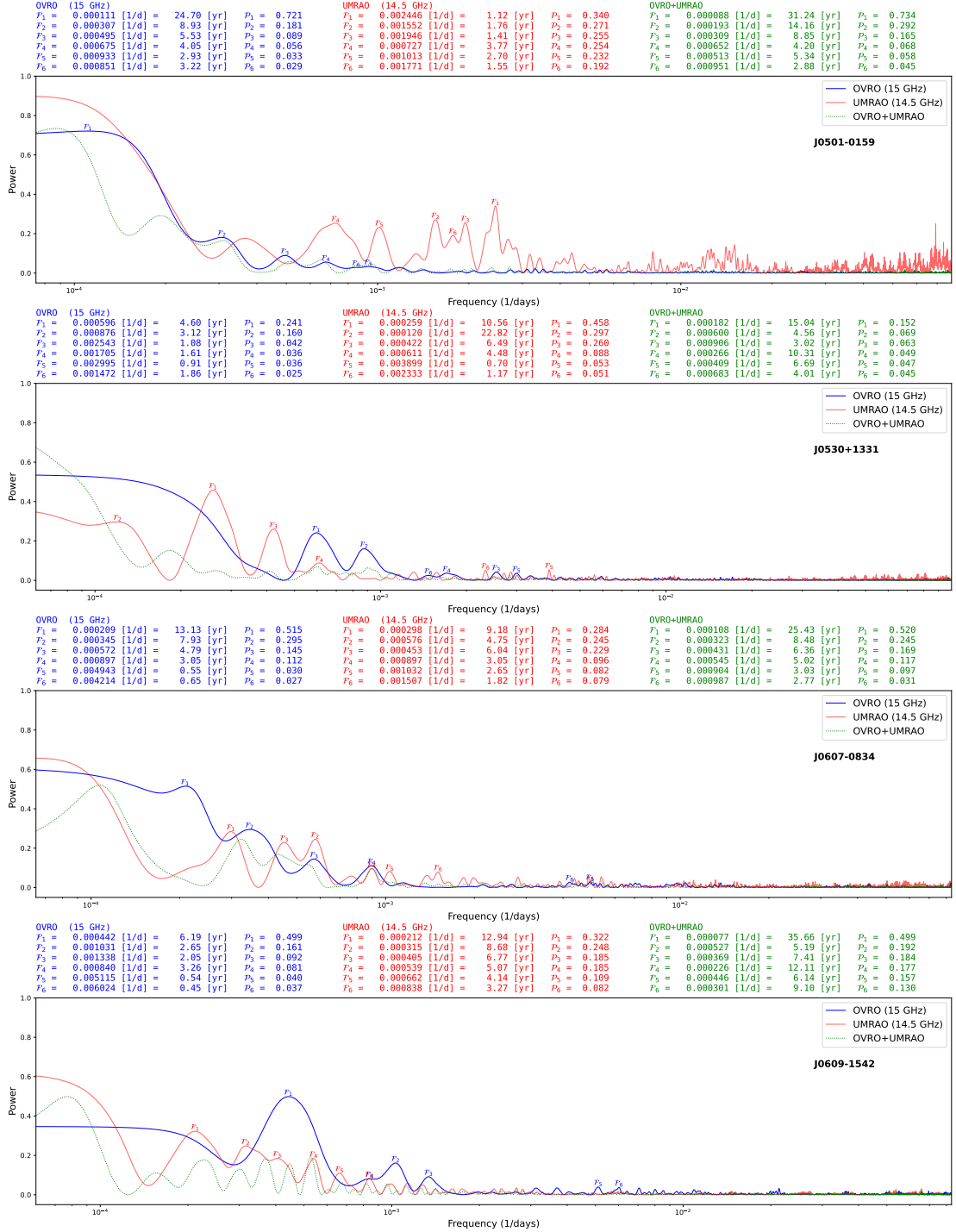


Figure B.7: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0501-0159, J0530+1331, J0607-0834, and J0609-1542.

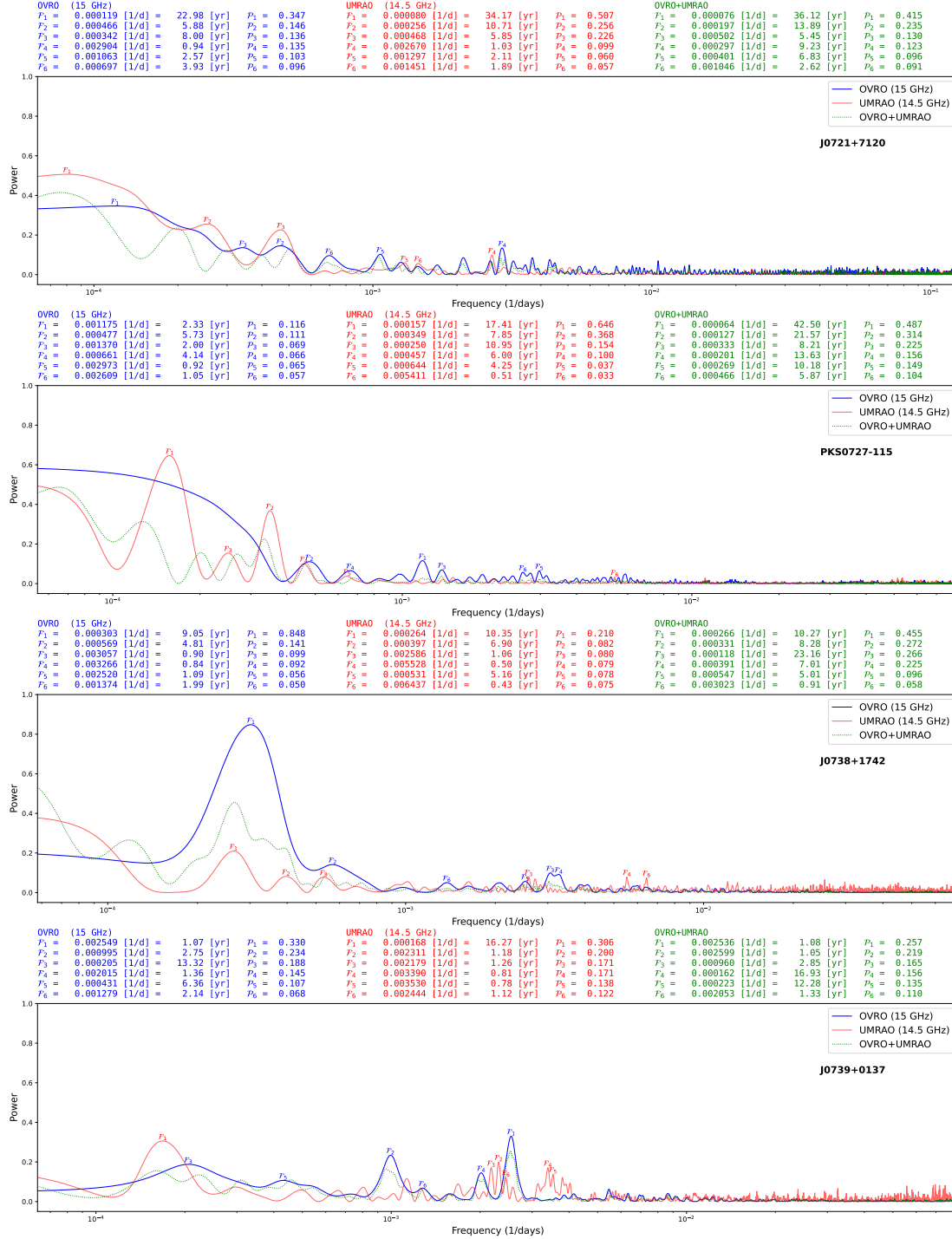


Figure B.8: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0721+7120, PKS0727-115, J0738+1742, and J0739+0137.

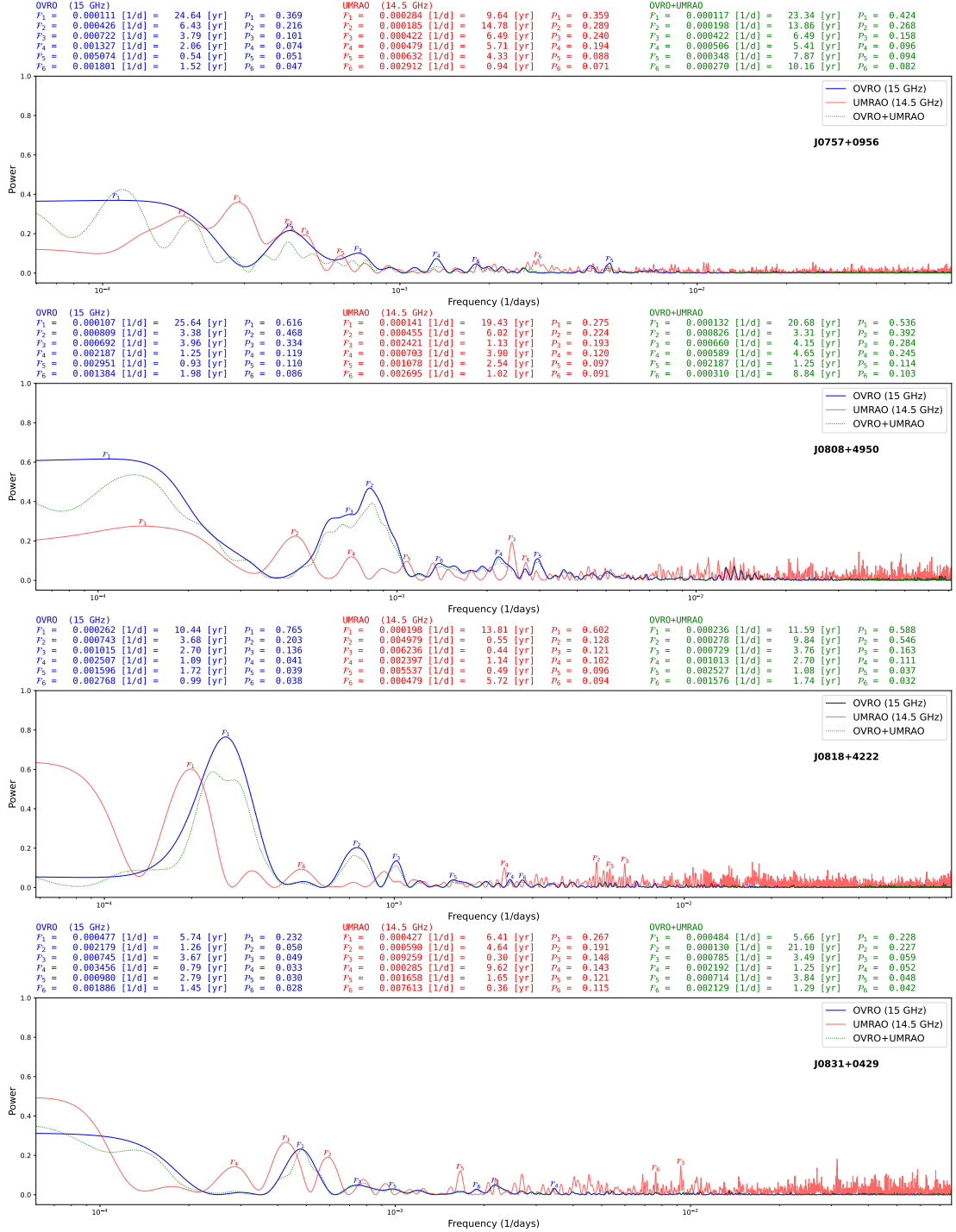


Figure B.9: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRao light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRao data. The sources shown, from top to bottom, are: J0757+0956, J0808+4950, J0818+4222, and J0831+0429.

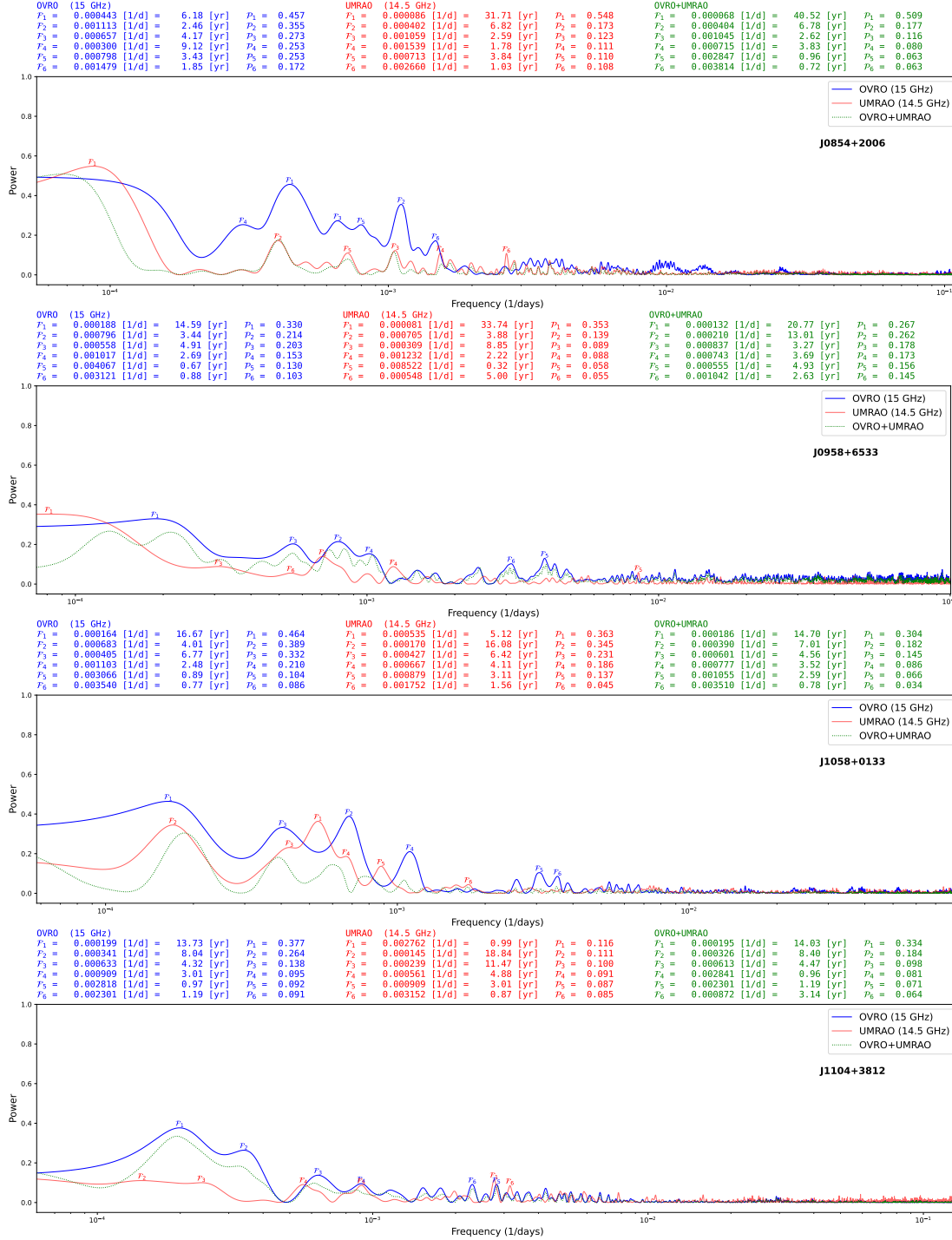


Figure B.10: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J0854+2006, J0958+6533, J1058+0133, and J1104+3812.

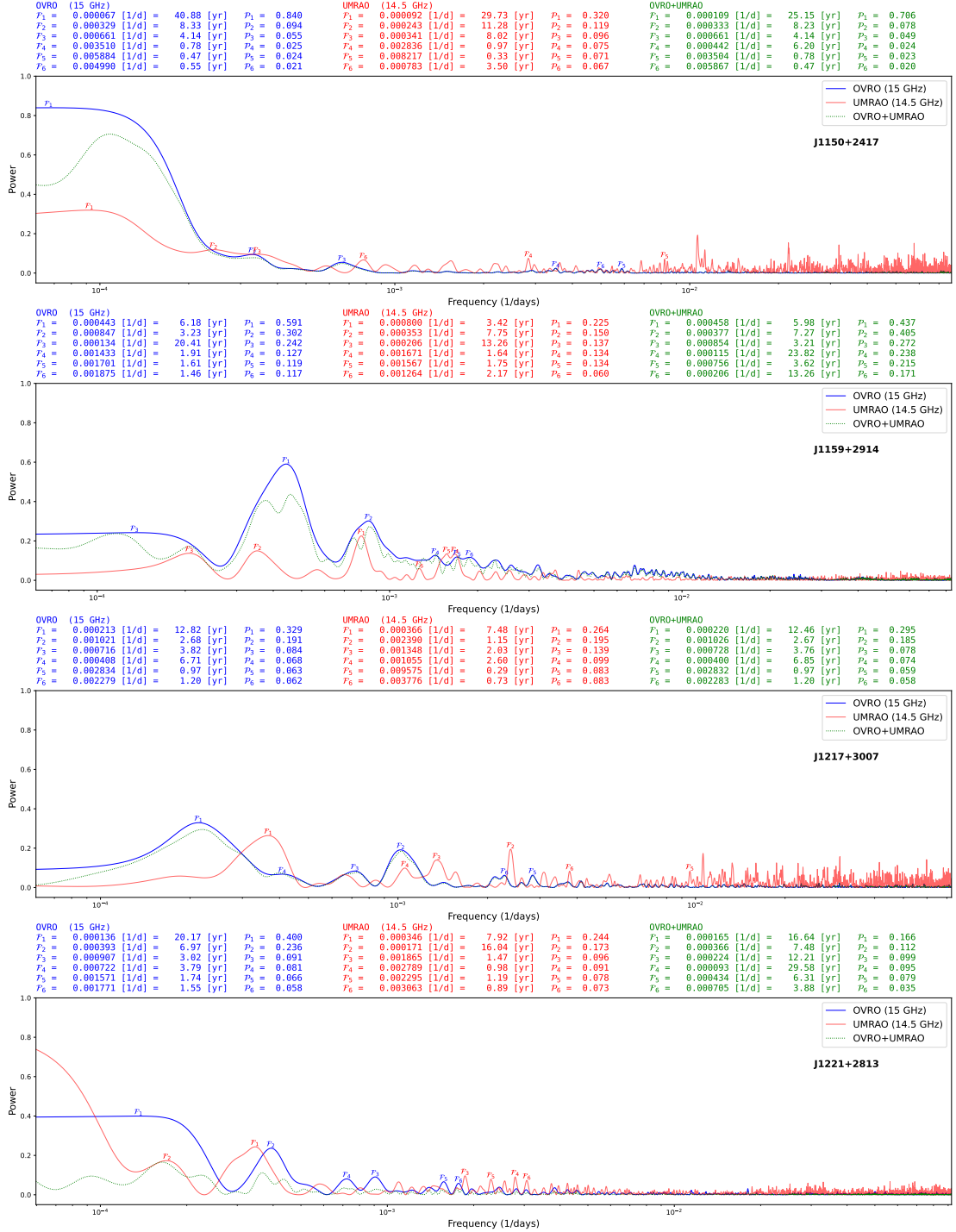


Figure B.11: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRao light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRao data. The sources shown, from top to bottom, are: J1150+2417, J1159+2914, J1217+3007, and J1221+2813.

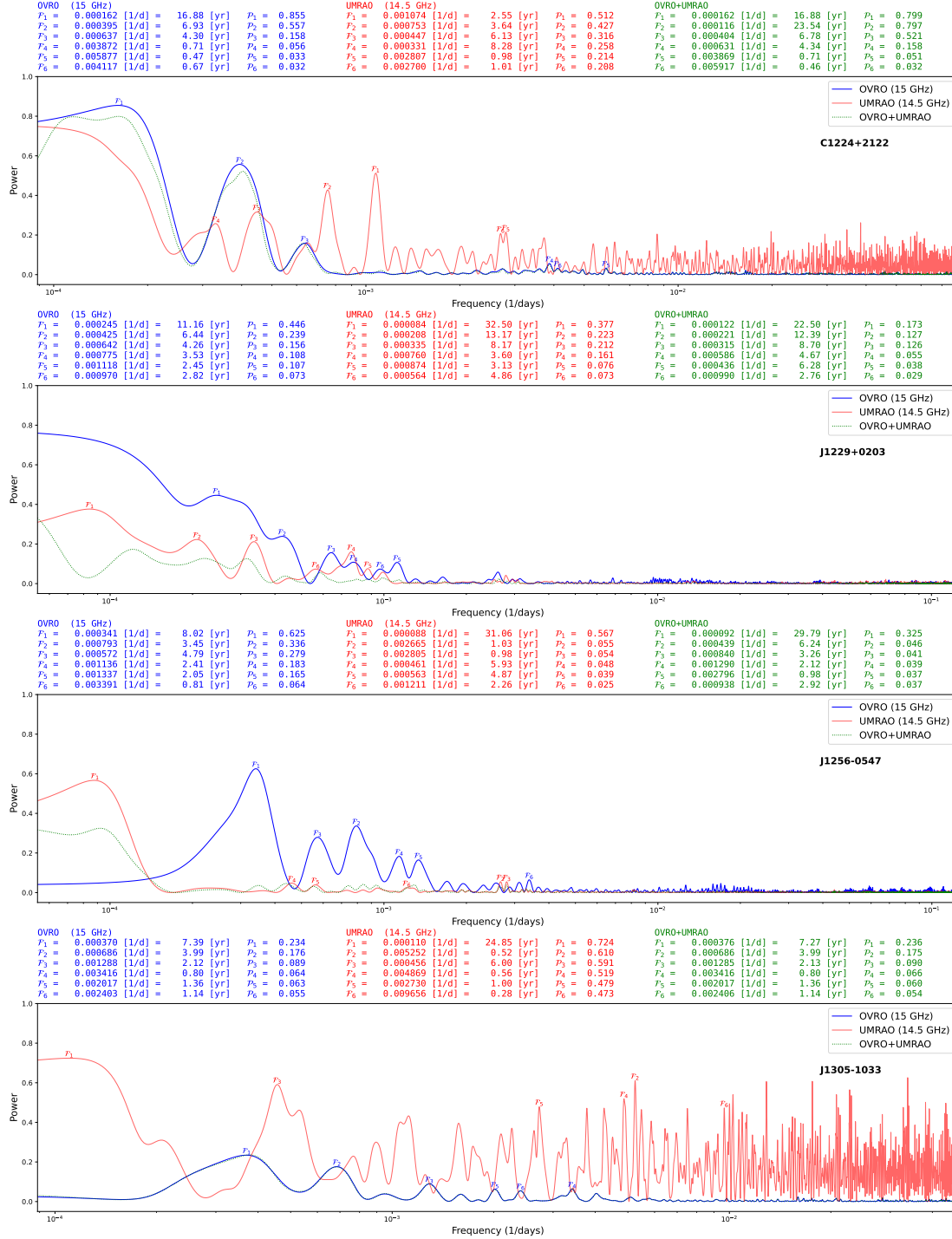


Figure B.12: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: C1224+2122, J1229+0203, J1256-0547, and J1305-1033.

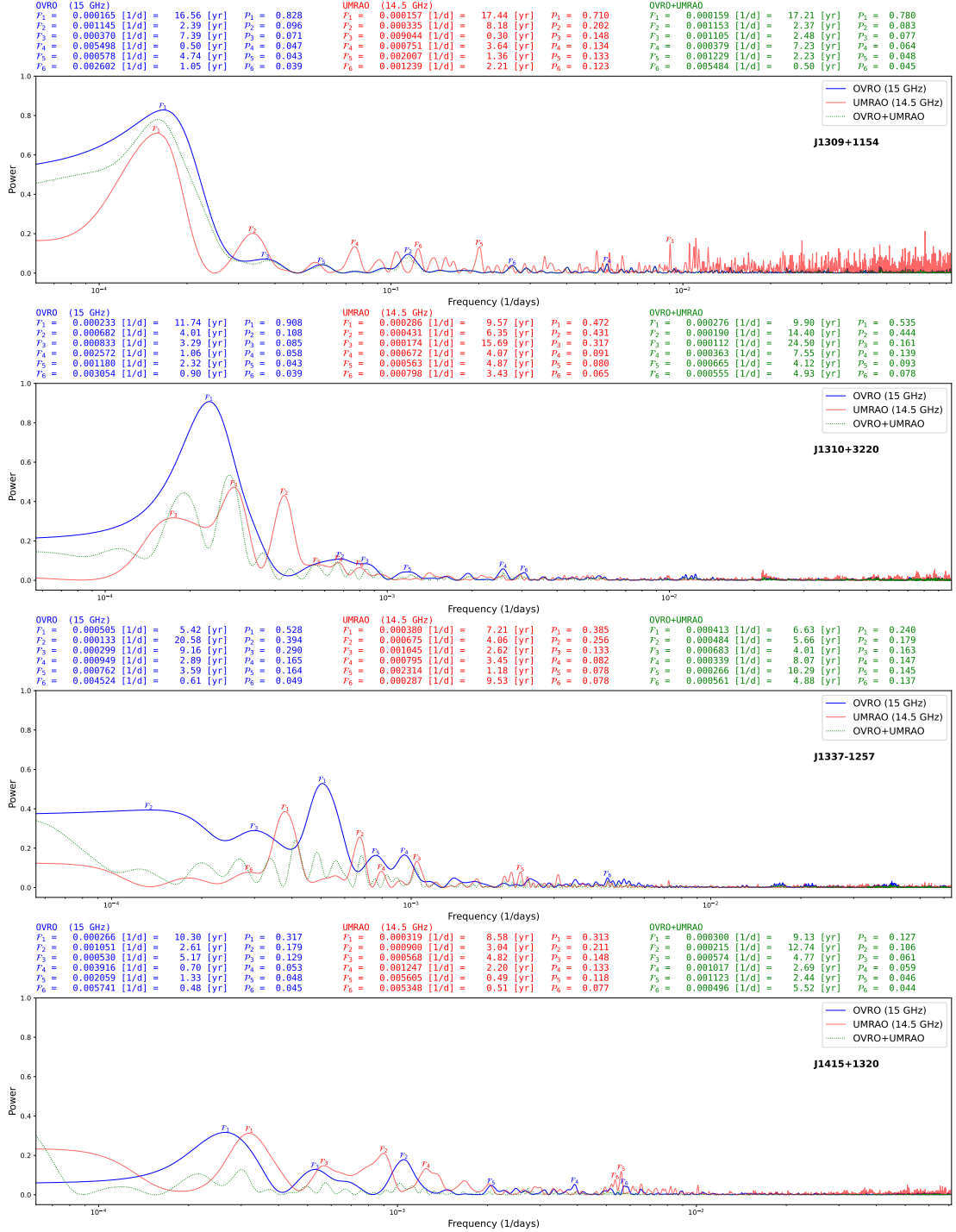


Figure B.13: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRao light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRao data. The sources shown, from top to bottom, are: J1309+1154, J1310+3220, J1337-1257, and J1415+1320.

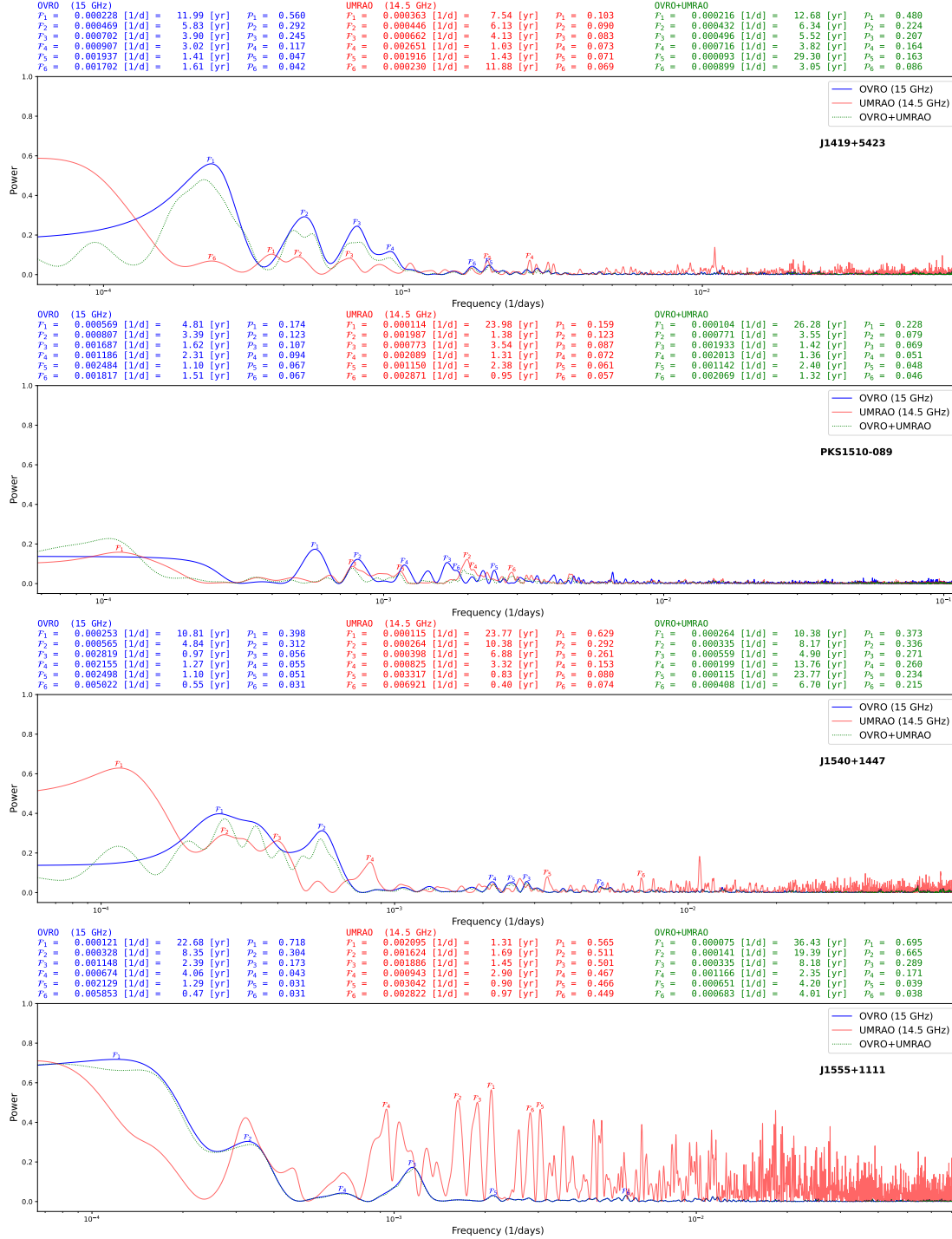


Figure B.14: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1419+5423, PKS1510-089, J1540+1447, and J1555+1111.

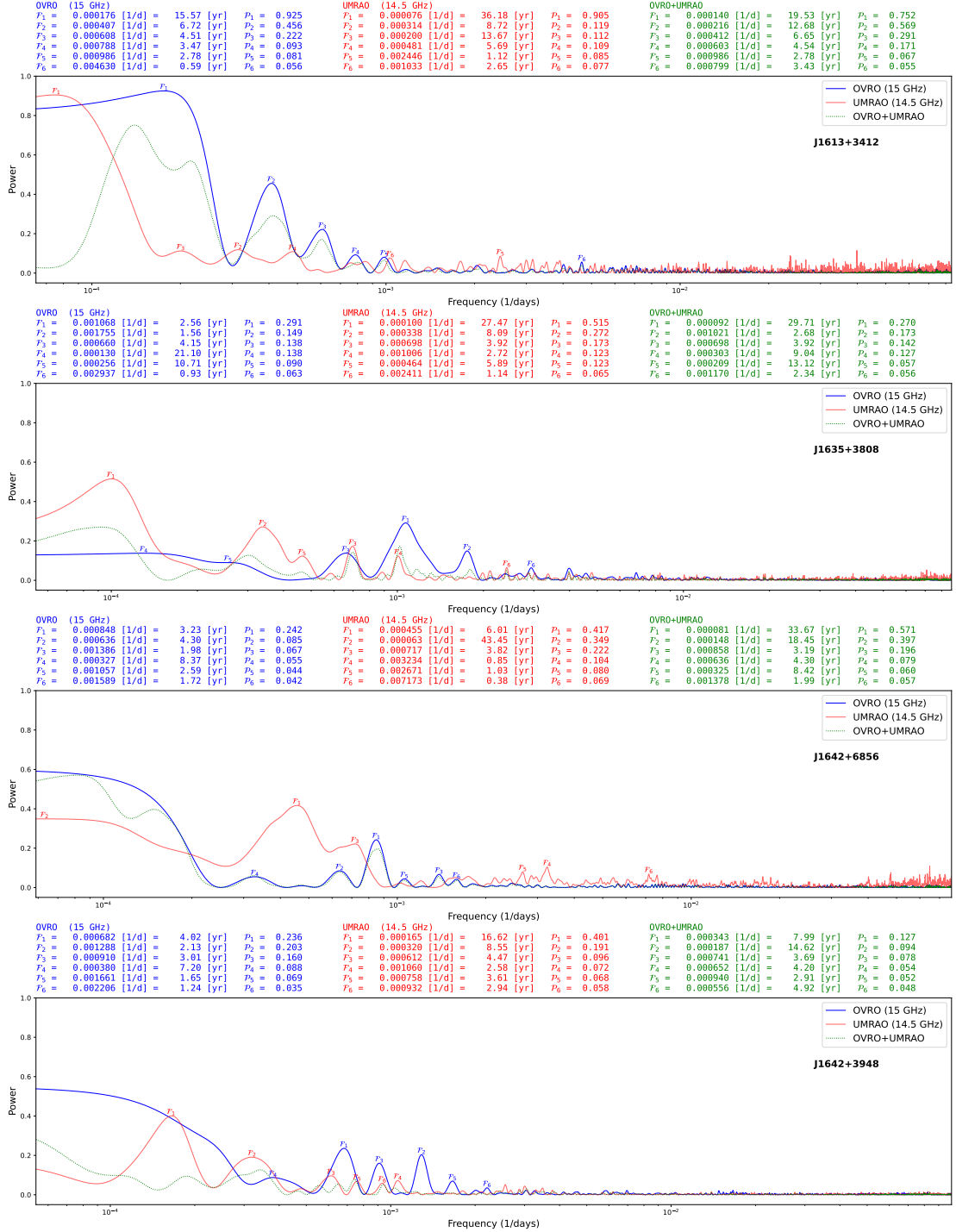


Figure B.15: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1613+3412, J1635+3808, J1642+6856, and J1642+3948.

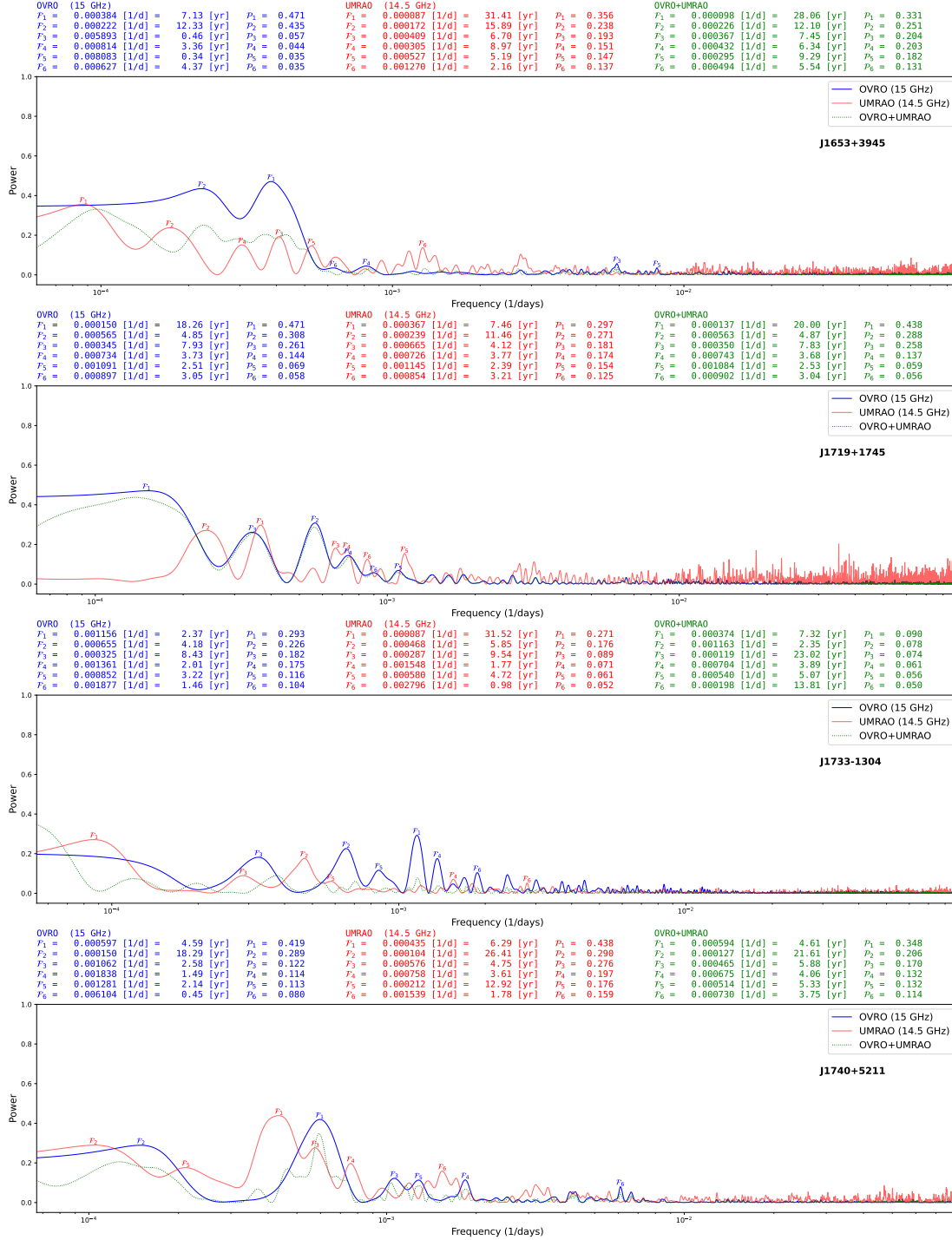


Figure B.16: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1653+3945, J1719+1745, J1733-1304, and J1740+5211.

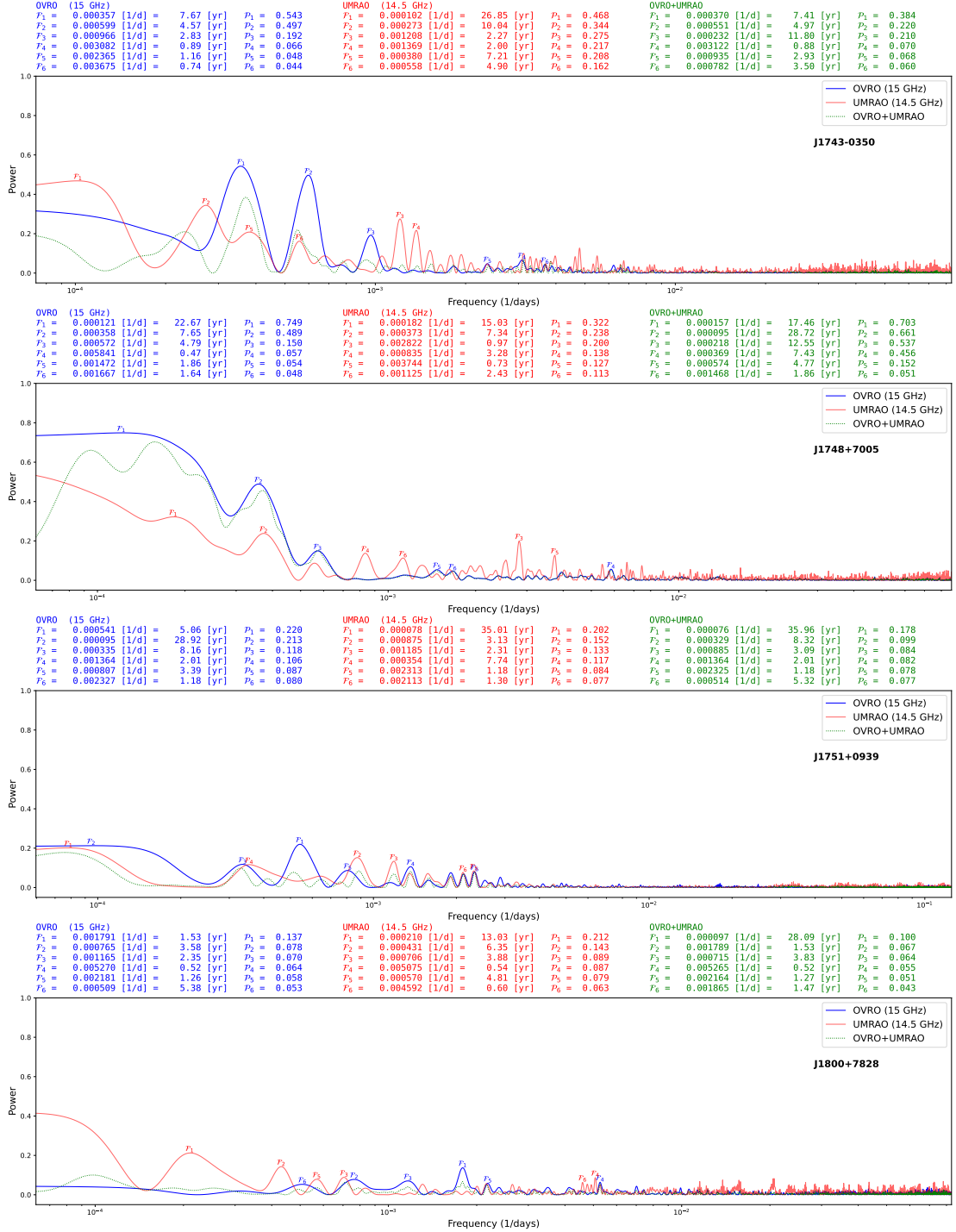


Figure B.17: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1743-0350, J1748+7005, J1751+0939, and J1800+7828.

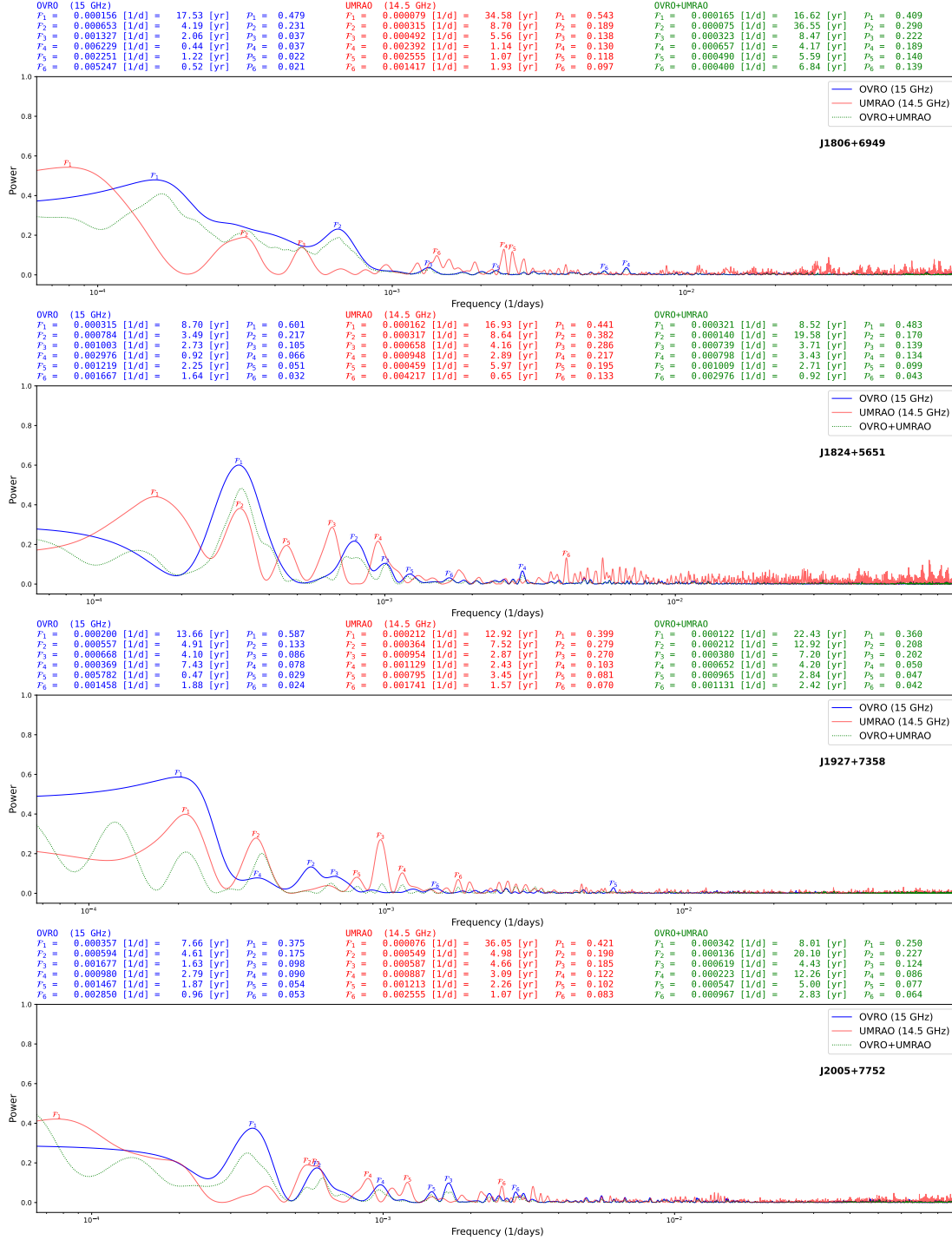


Figure B.18: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J1806+6949, J1824+5651, J1927+7358, and J2005+7752.

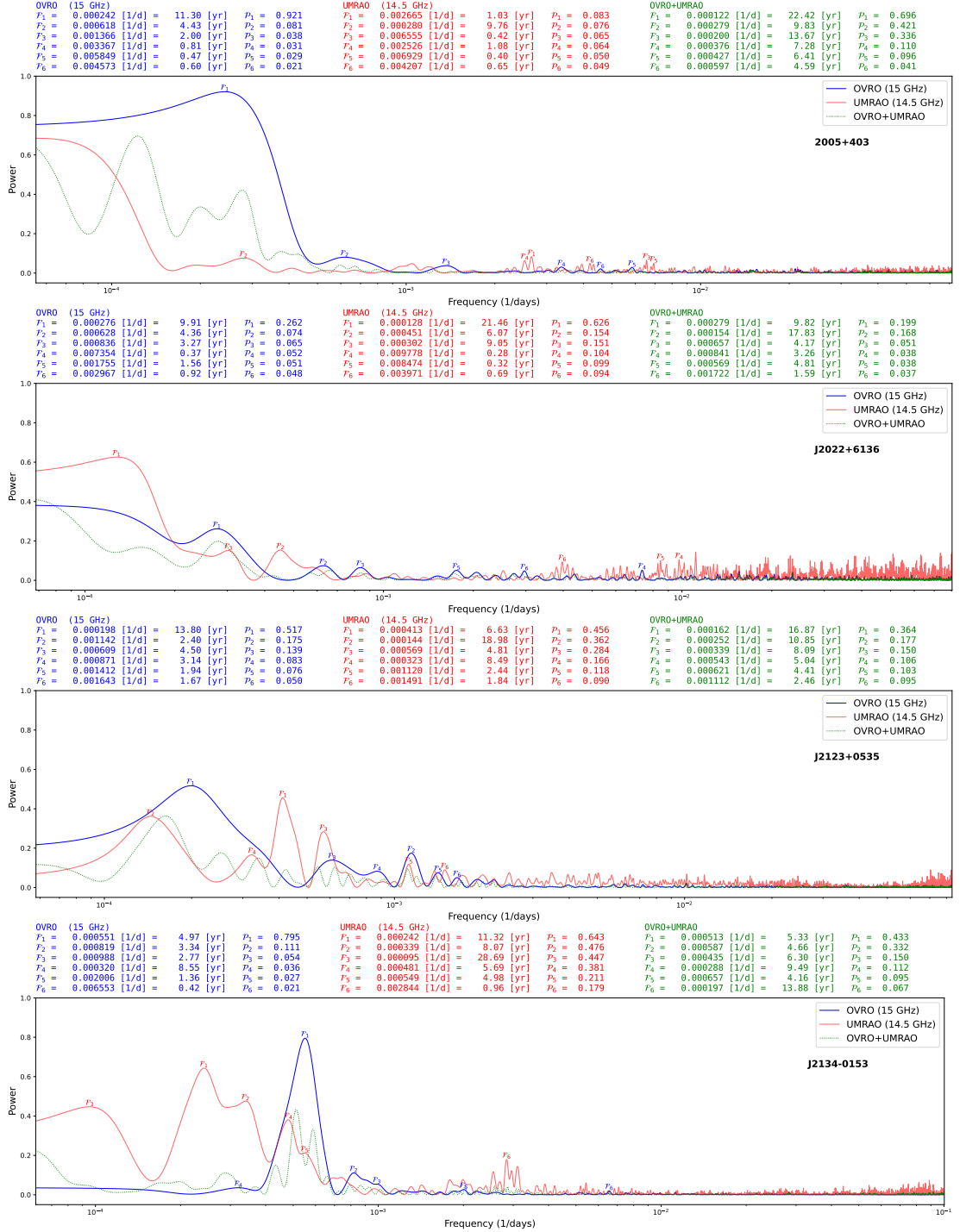


Figure B.19: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: 2005+403, J2022+6136, J2123+0535, and J2134-0153.

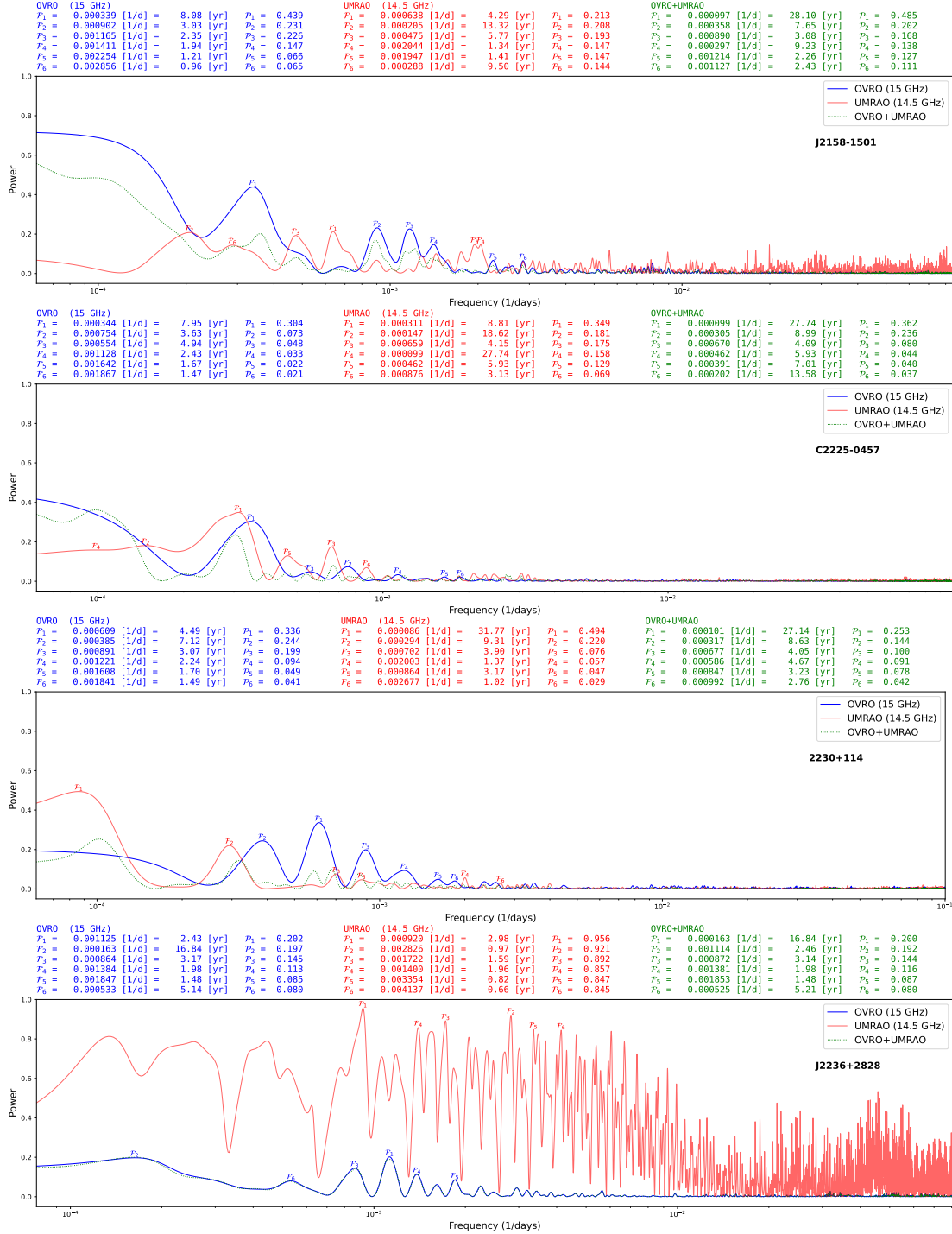


Figure B.20: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRAO at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J2158-1501, C2225-0457, 2230+114, and J2236+2828.

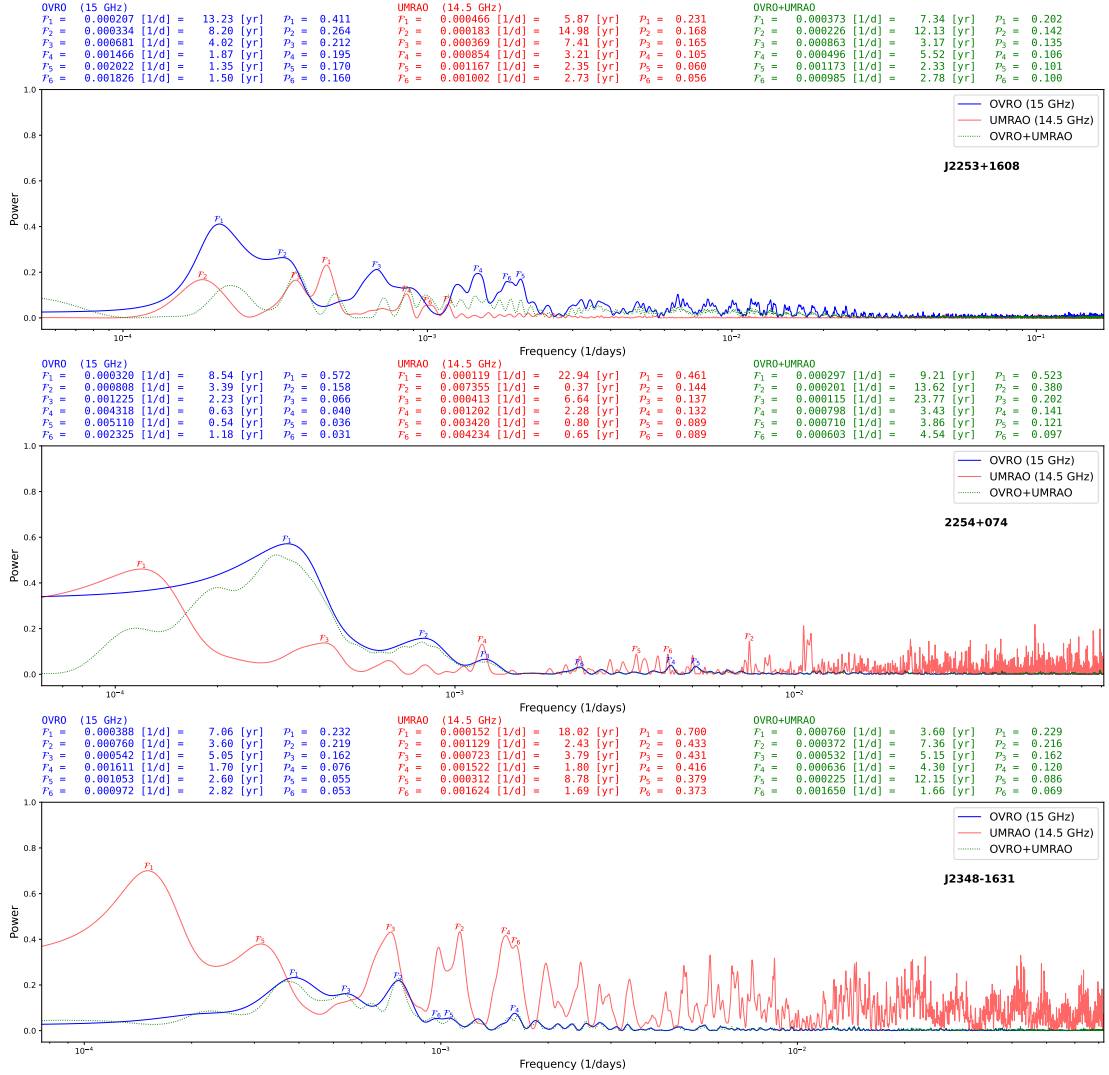


Figure B.21: Generalized Lomb–Scargle periodograms as a function of temporal frequency for the OVRO+UMRAO light curves. Each panel shows the overlaid GLSPs computed for OVRO at 15 GHz, UMRao at 14.5 GHz, and the combined OVRO+UMRAO data. The sources shown, from top to bottom, are: J2253+1608, 2254+074, and J2348-1631.

Appendix C

Synthetic Light Curves

C.0.1 Test 2: Same amplitude, different periods, random phases for each iteration.

1. Summary of Full Model (Test 2):

The final simulated data points are:

$$S_{\text{sim}}(t_i) = A \sin \left(2\pi \frac{t_i}{P_1} + \phi_1 \right) + A \sin \left(2\pi \frac{t_i}{P_2} + \phi_2 \right) + N(t_i) \quad (\text{C.1})$$

with

$$\left\{ \begin{array}{l} P_2 = \frac{1}{2}P_1 \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi) \text{ (new draw for every } P_1 \text{ value)} \\ N(t_i) \sim \mathcal{N}(0, (0.1A)^2) \\ t_i = \text{MJD}_i \end{array} \right.$$

2. Observations: This test (Figure C.1) shows that independently generating random phases for each period does not produce relevant differences compared with Test 1.

Note: In this test, for every new value of P_1 , the random phases ϕ_1 and ϕ_2 are *redrawn*, so each trial uses different phases.

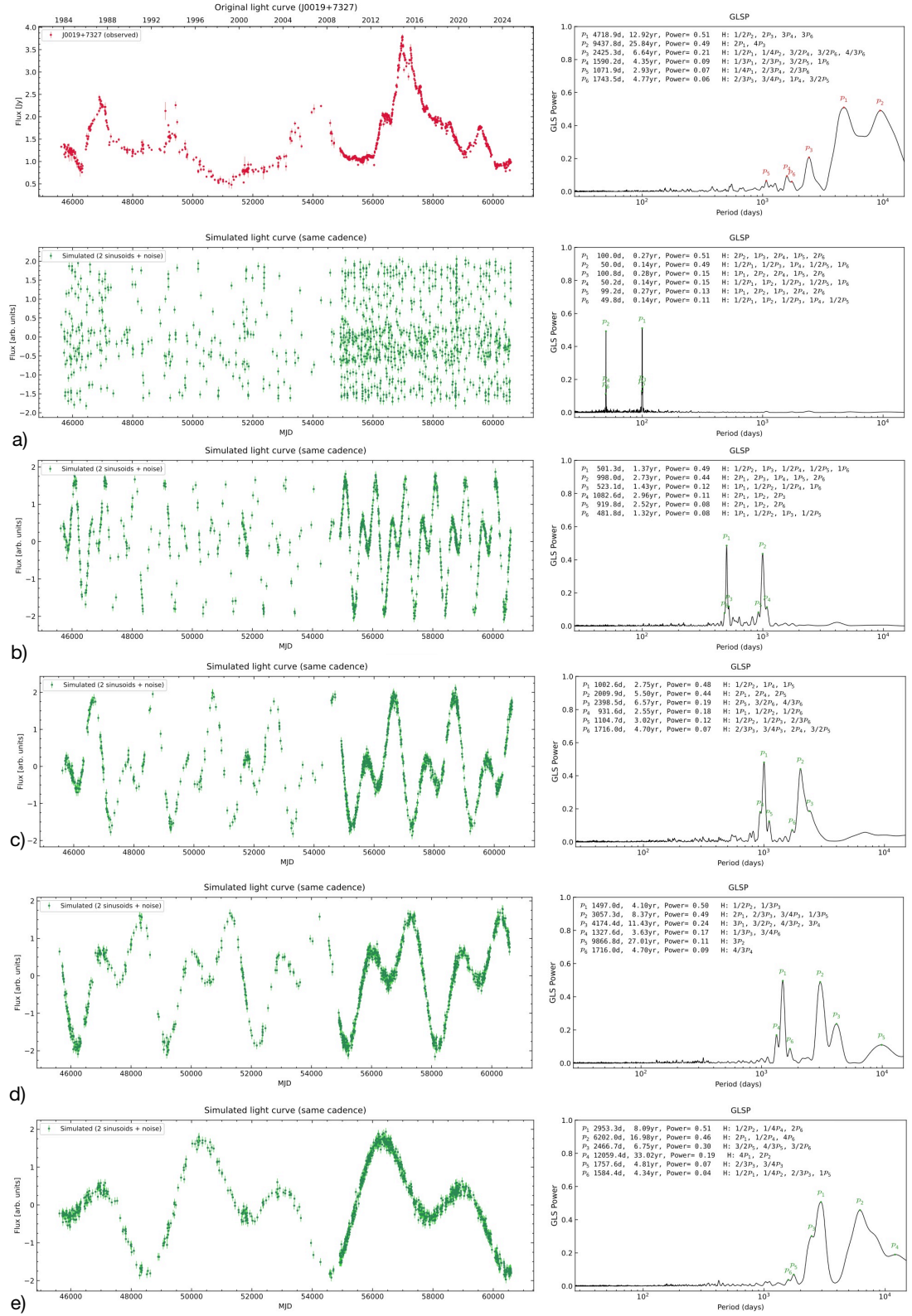


Figure C.1: Test 2: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 3000, e) 6000.

C.0.2 Test 3: Different amplitudes, periods, and random phases.

1. Summary of Full Model (Test 3):

$$S_{\text{sim}}(t_i) = A_1 \sin\left(2\pi\frac{t_i}{P_1} + \phi_1\right) + A_2 \sin\left(2\pi\frac{t_i}{P_2} + \phi_2\right) + N(t_i) \quad (\text{C.2})$$

with

$$\begin{cases} P_2 = \frac{1}{2}P_1 \\ A_1 = 2A_2 \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi) \\ N(t_i) \sim \mathcal{N}(0, (0.1A_1)^2) \\ t_i = \text{MJD}_i \end{cases}$$

2. Observations: This test (Figures C.2 and C.3) shows that the amplitude difference between the sinusoidal components strongly affects the power of their peaks in the GLSP. The higher-amplitude component dominates the periodogram, and, for periods between 2000 and 6000 days, the weaker component may no longer be distinguishable among the secondary peaks.

Note: For this test, several values of P_1 can be explored, but the same random phases ϕ_1 and ϕ_2 are used for all period choices within a given simulation run.

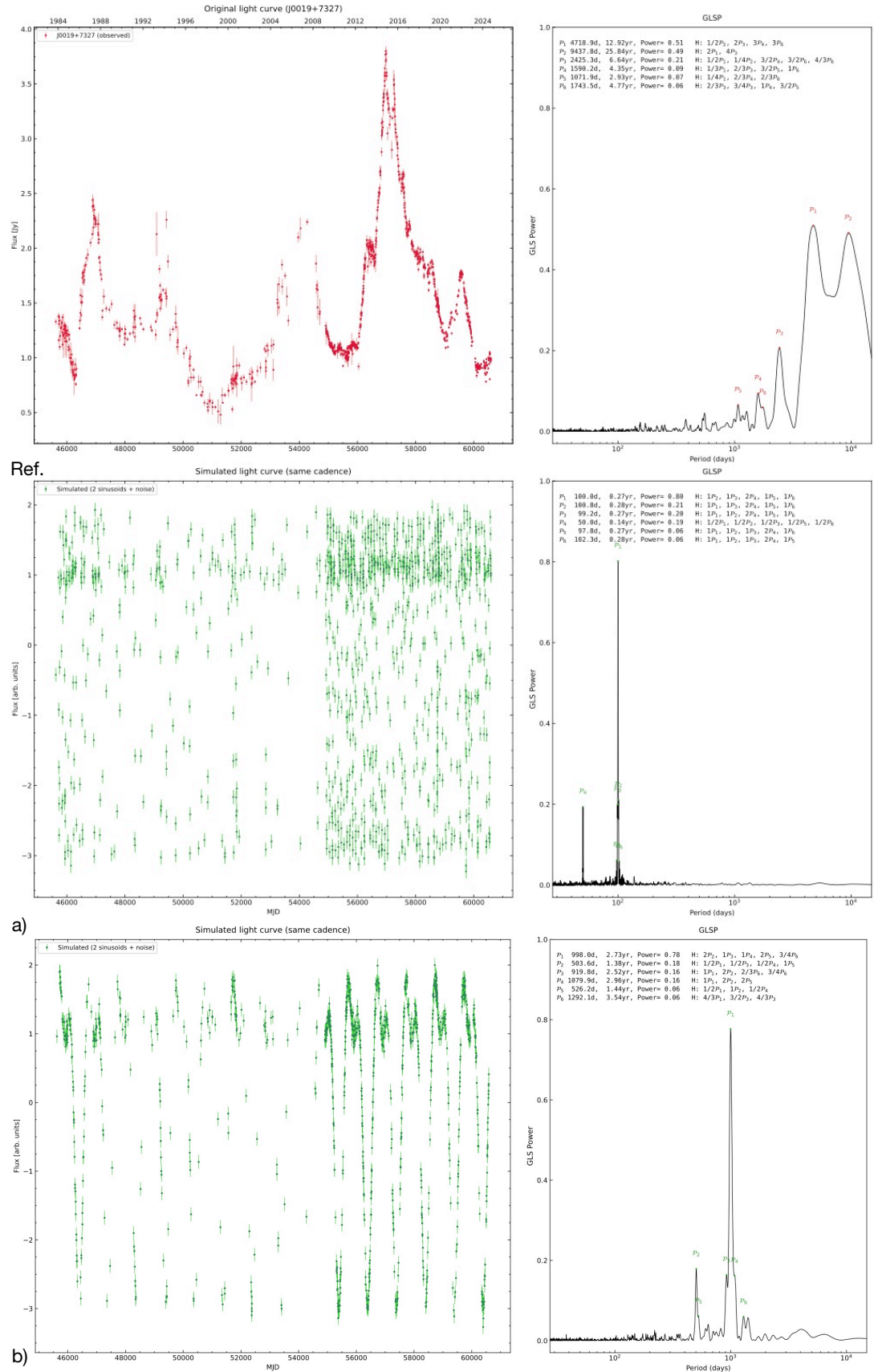


Figure C.2: Test 3: For a simulated period (in days): a) 100, b) 1000.

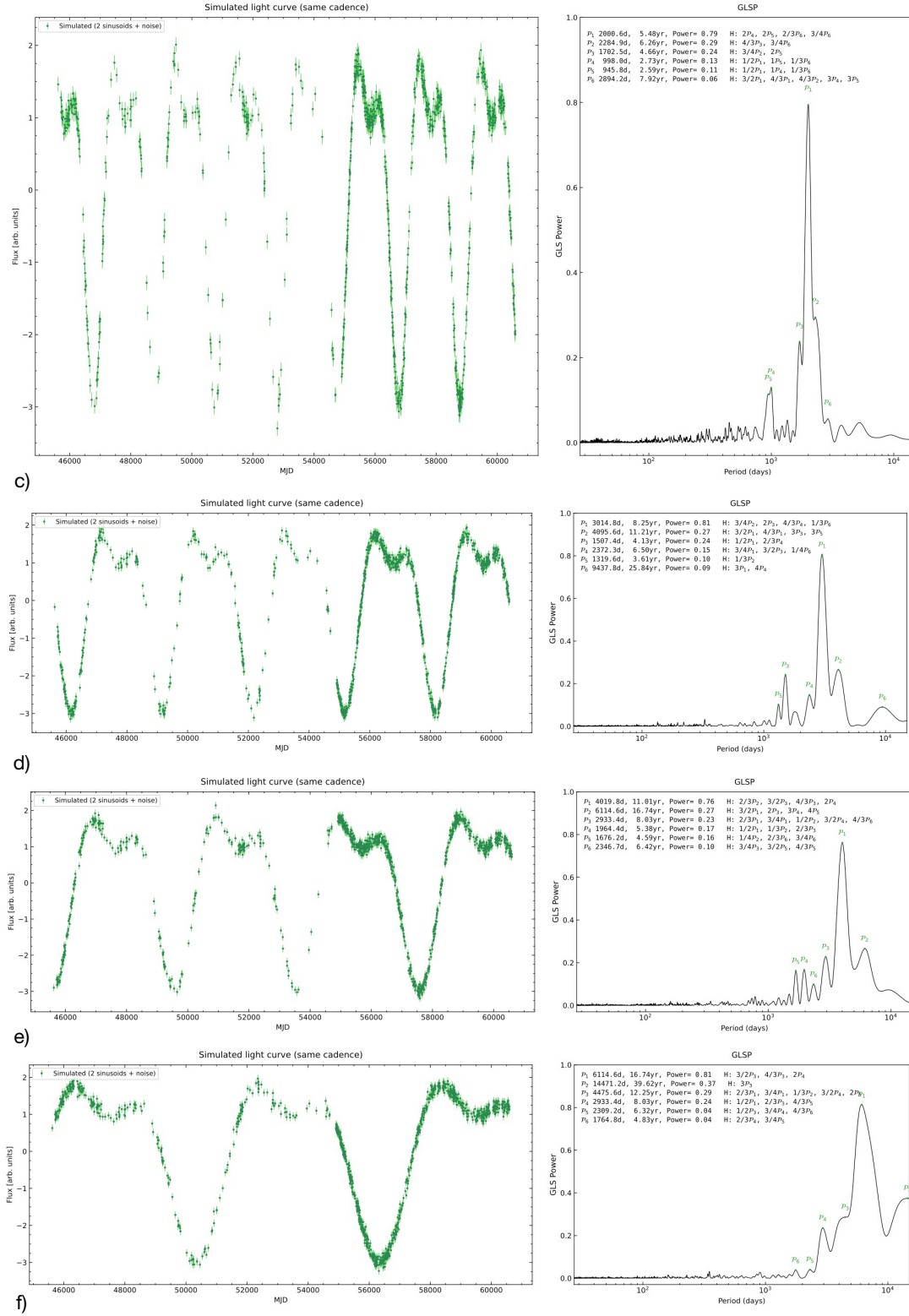


Figure C.3: Test 3: For a simulated period (in days): c) 2000, d) 3000, e) 4000, f) 6000.

C.0.3 Test 4: Same amplitude, different periods (5/7 factor), random phases.

1. Summary of Full Model (Test 4):

$$S_{\text{sim}}(t_i) = A \sin \left(2\pi \frac{t_i}{P_1} + \phi_1 \right) + A \sin \left(2\pi \frac{t_i}{P_2} + \phi_2 \right) + N(t_i) \quad (\text{C.3})$$

with

$$\begin{cases} P_1 = \frac{5}{7}P_2 \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi) \\ N(t_i) \sim \mathcal{N}(0, (0.1A)^2) \\ t_i = \text{MJD}_i \end{cases}$$

2. Observations: In this test (Figure C.4), for a period ratio of 5/7, a transition toward a single dominant period is observed at longer periods. Between 5000 and 6000 days, the GLSP changes from showing two peaks with powers below 0.4 to a single peak with a power close to 0.95.

Note: In this test, several values of P_2 can be explored, but the same random phases ϕ_1 and ϕ_2 are used for all period choices within a given simulation run.

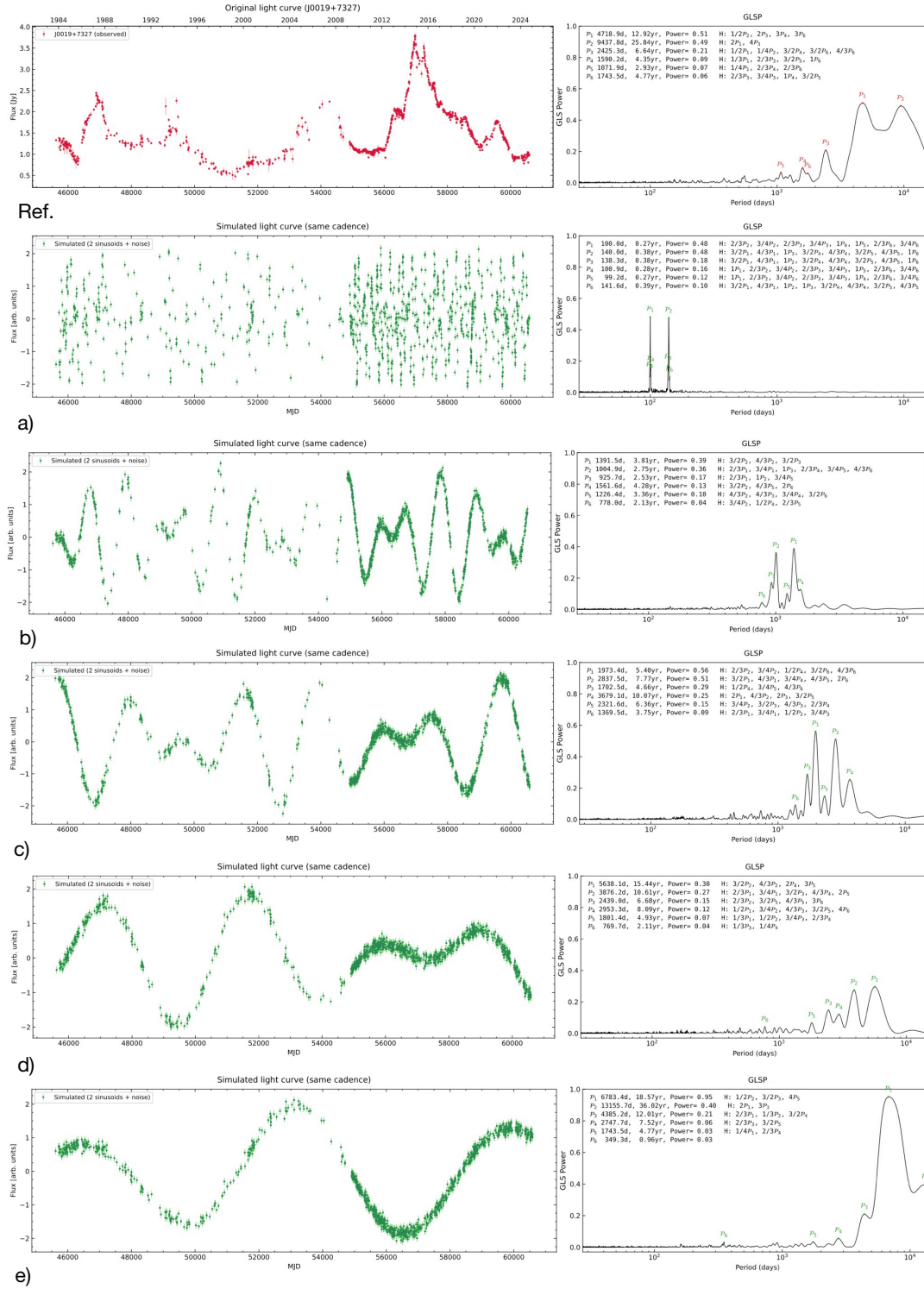


Figure C.4: Test 4: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 4000, e) 6000.

C.0.4 Test 5: Single sinusoidal signal with red noise.

1. Summary of Full Model (Test 5):

The final simulated data points are:

$$S_{\text{sim}}(t_i) = A \sin \left(2\pi \frac{t_i}{P} + \phi \right) + N_i \quad (\text{C.4})$$

with

$$\begin{cases} \phi \sim \mathcal{U}(0, 2\pi) \\ N_i = \alpha N_{i-1} + \epsilon_i, \epsilon_i \sim \mathcal{N}(0, (0.1A)^2) \\ t_i = \text{MJD}_i \end{cases}$$

2. Observations: In this test (Figure C.5), a single sinusoidal component combined with red noise consistently recovers the imposed period, with GLSP powers close to unity. The main peak is accompanied by lower-power secondary peaks, without a systematic change in the number of harmonic relationships.

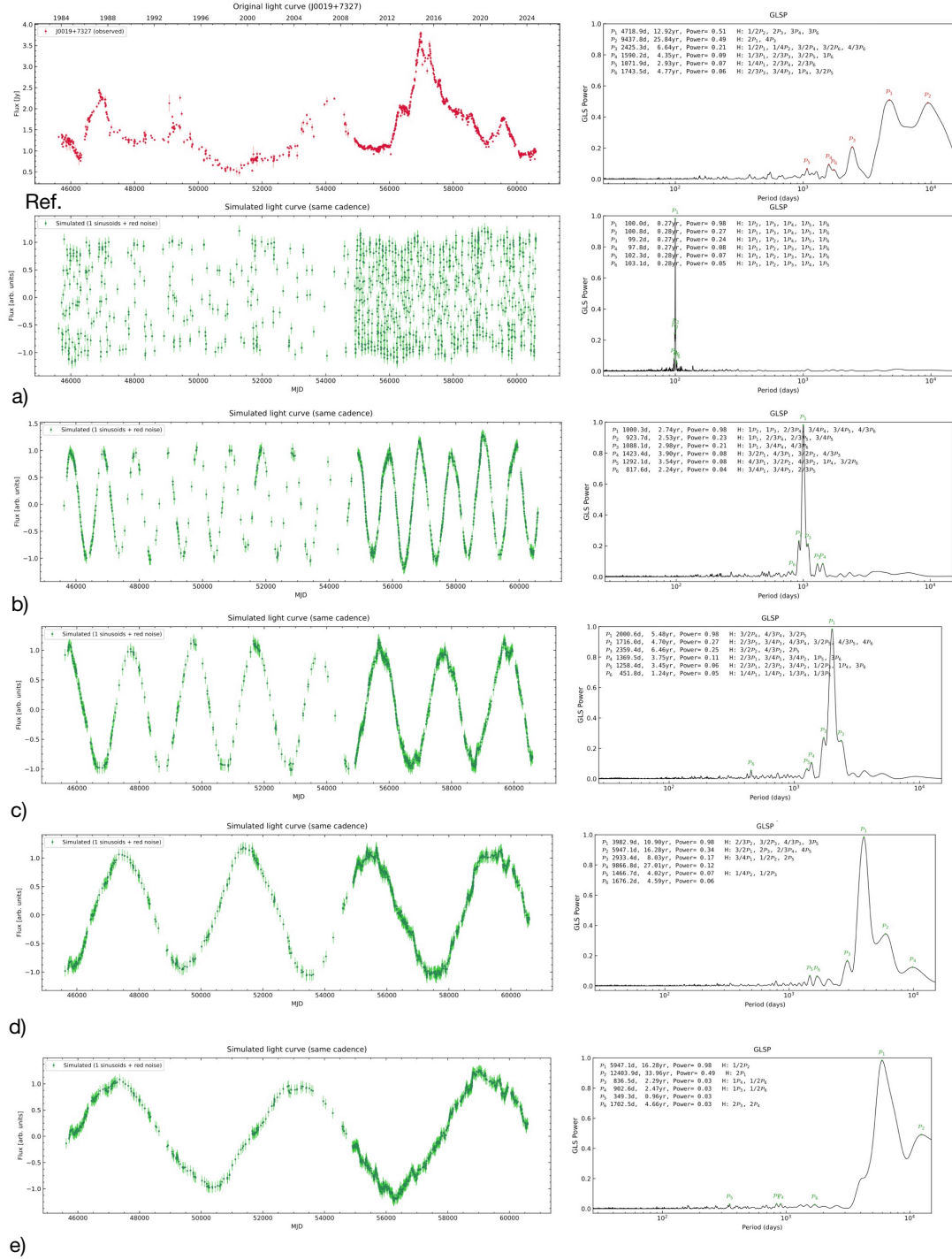


Figure C.5: Test 5: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 4000, e) 6000.

C.0.5 Test 6: Sum of two sinusoidal signals with varying amplitudes, periods, phases, and red noise.

1. Summary of Full Model:

The final simulated data points are:

$$S_{\text{sim}}(t_i) = A_1 \sin\left(2\pi \frac{t_i}{P_1} + \phi_1\right) + A_2 \sin\left(2\pi \frac{t_i}{P_2} + \phi_2\right) + N_i \quad (\text{C.5})$$

with

$$\begin{cases} A_1 = 1, & A_2 = 0.6 A_1 \\ P_1 = 2P_2 \\ \phi_1, \phi_2 \sim \mathcal{U}(0, 2\pi) \\ N_i = \alpha N_{i-1} + \epsilon_i, & \epsilon_i \sim \mathcal{N}(0, (0.5A_1)^2) \\ t_i = \text{MJD}_i \end{cases}$$

2. Observations: In this test (Figure C.6), the behavior is similar to that observed in Test 3, although with a greater contribution from noise. The GLSP preserves the period of the dominant component, and, for the 6000-day case, harmonic relationships close to 2:1 appear.

Note: The red noise is generated as a correlated process; setting α close to 1 produces stronger long-term correlations, while lower values approach white noise.

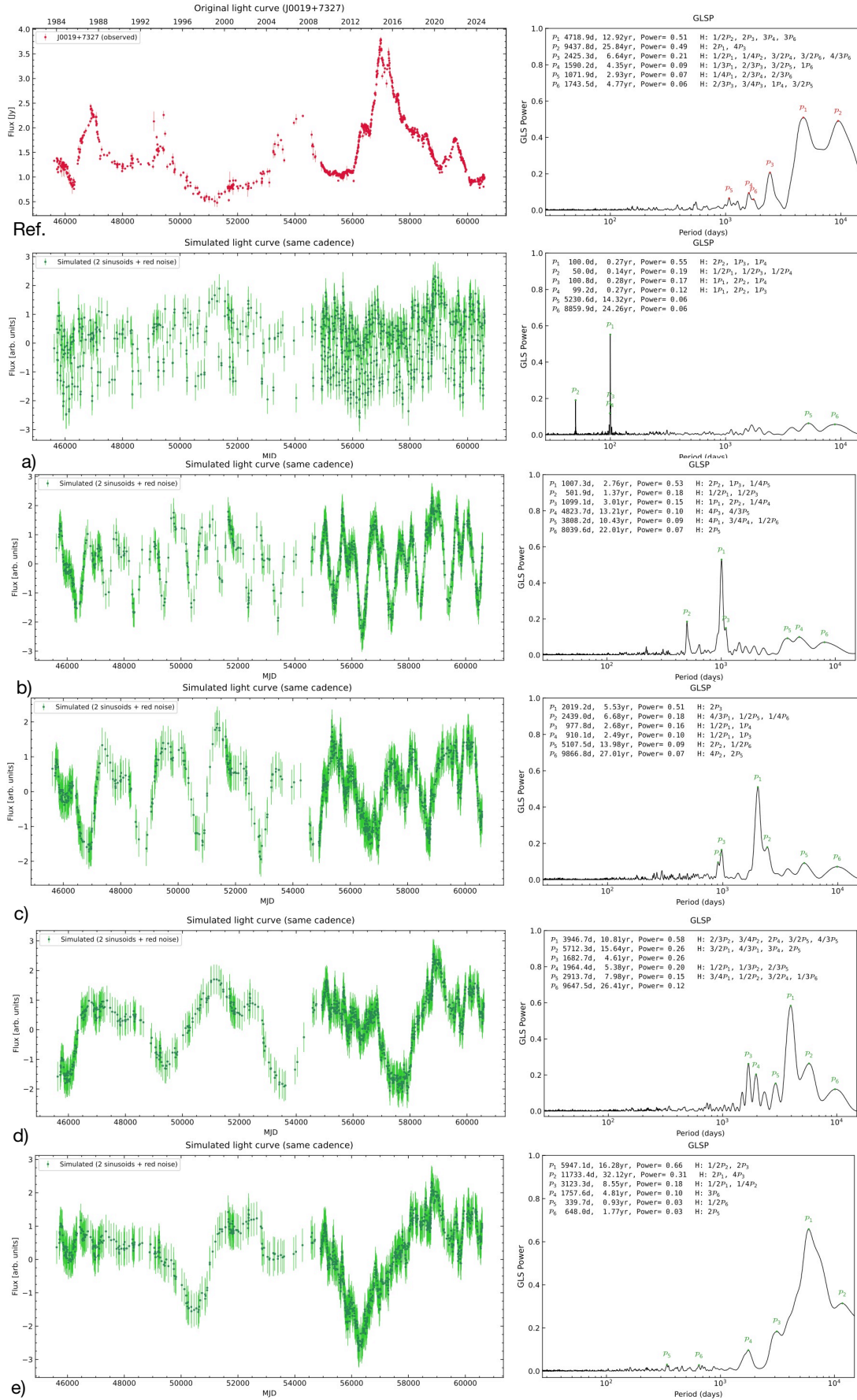


Figure C.6: Test 6: For a simulated period (in days): a) 100, b) 1000, c) 2000, d) 4000, e) 6000.

Appendix D

Observed PSDs and Slopes (β)

To estimate the PSD slope, we adopted the procedure used by [O'Neill et al. \(2022\)](#), based on the forward Monte Carlo approach of [Uttley et al. \(2002\)](#). Interpolation onto a regular time grid and the application of a Hann window follow the procedure described by [Max-Moerbeck et al. \(2014\)](#). The following panels, ordered by right ascension, present the results obtained for the combined OVRO and UMRAO light curves. The green line shows the PSD referred to as the observed PSD in this work. However, it does not correspond to the raw periodogram of the original time series, because the light curve was resampled onto a regular grid, its linear trend was removed, a Hann window was applied, and the resulting periodogram powers were subsequently averaged within logarithmic frequency bins. The intrinsic PSD is understood here as the underlying distribution of variability power in the source, before the effects introduced by the sampling, the finite duration of the light curve, and the processing applied to the data.

The slope of the intrinsic PSD was estimated by assuming a power-law model, $P(f) \propto f^{-\beta}$. For each value in a grid of trial slopes β , 1000 synthetic light curves were generated using the Timmer and Koenig method. The simulations were processed using the same resampling, detrending, Hann windowing, periodogram calculation, and logarithmic binning applied to the real data. The observed PSD was compared with each simulated ensemble using a χ^2 -like metric, and the optimal slope was defined as the value of β corresponding to the minimum of the χ^2 curve as a function of β . The p -value and confidence interval were calculated using a significance level of 0.05, whereas the uncertainty in β was estimated empirically through recovery tests in which simulations with known input slopes were subjected to the same complete estimation procedure. The estimated value of β and its uncertainty are indicated in the lower-left corner of each panel.

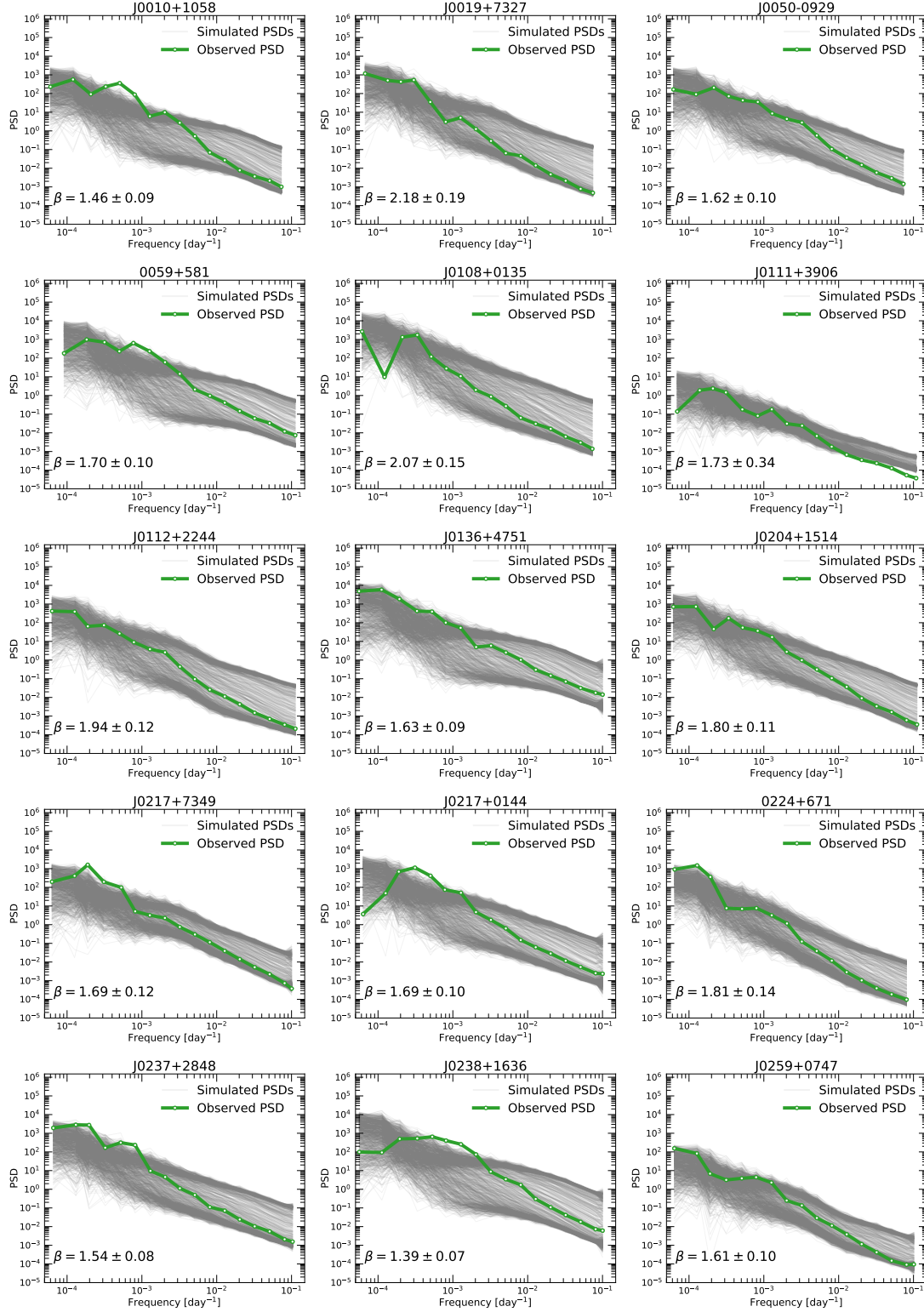
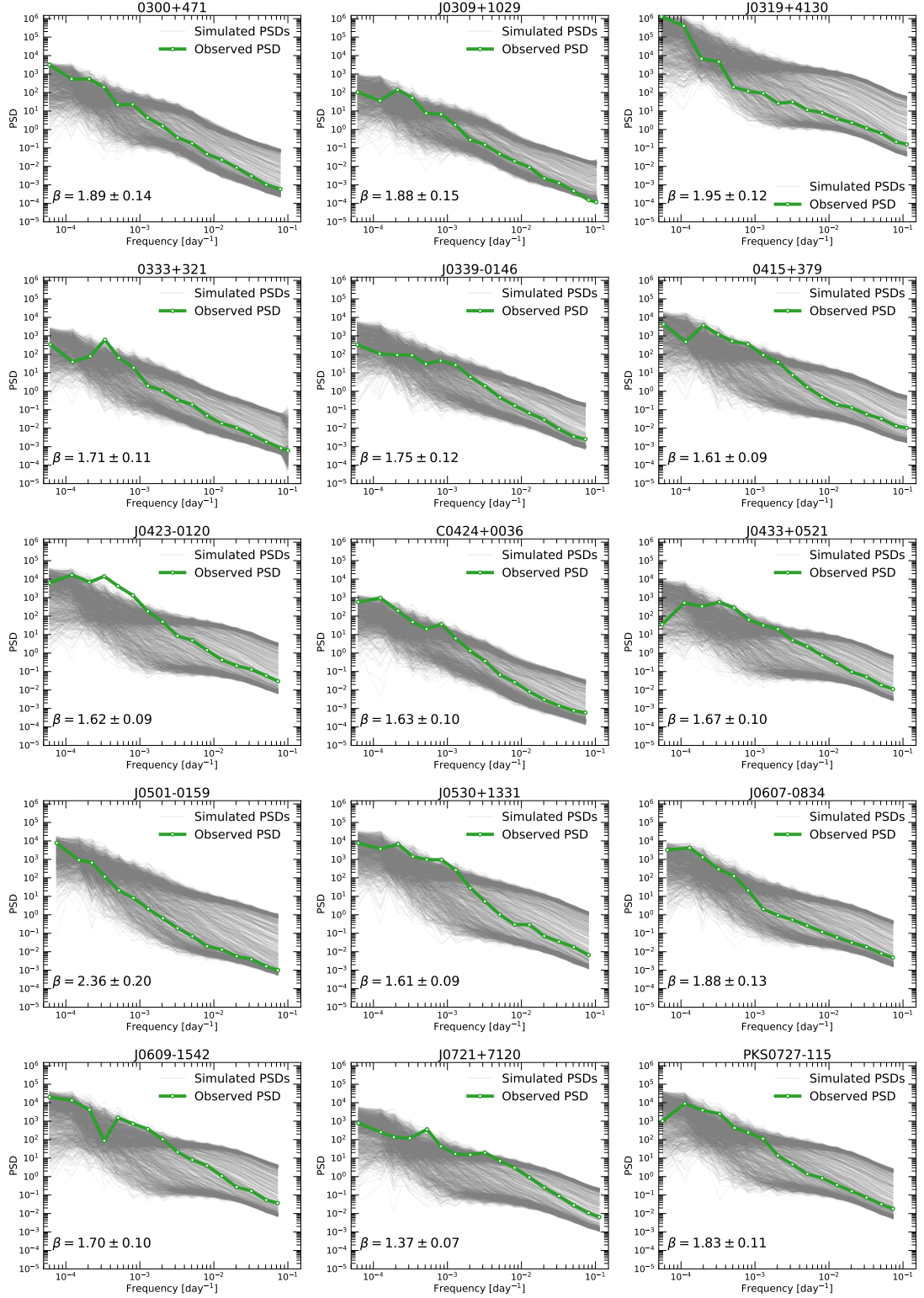
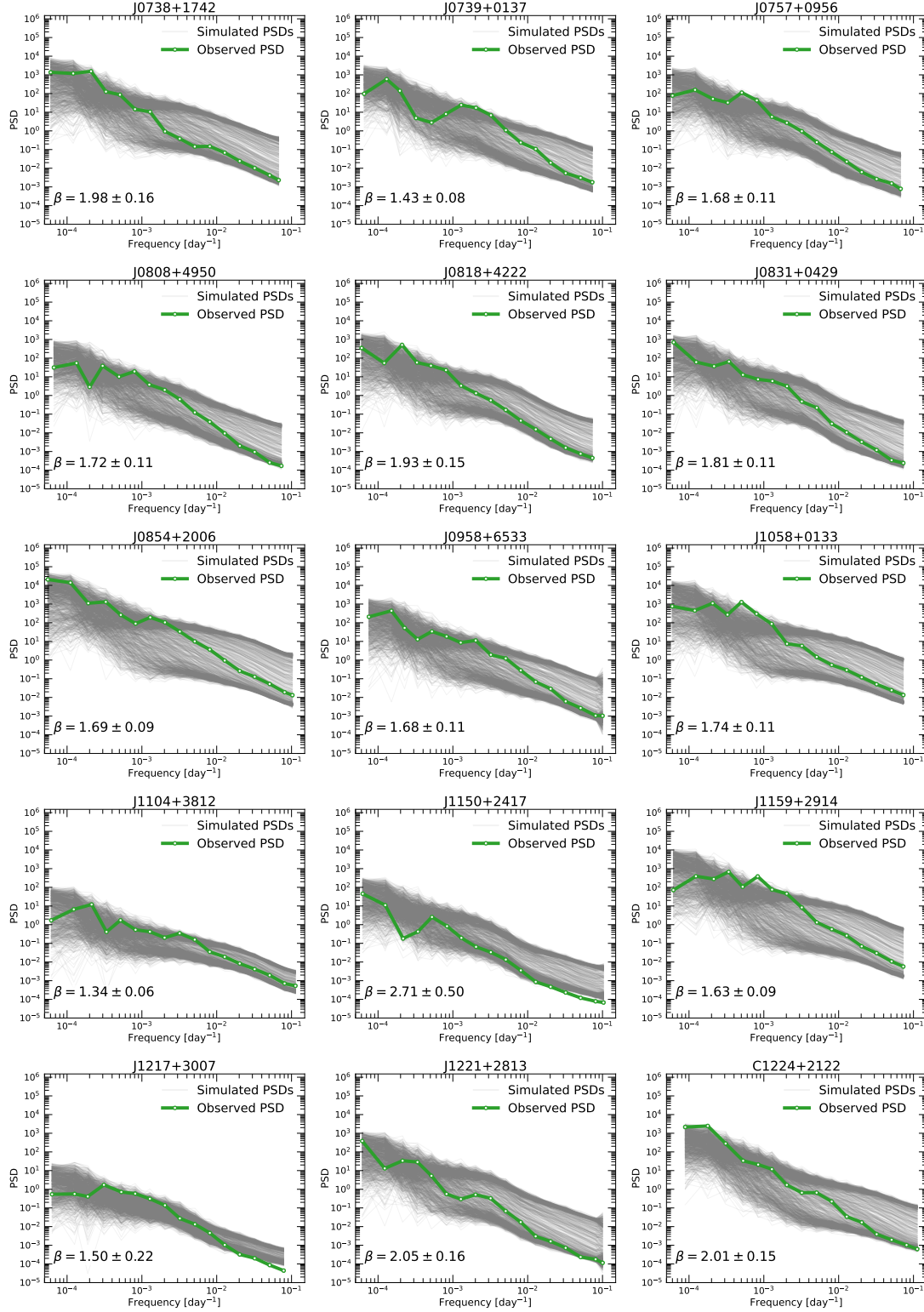
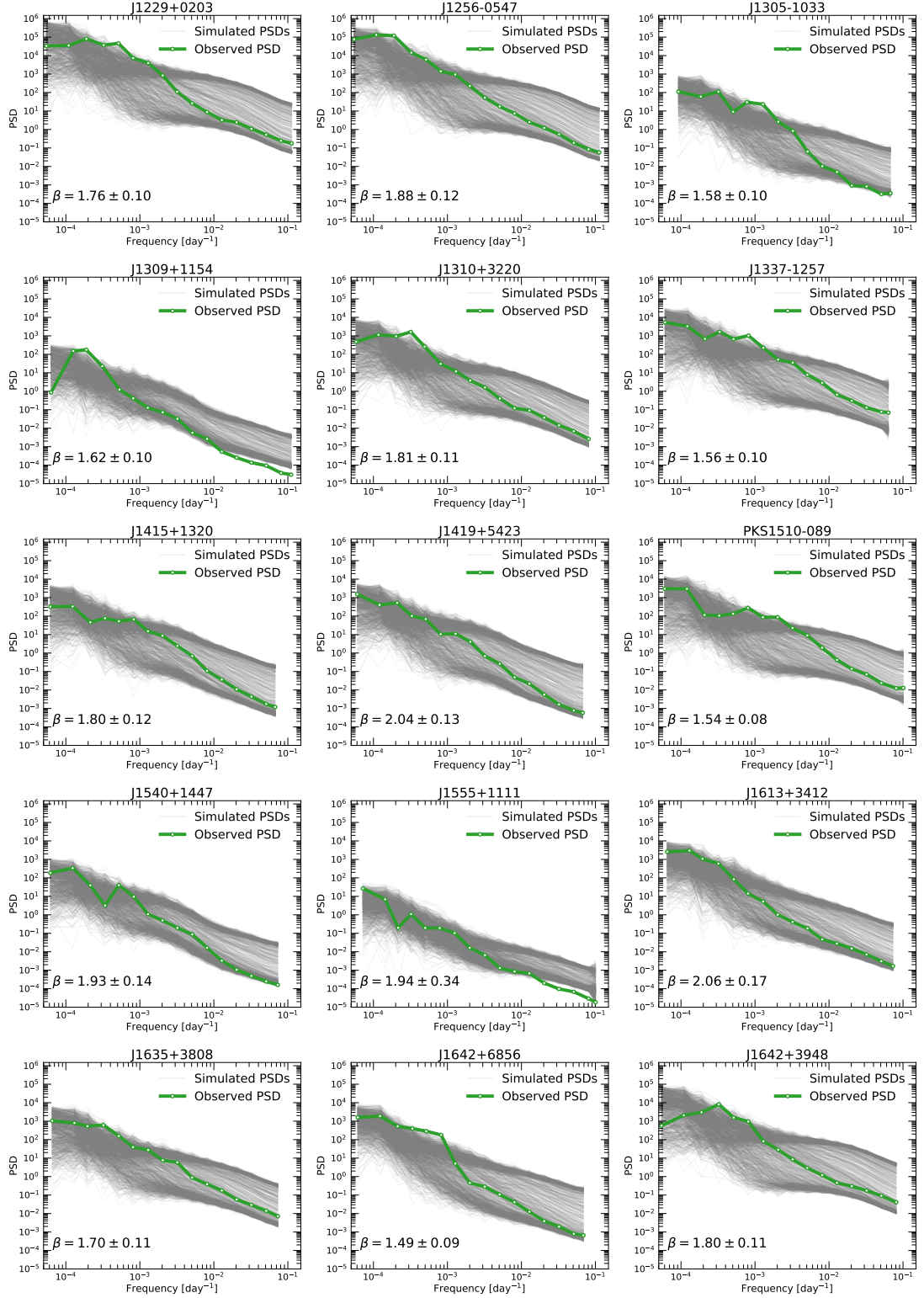
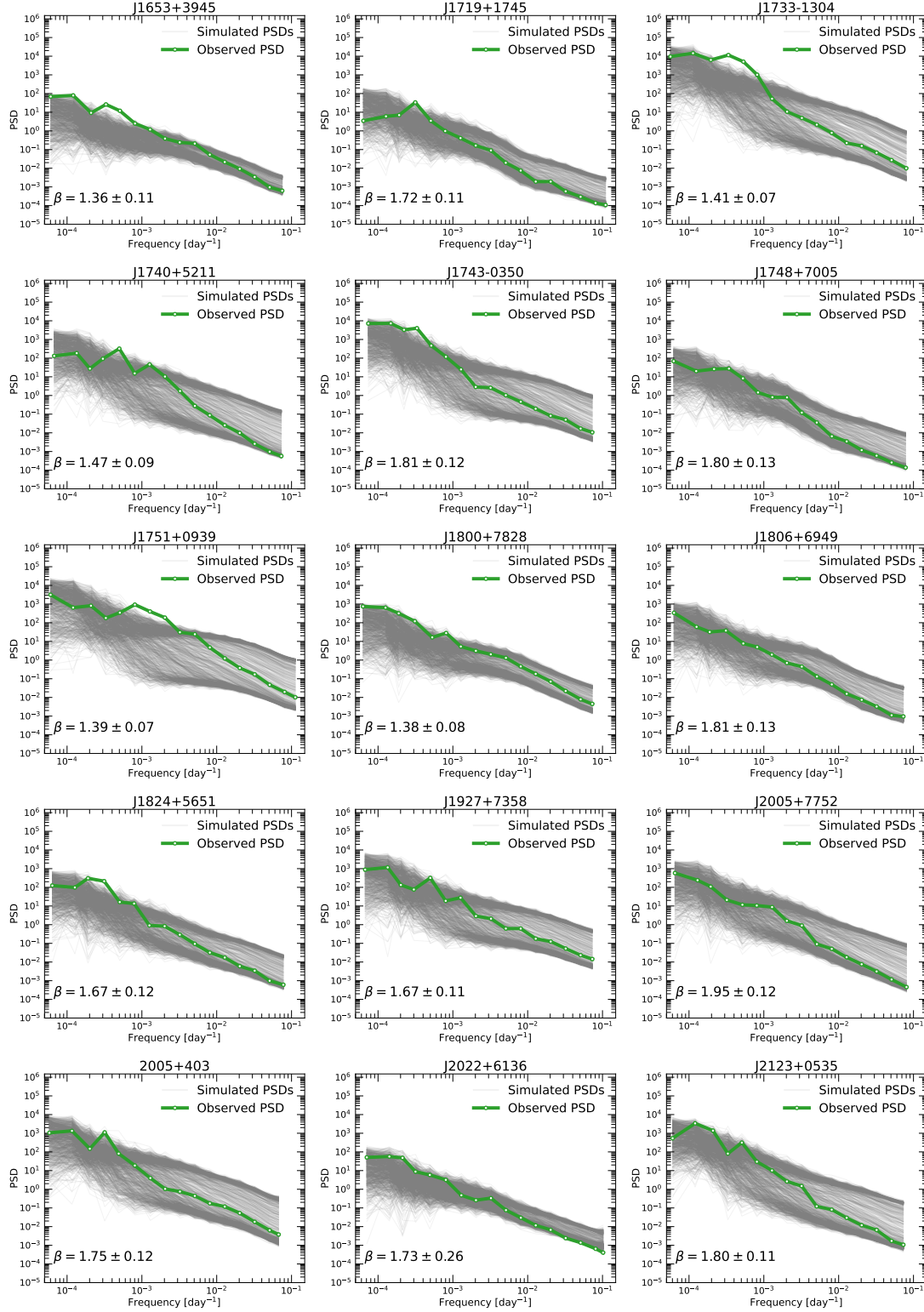


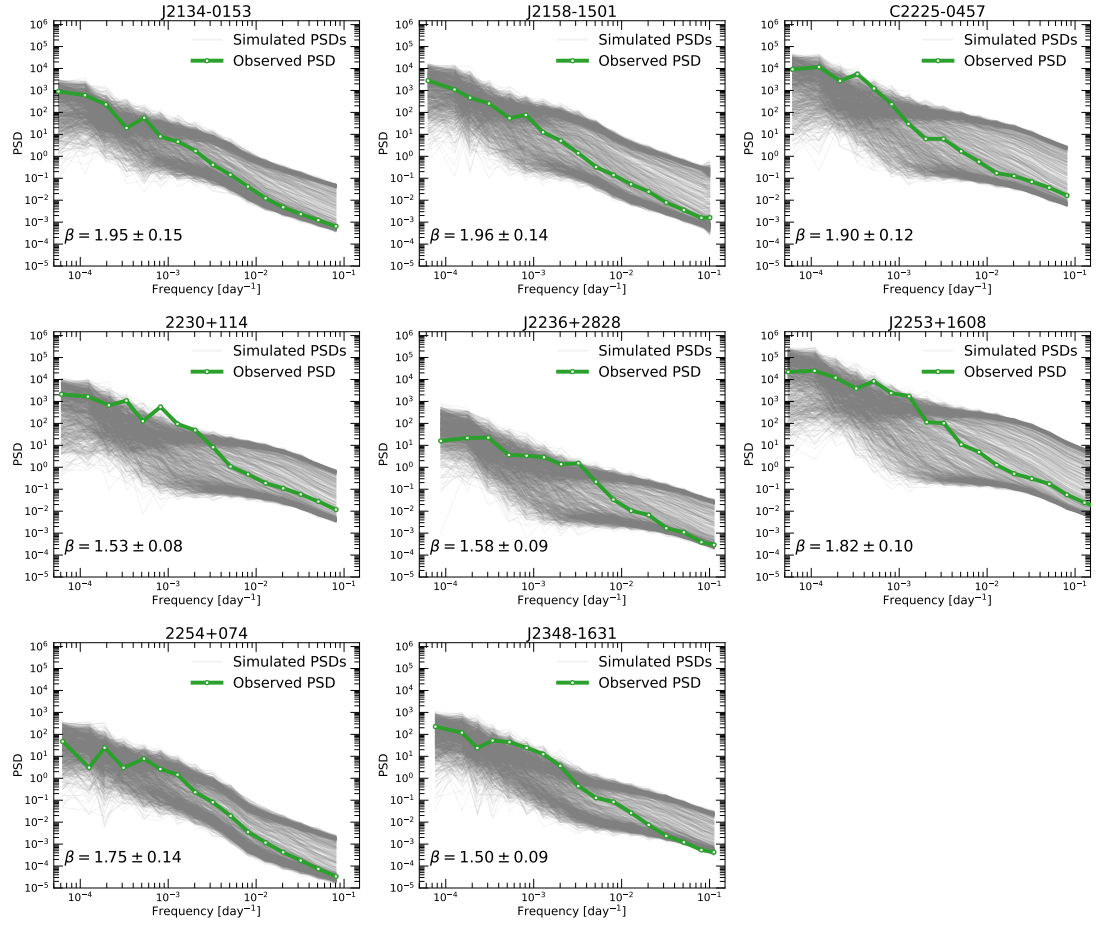
Figure D.1: Observed PSDs and estimated β values for sources 1–15 of the 83-source sample.

Figure D.2: Observed PSDs and estimated β values for sources 16–30 of the 83-source sample.

Figure D.3: Observed PSDs and estimated β values for sources 31–45 of the 83-source sample.

Figure D.4: Observed PSDs and estimated β values for sources 46–60 of the 83-source sample.

Figure D.5: Observed PSDs and estimated β values for sources 61–75 of the 83-source sample.

Figure D.6: Observed PSDs and estimated β values for sources 76–83 of the 83-source sample.

Appendix E

Sine Wave Fitting Results

As a complement to the main analyses presented in this thesis, a brief summary of the sine-wave fitting procedure (SWF) applied in [Molina et al. \(2026\)](#) is provided below. The methodology follows the approach previously employed in other studies of long-term variability ([O'Neill et al., 2022](#); [Kiehlmann et al., 2025](#); [de la Parra et al., 2025](#); [Hincks et al., 2026](#)).

For the 83 sources in the combined OVRO and UMRAO sample, the flux density was represented by the sinusoidal model

$$S(t) = A \sin \left[\frac{2\pi(t - t_0)}{P} - \phi_0 \right] + S_0, \quad (\text{E.1})$$

where P is the sine-wave period, A is its amplitude, ϕ_0 is the phase at $t = t_0$, and S_0 is the mean flux density. The reference time t_0 is defined as the temporal midpoint of the observations for each source.

The best-fit parameters were determined by maximizing the likelihood function

$$\ln \mathcal{L} = -\frac{1}{2} \sum_i \left[\frac{(S_i - S(t_i))^2}{\sigma_i^2 + \sigma_0^2} + \ln (\sigma_i^2 + \sigma_0^2) \right], \quad (\text{E.2})$$

where S_i and σ_i are, respectively, the observed flux density and its uncertainty at time t_i . The parameter σ_0 accounts for additional scatter in the light curve that is not represented by the original error bars, such as that associated with the intrinsic variability of the blazar. The posterior distributions of the model

parameters were obtained using a Markov chain Monte Carlo (MCMC) sampler.

J1309+1154 was treated as a special case. For this source, a linear trend was included in the sinusoidal model to account for the detrending of the combined light curve:

$$S(t) = S_0 + A \sin \left[\frac{2\pi(t - t_0)}{P} - \phi_0 \right] + \beta \frac{t - t_0}{365.25}, \quad (\text{E.3})$$

where β is the linear-trend coefficient, expressed in Jy yr^{-1} , and the factor 365.25 converts the time interval from days to years. Here, β refers to the temporal trend in flux density and should not be confused with the PSD slope. No time-dependent amplitude term was included, because its effect on the inferred periodicity was found to be negligible. The best-fit parameters for J1309+1154 are presented in Table E.1.

Table E.1: Best-fit SWF parameters for the combined light curve of J1309+1154 using the sine-plus-trend model.

Parameter	Sine + trend
P (days)	6551 ± 26
A (Jy)	0.2222 ± 0.0027
ϕ_0 (rad)	-1.138 ± 0.020
S_0 (Jy)	0.7862 ± 0.0032
β (Jy yr^{-1})	-0.00755 ± 0.00022
σ_0 (Jy)	0.0570 ± 0.0014

Table E.2: Best-fit Periods from Sine-wave Fits

Blazar	P (days) (UMRAO)	P (days) (OVRO)	P (days) (combined)	Source	P (days) (UMRAO)	P (days) (OVRO)	P (days) (combined)
J0010+1058	1868 ± 8	6920^{+400}_{-316}	1845 ± 5	J1217+3007	2518^{+65}_{-56}	4843 ± 86	4401 ± 47
J0019+7327	7064^{+143}_{-136}	4666^{+61}_{-57}	4616^{+46}_{-48}	J1221+2813	16720^{+34}_{-74}	5891^{+199}_{-174}	6309 ± 66
J0050-0929	6294 ± 244	16264^{+156}_{-323}	9076 ± 225	C1224+2122	10602^{+478}_{-769}	6213^{+110}_{-102}	5922 ± 46
0059+581	1632 ± 24	2697 ± 29	1699 ± 8	J1229+0203	2987 ± 13	18192^{+169}_{-344}	3065 ± 14
J0108+0135	6686 ± 102	12039^{+2846}_{-1765}	8694 ± 116	J1256-0547	12788^{+254}_{-238}	3118^{+30}_{-28}	5220 ± 64
J0111+3906	12075^{+1772}_{-1648}	14811^{+116}_{-242}	7702^{+130}_{-123}	J1305-1033	9956^{+905}_{-1127}	2567 ± 36	2516 ± 33
J0112+2244	8995 ± 335	15455^{+726}_{-1173}	9275 ± 104	J1309+1154	6391^{+115}_{-107}	6201 ± 48	6509 ± 32
J0136+4751	12655 ± 198	1478^{+14}_{-12}	13148^{+210}_{-198}	J1310+3220	3362 ± 26	4414 ± 21	3559 ± 18
J0204+1514	16077^{+178}_{-370}	16195^{+91}_{-194}	3908 ± 27	J1337-1257	2647 ± 21	2039 ± 14	2521 ± 11
J0217+7349	4865 ± 80	2676 ± 17	2786 ± 18	J1415+1320	3206 ± 73	3670 ± 60	3262 ± 16
J0217+0144	3007 ± 27	3971 ± 40	4082 ± 27	J1419+5423	16512^{+73}_{-154}	4159 ± 47	4384 ± 22
0224+671	14456^{+810}_{-1237}	1829 ± 14	1914 ± 6	PKS1510-089	9859 ± 319	1687^{+21}_{-20}	9780 ± 197
J0237+2848	7084 ± 129	9934^{+1031}_{-734}	4942 ± 38	J1540+1447	8264^{+307}_{-263}	2997^{+40}_{-38}	8408 ± 67
J0238+1636	1988 ± 12	2501^{+36}_{-34}	2062 ± 7	J1555+1111	14314^{+567}_{-756}	10789^{+759}_{-605}	6550^{+85}_{-79}
J0259+0747	14909^{+1104}_{-1637}	11530 ± 3357	13581^{+306}_{-290}	J1613+3412	13177^{+374}_{-341}	5348 ± 44	7180 ± 36
0300+471	16416^{+378}_{-528}	15379^{+438}_{-744}	6198 ± 37	J1635+3808	9853 ± 146	967 ± 6	982 ± 3
J0309+1029	3934^{+56}_{-53}	5508^{+110}_{-98}	6860 ± 37	J1642+6856	15952^{+734}_{-1232}	16632^{+246}_{-500}	6377 ± 68
J0319+4130	17186 ± 100	18347^{+109}_{-230}	18464^{+21}_{-44}	J1642+3948	5966 ± 39	17965^{+335}_{-655}	5608 ± 48
0333+321	15686^{+460}_{-685}	15456^{+644}_{-1091}	12781 ± 156	J1653+3945	12622^{+531}_{-437}	2503 ± 33	2305 ± 11
J0339-0146	4277 ± 74	5990 ± 73	6335 ± 36	J1719+1745	2763 ± 26	7531^{+332}_{-277}	7187 ± 75
0415+379	4186^{+49}_{-53}	1539 ± 12	3784 ± 30	J1733-1304	10261^{+416}_{-355}	2866 ± 39	3767 ± 32
J0423-0120	2979 ± 22	4537 ± 35	7716 ± 85	J1740+5211	9731^{+517}_{-444}	1733 ± 13	1723 ± 5
C0424+0036	7908 ± 168	16255^{+163}_{-338}	9094 ± 71	J1743-0350	9989^{+532}_{-431}	1650 ± 8	2745 ± 13
J0433+0521	3985 ± 32	5706^{+98}_{-93}	3932 ± 21	J1748+7005	16108^{+125}_{-263}	13770^{+1676}_{-1841}	6252 ± 34
J0501-0159	13226^{+114}_{-235}	11151^{+996}_{-802}	6074 ± 87	J1751+0939	16315^{+321}_{-556}	14755^{+1404}_{-1940}	848 ± 3
J0530+1331	3761 ± 40	15895^{+150}_{-311}	3953^{+40}_{-38}	J1800+7828	13780^{+1065}_{-910}	1944^{+48}_{-43}	9821 ± 198
J0607-0834	12873^{+678}_{-572}	4794 ± 56	9242 ± 72	J1806+6949	14235^{+946}_{-831}	6612^{+139}_{-129}	6097 ± 69
J0609-1542	16442^{+131}_{-271}	2198 ± 23	4184 ± 33	J1824+5651	6840^{+201}_{-176}	3355 ± 25	3022 ± 17
J0721+7120	13095^{+646}_{-541}	12042^{+2175}_{-1680}	14777 ± 224	J1927+7358	4855^{+91}_{-84}	5133^{+130}_{-115}	7924 ± 67
PKS0727-115	6534 ± 35	17880^{+238}_{-491}	7356 ± 54	J2005+7752	11496^{+436}_{-404}	2751 ± 26	6749 ± 81
J0738+1742	11531^{+509}_{-444}	3265 ± 22	4252 ± 32	2005+403	17638^{+310}_{-486}	4263 ± 44	8768 ± 45
J0739+0137	6096 ± 119	5298^{+119}_{-110}	6536 ± 68	J2022+6136	7430^{+284}_{-256}	10956^{+1554}_{-1121}	6262 ± 95
J0757+0956	3637^{+62}_{-55}	7801^{+940}_{-629}	8138 ± 74	J2123+0535	2403 ± 15	5961^{+125}_{-117}	7184 ± 89
J0808+4950	8061^{+1980}_{-1014}	11426^{+2707}_{-1929}	4260 ± 38	PKS 2131-021	10735^{+1291}_{-639}	1787 ± 5	1723 ± 3
J0818+4222	14068^{+690}_{-580}	3800 ± 24	4632 ± 24	J2158-1501	2013 ± 33	16044^{+174}_{-365}	16246^{+24}_{-52}
J0831+0429	12916^{+980}_{-748}	14549^{+1342}_{-1796}	6501 ± 76	C2225-0457	3097 ± 16	16320^{+74}_{-158}	9066 ± 163
J0854+2006	10276 ± 100	2124 ± 25	12569^{+177}_{-167}	2230+114	12002^{+237}_{-220}	15389^{+746}_{-1354}	1464 ± 5
J0958+6533	8941^{+516}_{-422}	13226^{+275}_{-554}	6741 ± 103	J2236+2828	1087^{+3}_{-3}	6578^{+208}_{-181}	6504^{+161}_{-147}
J1058+0133	6152 ± 86	6490^{+236}_{-205}	5395 ± 62	J2253+1608	2112 ± 7	2768 ± 21	2735 ± 10
J1104+3812	16211^{+292}_{-576}	4908 ± 73	5116^{+65}_{-68}	2254+074	8179^{+649}_{-467}	3066^{+51}_{-48}	3466^{+39}_{-41}
J1150+2417	11523^{+2290}_{-1505}	13457^{+1028}_{-837}	9333 ± 94	J2348-1631	1384^{+20}_{-16}	1283 ± 9	1306 ± 6
J1159+2914	1237 ± 6	2396 ± 23	2760 ± 13	...	...	...	...

Note. PKS 2131-021 includes Haystack data obtained between 1975 and 1982 (O'Neill et al. (2022) and Kiehlmann et al. (2025)).

Appendix F

Proposed WWZ Metric as a Visual Guide for Inspecting a Sample of Light Curves

For the globally normalized mosaic (4.21), the WWZ map of each source is scaled relative to the maximum WWZ value found across the entire sample. If $Z_j(P, t)$ represents the WWZ power of source j , then

$$Z_{\max, \text{global}} = \max_j \left[\max_{P, t} Z_j(P, t) \right]. \quad (\text{F.1})$$

The map displayed in the mosaic therefore corresponds to

$$Z_{j, \text{norm}}(P, t) = \frac{Z_j(P, t)}{Z_{\max, \text{global}}}. \quad (\text{F.2})$$

In this way, the color bar allows the relative intensity of the WWZ structures to be compared directly among different sources.

The curve on the left-hand side represents a diagnostic metric of temporal coherence. For each source, the maximum WWZ value within the analyzed period range is first identified:

$$Z_{0,j} = Z_j(P_{0,j}, t_{0,j}) = \max_{P, t} Z_j(P, t), \quad (\text{F.3})$$

where $P_{0,j}$ and $t_{0,j}$ are, respectively, the period and time associated with this

maximum.

To evaluate whether the signal remains close to the same period, a local window around $P_{0,j}$ is defined:

$$\mathcal{W}_j = \left\{ P : \frac{|P - P_{0,j}|}{P_{0,j}} \leq \epsilon \right\}, \quad (\text{F.4})$$

where ϵ controls the relative width of the period window. From this window, a local WWZ-power time series is constructed,

$$R_j(t_k) = \max_{P \in \mathcal{W}_j} Z_j(P, t_k), \quad (\text{F.5})$$

which measures, at each time t_k , the maximum power near the period $P_{0,j}$.

The temporal evaluation is restricted to the actual observational coverage of each source. For this purpose, we define

$$\mathcal{C}_j = \{t_k : t_{\text{start},j} \leq t_k \leq t_{\text{end},j}\}, \quad (\text{F.6})$$

where $t_{\text{start},j}$ and $t_{\text{end},j}$ are the first and last observed times of source j , respectively. Within this coverage, times for which the local power exceeds a fraction q of the maximum value of the source are considered active:

$$\mathcal{A}_j = \{t_k \in \mathcal{C}_j : R_j(t_k) \geq q Z_{0,j}\}. \quad (\text{F.7})$$

The temporal persistence is defined as

$$S_{T,j} = \frac{|\mathcal{A}_j|}{|\mathcal{C}_j|}. \quad (\text{F.8})$$

Therefore, $S_{T,j}$ measures the fraction of the temporal coverage during which the WWZ power remains elevated near the period $P_{0,j}$.

In addition to the overall persistence, the temporal continuity is measured. Let $\mathcal{L}_j$ be the longest continuous interval contained in $\mathcal{A}_j$. The continuity is defined as

$$S_{\text{cont},j} = \frac{|\mathcal{L}_j|}{|\mathcal{A}_j|}. \quad (\text{F.9})$$

This factor penalizes signals that appear fragmented in time, even when the total

fraction of active times is relatively high.

The number of sustained cycles is estimated from the temporal duration of the longest continuous interval. If $\Delta t_{\text{longest},j}$ is the duration of $\mathcal{L}_j$, then

$$N_{\text{cyc,long},j} = \frac{\Delta t_{\text{longest},j}}{P_{0,j}}. \quad (\text{F.10})$$

The contribution associated with the number of cycles is normalized as

$$S_{\text{cyc},j} = \min \left[\frac{N_{\text{cyc,long},j}}{N_{\text{cyc,ref}}}, 1 \right], \quad (\text{F.11})$$

where $N_{\text{cyc,ref}}$ is the number of cycles used as a reference for considering that a signal is sustained over a sufficient fraction of the temporal baseline.

A contrast term is also included. First, the typical power level around the evaluated period is defined as

$$B_j = \text{median}_{t_k \in \mathcal{C}_j} [R_j(t_k)]. \quad (\text{F.12})$$

The contrast is then defined as

$$S_{\text{contrast},j} = \frac{Z_{0,j} - B_j}{Z_{0,j}}. \quad (\text{F.13})$$

This factor measures how strongly the WWZ maximum stands out relative to the typical power level of the temporal ridge. A high value indicates a well-defined structure, whereas a low value indicates a more diffuse signal or one that is less clearly separated from the local background.

To incorporate the minimum-cycle criterion, the observed temporal baseline of each source is defined as

$$T_{\text{obs},j} = t_{\text{end},j} - t_{\text{start},j}. \quad (\text{F.14})$$

Optionally, a cycle-based rejection factor can be applied,

$$G_j = \begin{cases} 0, & \text{if } P_{0,j} > T_{\text{obs},j}/N_{\text{cyc,ref}}, \\ 1, & \text{if } P_{0,j} \leq T_{\text{obs},j}/N_{\text{cyc,ref}}. \end{cases} \quad (\text{F.15})$$

When this rejection criterion is not applied, $G_j = 1$ is adopted for all sources.

The maximum power can be incorporated either linearly or through a saturating

transformation. This is represented by a general function Φ , defined as

$$\Phi(Z_{0,j}) = Z_{0,j} \quad (\text{F.16})$$

for the linear case, or alternatively as

$$\Phi(Z_{0,j}) = \log(1 + Z_{0,j}) \quad (\text{F.17})$$

to reduce the relative weight of very intense but temporally localized WWZ maxima.

Using the definitions above, the raw coherence metric is defined as

$$M_{\text{raw},j} = G_j \Phi(Z_{0,j}) S_{T,j}^\alpha S_{\text{cont},j} S_{\text{cyc},j} S_{\text{contrast},j}, \quad (\text{F.18})$$

where α controls the weight assigned to temporal persistence. This metric favors sources with high WWZ power, temporal persistence, continuity, cycle coverage, and contrast relative to the local background.

Finally, the metric is normalized relative to the maximum value obtained within the sample:

$$M_j = \frac{M_{\text{raw},j}}{\max_j M_{\text{raw},j}}. \quad (\text{F.19})$$

The lateral curve represents this normalized value through a visual score,

$$\text{Score}_j = S_{\min} + (S_{\max} - S_{\min}) M_j. \quad (\text{F.20})$$

Thus, the lateral curve does not measure only the maximum WWZ power, but rather a diagnostic combination of power, temporal persistence, continuity, number of cycles, and contrast.

Metric calibration

The metric defined in equation (F.18) depends on adjustable parameters that control the strictness of the diagnostic inspection:

- ϵ : controls the relative width of the period window defined in equation (F.4). Small values require the signal to remain very close to $P_{0,j}$, whereas larger values allow for drift or broadening of the ridge.
- q : sets the relative threshold used to define the active times in equation (F.7). High values select only regions of strong WWZ power, whereas low values allow weaker structures to be included.
- $N_{\text{cyc,ref}}$: establishes the reference number of cycles used in equation (F.11). It also determines the optional rejection criterion defined in equation (F.15).
- G_j : activates or deactivates the cycle-based rejection criterion. If the rejection criterion is applied, sources satisfying $P_{0,j} > T_{\text{obs},j}/N_{\text{cyc,ref}}$ receive $G_j = 0$. If it is not applied, $G_j = 1$ is used for the entire sample.
- α : controls the weight of temporal persistence in equation (F.18). Larger values more strongly penalize signals that are brief or localized in time.
- $\Phi(Z_{0,j})$: defines how the maximum WWZ power enters the metric. The linear form in equation (F.16) directly preserves the amplitude of the maximum, whereas the saturated form in equation (F.17) reduces the weight of extremely intense maxima.
- S_{min} and S_{max} : define the visual range of the score in equation (F.20). These parameters modify only the display scale of the lateral curve and do not alter the relative ranking of the sources.

The calibration of this proposed metric uses our best candidates, J1309+1154 and J2134–0153, as references. For example, the period precision of J2134–0153 and the max power, persistence, and number of cycles of J1309+1154 are adopted as minimum reference values.

Appendix G

Periodicity Analysis Methods

The methods used for the preliminary periodicity analysis are described below, followed by the results obtained for the 83 sources in the combined OVRO+UMRAO sample.

G.0.1 Generalized Lomb–Scargle periodogram

Periodicity analysis in astronomical light curves requires methods capable of handling irregular sampling and incomplete temporal windows. In a uniformly sampled series, the Discrete Fourier Transform is defined on a regular time grid,

$$\hat{g}_k = \sum_{j=0}^{N-1} g_j e^{-2\pi i k j / N}, \quad (\text{G.1})$$

where the basis functions

$$\phi_k(t_j) = e^{2\pi i k j / N} \quad (\text{G.2})$$

are orthogonal over the grid,

$$\langle \phi_k, \phi_l \rangle = \sum_{j=0}^{N-1} \phi_k(t_j) \overline{\phi_l(t_j)} = N \delta_{kl}. \quad (\text{G.3})$$

This orthogonality allows the contributions from different frequencies to be separated. In addition, for a constant time step Δt , the Nyquist limit is given by

$$f_{\text{Ny}} = \frac{1}{2\Delta t}. \quad (\text{G.4})$$

Therefore, the FFT is appropriate when the series lies on a regular grid. In real light curves, the observation times are irregular and may contain long gaps. In that case, directly applying an FFT requires interpolating, rebinning, or completing the time series, which modifies the window function and may introduce spurious spectral power.

If the sampling is described by a window function

$$W(t) = \sum_{i=1}^N \delta(t - t_i), \quad (\text{G.5})$$

the observed signal can be written as

$$y_{\text{obs}}(t) = y(t)W(t). \quad (\text{G.6})$$

By the convolution theorem, its transform is given by

$$\mathcal{F}\{y_{\text{obs}}\} = \mathcal{F}\{y\} * \mathcal{F}\{W\}. \quad (\text{G.7})$$

For uniform sampling, $W(t)$ corresponds to a regular Dirac comb; for irregular sampling, its transform has a more complex structure, which may produce aliasing and an artificial redistribution of power in the spectrum.

The Lomb–Scargle periodogram addresses this problem by reformulating spectral analysis as a sinusoidal least-squares fit at each trial frequency. For an angular frequency $\omega = 2\pi f$, the basic model is

$$y(t_i) = A \cos(\omega t_i) + B \sin(\omega t_i). \quad (\text{G.8})$$

The parameters A and B are obtained by minimizing

$$\chi^2(\omega) = \sum_{i=1}^N [y_i - A \cos(\omega t_i) - B \sin(\omega t_i)]^2. \quad (\text{G.9})$$

The cross term between the sine and cosine components is

$$\sum_{i=1}^N \cos(\omega t_i) \sin(\omega t_i). \quad (\text{G.10})$$

On a uniform grid, this term vanishes because of orthogonality, but for irregular sampling, in general,

$$\sum_{i=1}^N \cos(\omega t_i) \sin(\omega t_i) \neq 0. \quad (\text{G.11})$$

For this reason, the sine and cosine components are not directly orthogonal over the observed times.

Scargle introduced a frequency-dependent time shift, $\tau(\omega)$, defined so that the shifted sine and cosine functions are orthogonal:

$$\sum_{i=1}^N \cos[\omega(t_i - \tau)] \sin[\omega(t_i - \tau)] = 0. \quad (\text{G.12})$$

This condition leads to

$$\tan(2\omega\tau) = \frac{\sum_i \sin(2\omega t_i)}{\sum_i \cos(2\omega t_i)}. \quad (\text{G.13})$$

With this shift, and using centered data $\tilde{y}_i = y_i - \bar{y}$, the Lomb–Scargle periodogram can be written as

$$P_{\text{LS}}(\omega) = \frac{1}{2\sigma_y^2} \left[\frac{(\sum_i \tilde{y}_i \cos[\omega(t_i - \tau)])^2}{\sum_i \cos^2[\omega(t_i - \tau)]} + \frac{(\sum_i \tilde{y}_i \sin[\omega(t_i - \tau)])^2}{\sum_i \sin^2[\omega(t_i - \tau)]} \right], \quad (\text{G.14})$$

where σ_y^2 is the variance of the data. This expression measures, at each frequency, how well a sinusoidal combination describes the observed variability.

The generalized Lomb–Scargle periodogram extends this formulation by incorporating a floating mean and weights associated with the observational uncertainties. At each frequency, the model

$$y(t_i) = a \cos(\omega t_i) + b \sin(\omega t_i) + c, \quad (\text{G.15})$$

is fitted through weighted minimization

$$\chi^2(\omega) = \sum_{i=1}^N \left[\frac{y_i - a \cos(\omega t_i) - b \sin(\omega t_i) - c}{\sigma_i} \right]^2. \quad (\text{G.16})$$

The constant term c allows the mean of the model to adjust to the data at each frequency, while the uncertainties σ_i weight the contribution of each measurement.

In this work, a normalized GLSP power was used. Under this normalization, the power at a frequency ω is interpreted as the relative reduction in χ^2 between a constant model and the best sinusoidal model at that frequency:

$$P_{\text{GLSP}}(\omega) = \frac{\chi_0^2 - \chi^2(\omega)}{\chi_0^2}. \quad (\text{G.17})$$

Here, χ_0^2 corresponds to the reference fit associated with a constant model, and $\chi^2(\omega)$ corresponds to the weighted sinusoidal fit. Therefore, the GLSP power does not represent a physical power of the source, but rather a statistic that quantifies how much the sinusoidal model improves upon the constant model. With the standard normalization used, the power is dimensionless and takes values between 0 and 1, where larger values indicate a greater reduction in χ^2 . An example of this type of analysis is shown in Figure G.1.

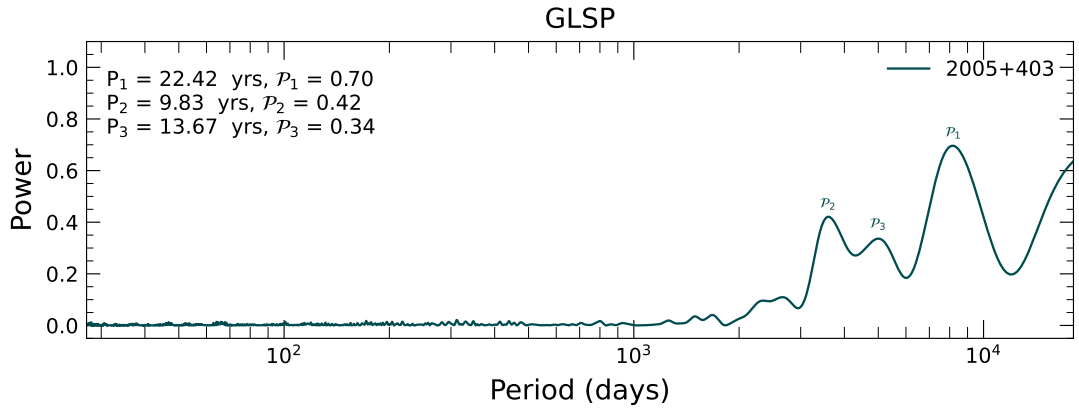


Figure G.1: GLSP example. The detected main period, corresponding to the highest peak, is 22.42 years, with a power of 0.7.

G.0.2 Sinusoidal least squares fitting

Sinusoidal least squares fitting was used as a diagnostic tool to visualize, in the time domain, the periods previously identified with the GLSP. In the previous stage, the GLSP analysis produced, for each source, results containing the main periods P_i and their associated powers, with $i = 1, \dots, 6$. In this preliminary stage, the first three periods, P_1 , P_2 , and P_3 , were used as references to perform the sinusoidal fits. The GLSP powers do not enter the minimization and are used only as auxiliary information in the plot legends.

For a light curve with flux density measurements $S_j = S_\nu(t_j)$ and uncertainties σ_j , the model fitted is

$$S_\nu^{\text{model}}(t_j) = A_i \sin \left(2\pi \frac{t_j}{P_i} + \phi_i \right) + C_i, \quad (\text{G.18})$$

where A_i is the amplitude, ϕ_i is the phase, C_i is the mean level of the light curve, and P_i is the period used in the fit. For each candidate period, P_i is restricted to a very narrow interval around the value provided by the GLSP,

$$P_i - 10^{-5} \leq P \leq P_i + 10^{-5}. \quad (\text{G.19})$$

Therefore, the fit mainly optimizes the amplitude, phase, and mean level of the sinusoid associated with the selected period.

The model parameters are obtained through a nonlinear weighted least squares minimization. For each candidate period, the minimized quantity is

$$\chi_i^2 = \sum_{j=1}^N \left[\frac{S_j - A_i \sin \left(2\pi \frac{t_j}{P_i} + \phi_i \right) - C_i}{\sigma_j} \right]^2. \quad (\text{G.20})$$

In the implementation, this minimization was performed in Python using the `curve_fit` routine from `scipy.optimize`, with the amplitude, period, phase, and mean level used as initial values.

Each period considered generates a sinusoidal curve fitted to the observed data, allowing a visual inspection of whether the time scale suggested by the GLSP describes a coherent modulation in the light curve.

G.0.3 Weighted Wavelet Z-Transform (WWZ)

As mentioned above, the analysis of variability in astrophysical time series faces two fundamental obstacles: the data are almost never uniformly sampled in time, and the physical signals of interest evolve their period, amplitude, and phase over long baselines. Classical Fourier analysis is optimal only when the signal is strictly periodic and the sampling perfectly regular. In practice, neither condition holds: astronomical monitoring campaigns are interrupted by seasonal gaps, weather, and instrumental constraints, while real astrophysical oscillations drift in their parameters over years or decades. The Weighted Wavelet Z-transform (WWZ), introduced by Foster (1996), was developed precisely to overcome these limitations, providing a time-frequency analysis tool that is robust to irregular sampling and sensitive to the temporal evolution of periodic signals.

The wavelet transform generalizes Fourier analysis by localizing the spectral analysis around each instant τ , rather than averaging over the entire time series. The continuous wavelet transform of a signal $x(t)$ is defined as

$$W(\omega, \tau; x(t)) = \omega^{1/2} \int x(t) f^*(\omega(t - \tau)) dt, \quad (\text{G.21})$$

where f^* denotes the complex conjugate of the analyzing wavelet, or mother wavelet f , evaluated at the argument $z \equiv \omega(t - \tau)$, and ω is the trial frequency. Foster adopts the abbreviated Morlet wavelet

$$f(z) = e^{iz - cz^2} = e^{i\omega(t - \tau) - c\omega^2(t - \tau)^2}, \quad (\text{G.22})$$

which combines a purely oscillatory term $e^{i\omega(t - \tau)}$ with a Gaussian decay $e^{-c\omega^2(t - \tau)^2}$ that focuses the analysis within a frequency-dependent temporal window. The constant $c = 0.0125$ controls the decay rate of this window.

When the discrete wavelet transform (DWT) is applied directly to unevenly sampled data, however, severe problems arise. The local data number

$$n(\omega, \tau) = \sum_{\alpha} e^{-c\omega^2(t_{\alpha} - \tau)^2} \quad (\text{G.23})$$

fluctuates strongly with position τ , so the transform response depends more on how many data points happen to fall near a given instant than on any actual periodic

signal. False peaks appear at spurious frequencies, and amplitude estimates are systematically biased because the Morlet wavelet does not resemble the undamped sinusoidal signal being sought: projecting onto a decaying trial function inevitably distorts the recovered amplitude and phase. Alternative rescalings such as the discrete wavelet power (DWP) and discrete wavelet amplitude (DWA) mitigate these effects only partially.

The key conceptual advance of [Foster \(1996\)](#) is to reinterpret the wavelet transform as a *weighted projection* onto purely oscillatory trial functions, rather than a projection onto the decaying Morlet wavelet itself. Instead of projecting onto the decaying function $\phi(t) = e^{i\omega(t-\tau)-c\omega^2(t-\tau)^2}$, one projects onto the three undamped trial functions

$$\phi_1(t) = 1, \quad \phi_2(t) = \cos(\omega(t - \tau)), \quad \phi_3(t) = \sin(\omega(t - \tau)), \quad (\text{G.24})$$

assigning to each observation α a statistical weight

$$w_\alpha = e^{-c\omega^2(t_\alpha - \tau)^2}. \quad (\text{G.25})$$

The Gaussian decay now acts as a weighting kernel rather than being incorporated into the trial functions themselves, so that nearby observations contribute strongly to the local fit while distant ones are down-weighted, without distorting the functional form of the model. The weighted projection minimizes the sum of weighted squared residuals, yielding the best-fit coefficients y_1, y_2, y_3 of the local model $y(t) = y_1 + y_2 \cos(\omega(t - \tau)) + y_3 \sin(\omega(t - \tau))$. The inclusion of the constant term y_1 is essential: with irregular sampling, the cosine and sine functions do not have zero weighted mean, so omitting this term introduces a systematic error in the zero-point of the signal, corrupting both the period and amplitude estimates.

A central quantity in the formalism is the *effective number of data points*

$$N_{\text{eff}} = \frac{(\sum_\alpha w_\alpha)^2}{\sum_\alpha w_\alpha^2} = \frac{n^2(\omega, \tau)}{n(\sqrt{2}\omega, \tau)}, \quad (\text{G.26})$$

which quantifies how many observations genuinely contribute to the analysis at each (ω, τ) pair, and replaces the total data count N in all statistical formulas, automatically correcting for density fluctuations in the sampling.

From the weighted projection, one defines the weighted wavelet transform (WWT) as the ratio of the weighted variance of the model V_y to twice the weighted variance of the data V_x :

$$\text{WWT} = \frac{(N_{\text{eff}} - 1) V_y}{2 V_x}. \quad (\text{G.27})$$

Under the null hypothesis of pure noise, the WWT follows approximately a chi-square distribution with two degrees of freedom and expected value unity. However, the WWT has a residual deficiency: at lower frequencies the Gaussian window is wider and more data are incorporated, so N_{eff} grows, causing the WWT to increase with decreasing ω even when the fit is no better. This produces spurious power at low frequencies. To remove this dependence on the effective sample size, Foster applies the Z-statistic, yielding the *weighted wavelet Z-transform*:

$$\text{WWZ} = \frac{(N_{\text{eff}} - 3) V_y}{2 (V_x - V_y)}. \quad (\text{G.28})$$

The denominator $V_x - V_y$ is the residual variance, that is, the fraction of the data variability not accounted for by the periodic model. When the signal is strong, $V_y \approx V_x$ and the WWZ grows large; when the data are pure noise, $V_y \approx 0$ and the WWZ remains near unity. The WWZ follows approximately an F -distribution with $N_{\text{eff}} - 3$ and 2 degrees of freedom, with expected value equal to 1 under the null hypothesis; consequently, large values signal statistically significant periodicity at that (ω, τ) pair.

Once the dominant frequency has been located from the peak of the WWZ, the instantaneous amplitude of the oscillation is recovered through the *weighted wavelet amplitude*

$$\text{WWA} = \sqrt{y_2^2 + y_3^2}, \quad (\text{G.29})$$

where y_2 and y_3 are the sine and cosine projection coefficients. The WWA provides the signal amplitude in the same physical units as the original data, evaluated locally around the instant τ .

In practice, the WWZ is computed over a two-dimensional grid of trial frequencies ω and time positions τ . At each grid point, the Gaussian weights are evaluated, the weighted projection is performed onto the three trial functions, and the WWZ and WWA statistics are computed from the resulting variances. The peak of the WWZ as a function of ω at each fixed τ identifies the instantaneous dominant period;

its value at that peak measures the statistical significance of the periodicity; and the corresponding WWA gives the local amplitude. The result is a time-frequency power map that reveals how the periodicity, its significance, and its amplitude evolve across the entire observational baseline. Foster (1996) demonstrates through synthetic signals with the same irregular sampling as the variable stars R Aquilae and FS Comae that the WWZ/WWA combination recovers the true period with a precision substantially superior to all standard DWT-based alternatives, even in the presence of large data gaps, with the sole caveat that reliable inference is impossible when the gap duration exceeds the period of the signal under study.

Examples of three-dimensional representations of the WWZ maps are shown in Figure G.2. Both the vertical axis and the color scale represent the WWZ power, with higher values corresponding to greater heights and colors approaching red. The dimensionless parameter c controls the decay of the Gaussian weights, $w(t) = \exp[-c\omega^2(t-\tau)^2]$, where τ is the center of the temporal window and ω is the trial angular frequency. At a given frequency, smaller values of c produce broader temporal windows, favoring frequency resolution, whereas larger values produce narrower windows, favoring temporal localization. The value $c = 0.0125$ adopted in Foster (1996) closely approximates the commonly used choice $(8\pi^2)^{-1} \simeq 0.012665$. This choice gives the Gaussian window a standard deviation approximately equal to one trial period, so that the weights decrease appreciably over a single cycle. Figures G.3, G.4, and G.5 illustrate the resulting changes in the WWZ maps of J0818+4222 for $c = 0.003$, 0.0125 , and 0.1 , respectively.

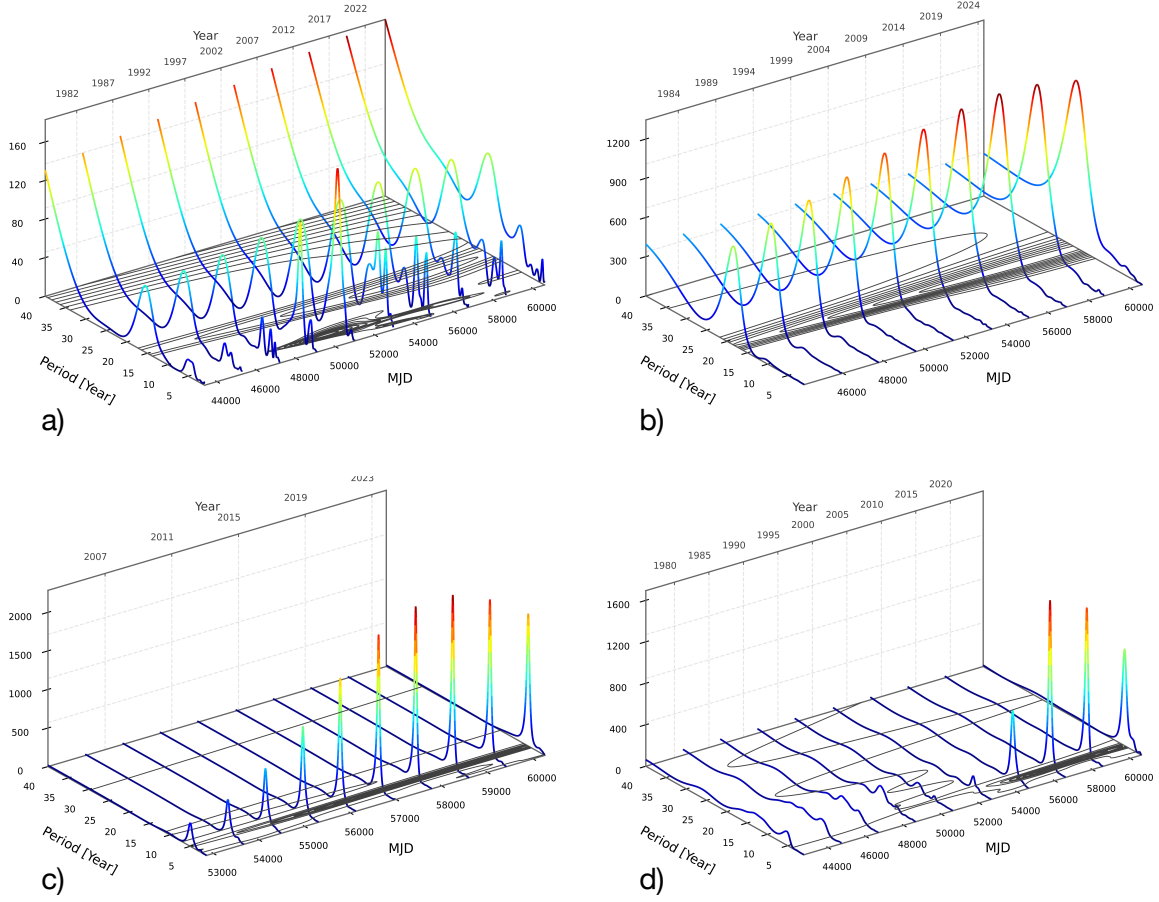


Figure G.2: 3D views of the WWZ results. Panel (a), corresponding to the source J1058+0133, shows a single peak near 1997 and a very low-power periodic feature at a period of approximately 15 years. Panel (b), corresponding to J1309+1154, shows a clear periodic feature near 18 years that persists throughout the temporal baseline. Panels (c) and (d) correspond to PKS 2131–021 (J2134–0153): panel (c) covers the interval from July 2003 to November 2023, while panel (d) shows the full temporal range.

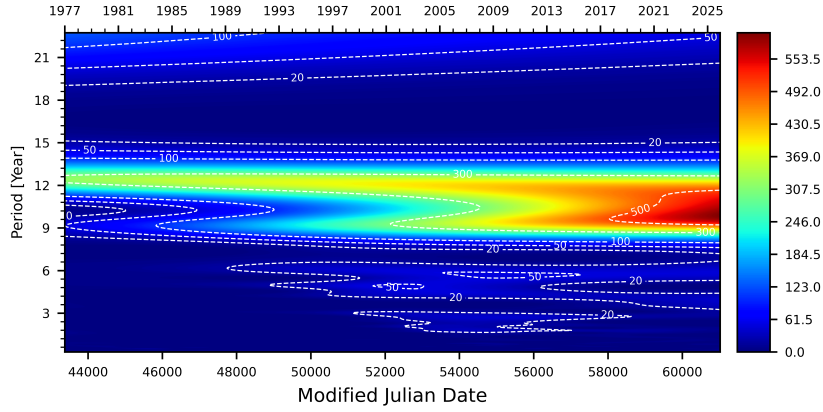


Figure G.3: Example of a Weighted Wavelet Z-transform (WWZ) map of J0818+4222 with contours, computed using a decay parameter of $c = 0.003$. The map exhibits limited temporal localization but good frequency (or period) resolution.

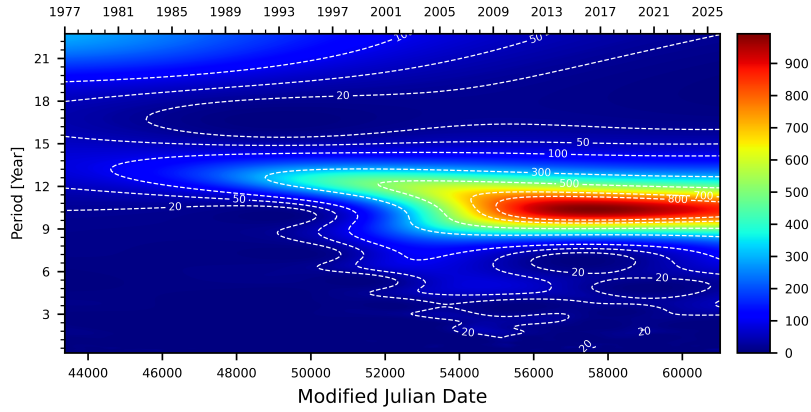


Figure G.4: Example of a Weighted Wavelet Z-transform (WWZ) map of J0818+4222 with contours, showing a periodic signal that has persisted over the past ~ 11.1 years. The adopted decay parameter is $c = 0.0125$, a commonly used value originally adopted by Foster (1996).

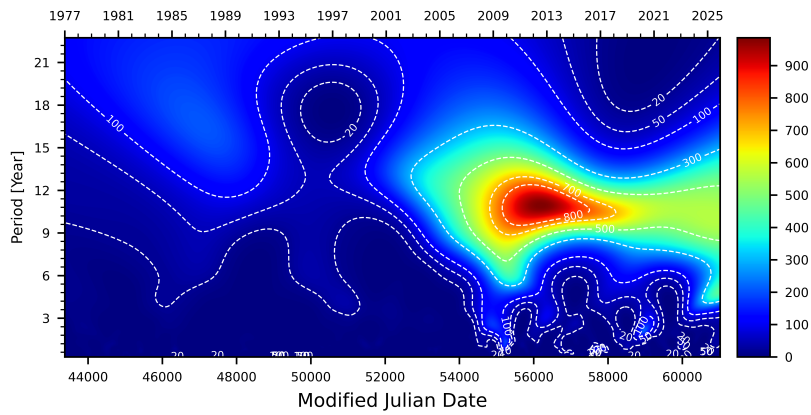


Figure G.5: Example of a Weighted Wavelet Z-transform (WWZ) map of J0818+4222 with contours, computed using a decay parameter of $c = 0.1$. The map exhibits better temporal localization but poorer frequency (or period) resolution.

Appendix H

A hint of a harmonic in PKS J1309+1154

The evidence in favor of a harmonic in PKS J1309+1154 (Fig. 5.3) is not based solely on the appearance of a periodic signal in the residual after subtracting the fundamental component and the linear trend. Although the residual shows a modulation compatible with a first harmonic, with a frequency approximately twice that of the fundamental and in phase with it, its local significance of 2.3σ should be regarded as suggestive rather than conclusive.

The most relevant argument comes from the direct inspection of the original light curve. Adopting a fundamental period of approximately 6550 days, the curve can be divided into 11 regions delimited by the zero crossings of the expected harmonic. In all these regions, the data alternate coherently from one side to the other with respect to the fundamental plus trend fit, following the behavior predicted by a model that includes the fundamental component and its first harmonic. The probability that this configuration occurs randomly is $p = 4.9 \times 10^{-4}$, equivalent to a significance of 3.3σ (Readhead et al., 2026). This phase-coherent alternation constitutes an important argument against the possibility that the harmonic is simply an artifact produced by the residual estimation.

In addition, the inclusion of the harmonic reproduces morphological features of the full light curve that are not adequately described by a single sinusoidal component. In particular, the fundamental plus harmonic model explains the flattening of the minima during intervals of approximately 8 years, as well as the presence of more

pronounced maxima. These structures appear directly in the observed variability and therefore provide an independent verification of the residual signal. In a complementary way, the folded light curve using the fundamental period shows that the deviations with respect to the main component repeat from cycle to cycle, reinforcing the phase coherence of the harmonic over the available temporal baseline.

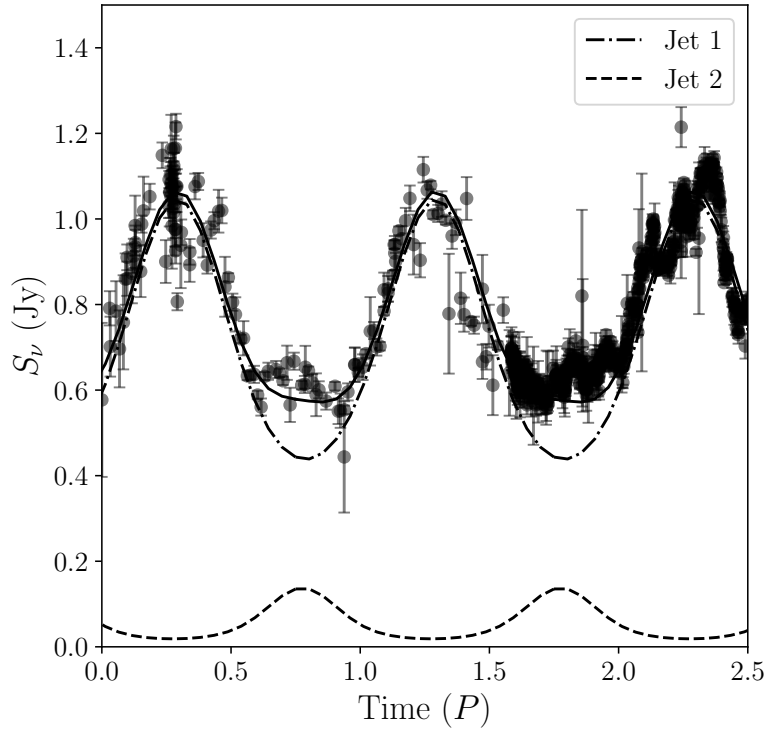


Figure H.1: Schematic model of the SMBHB system proposed by [Readhead et al. \(2026\)](#), in which both black holes contribute relativistic jets of different intensities. The dominant jet produces the main periodic modulation, while the weaker component can introduce an additional modulation compatible with the observed harmonic structure. Reproduced from [Readhead et al. \(2026\)](#).

The WWZ analysis provides an additional perspective in the time-period domain. The fundamental periodicity remains visible over nearly the entire temporal baseline, while the harmonic appears as a weaker but coherent structure, partially masked by non-sinusoidal variability intrinsic to the source. This difference is expected: the harmonic has a smaller amplitude than the fundamental component and may be affected by contributions from jet regions not directly associated with the main periodic modulation.

Finally, the model proposed in [Readhead et al. \(2026\)](#), Fig. H.1, provides a possible physical framework for interpreting this structure. In this scenario, the

detrended light curve can be reproduced through a modified version of the jet model in an SMBHB system, in which both black holes contribute jets of different intensities. The dominant component produces the main modulation, while a secondary contribution may generate a structure compatible with an additional maximum or a harmonic component. This model does not constitute a unique demonstration of the physical nature of the system, but it shows that the presence of the harmonic can be plausibly incorporated within an SMBHB interpretation with relativistic emission modulated by geometric and Doppler effects.

Taken together, the residual signal, the coherent alternation in the original light curve, the repetition of the deviations in the folded light curve, the persistence in the WWZ Fig. 5.4, and the existence of a plausible physical model all converge toward the same interpretation: the fundamental periodicity and its first harmonic are difficult to explain as random products of a steep PSD or as artifacts of the subtraction procedure. Instead, they favor the presence of an intrinsic periodicity in PKS J1309+1154, probably associated with the central region of the blazar.

Appendix I

Optical and infrared observations of PKS J1309+1154

Optical and infrared observations of PKS J1309+1154 provide an additional test of whether the sinusoidal modulation detected at radio wavelengths is also present at higher frequencies. The optical data come from the Catalina Real Time Transient Survey (CRTS) and the Zwicky Transient Facility (ZTF), covering a temporal baseline of approximately 19.25 yr, although with a gap of about 800 days between both surveys. In addition, multiepoch infrared photometry from ALLWISE and NEOWISE in the W1 and W2 bands is included for comparison, as shown in Figure I.1.

In contrast to the UMRAO+OVRO radio light curve, the optical data do not show a clear sinusoidal modulation at the radio period of 17.9 yr. Although long-term variability is present, the characteristic timescales suggested by different analyses are not individually consistent with the radio periodicity. The combined CRTS and ZTF *r*-band data suggest a longer timescale of about 24 yr, while detrending the optical light curves produces weaker signals near ~ 7.5 yr and ~ 17.1 yr. A multiband Lomb-Scargle analysis of the optical data also yields a signal near 11 yr. This diversity of timescales indicates that the optical evidence is ambiguous.

Therefore, the optical and infrared data do not provide an independent confirmation of the radio periodicity at this stage. This may be due to the shorter temporal baseline compared with the radio monitoring, the gap between CRTS and ZTF, the heterogeneous filter coverage, secular trends, and additional

non-sinusoidal variability intrinsic to the blazar. Nevertheless, these data do not rule out a physical connection with the radio modulation. Some optical and infrared features may be broadly compatible with phase shifts observed in other SMBHB candidates, but a longer and more homogeneous temporal baseline is required before a coherent optical or infrared modulation can be established.

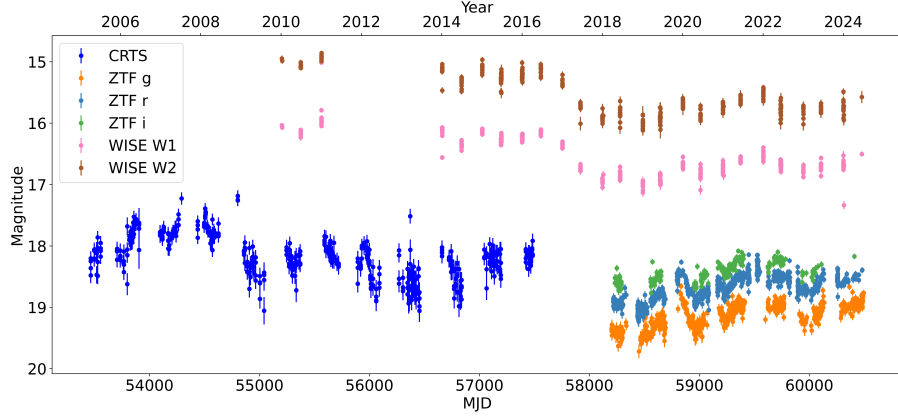


Figure I.1: Optical and near-infrared light curves of PKS J1309+1154. The optical data include CRTS and ZTF observations in the *gri* bands, while the infrared data correspond to WISE W1 and W2 photometry. The W1 and W2 magnitudes are shifted upward by 3 mag for visual comparison. Although long-term variability is present, the current optical and infrared data do not show a clear coherent modulation at the 17.9 yr radio period [Molina et al. \(2026\)](#).