Departamento de
Ingeniería Industrial
Universidad de Concepción

USO Y VALOR DEL TIEMPO PARA GRUPOS HETEROGÉNEOS DE PARTICIPACIÓN EN ACTIVIDADES Y RESTRICCIONES DE TIEMPO

Por:
Pablo Ignacio Reyes Polanco

Tesis presentada a la Facultad de Ingeniería de la Universidad de Concepción
para optar al grado de Magíster en Ingeniería Industrial

Enero 2025
Concepción, Chile

Profesor Guía:
Dr. Sebastián Astroza Tagle

Acknowledgments

Este trabajo fue parcialmente apoyado por la infraestructura de supercómputo del NLHPC (CCSS210001) y por la ANID mediante Fondecyt 1221724 y la beca magister nacional 2023-22230730.

Resumen

En este trabajo se estudian los modelos microeconómicos de uso del tiempo con tiempo gastos comprometidos; y se abordan sus limitaciones actuales respecto a sus restricciones de tiempo, las que suelen asumirse comunes para todos los individuos, quienes participan del mismo conjunto de actividades. Las diferencias de preferencias entre personas hacen que la formulación endógena de restricciones técnicas para el mínimo tiempo asignado a actividades (tiempo comprometido) y la selección de conjuntos de actividades a realiza una necesidad para entender el panorama completo de la asignación del tiempo. Se propone un enfoque de clases latentes confirmatorias para la segmentación endógena de individuos con el fin de modelar diferentes comportamientos en actividades restringidas y participación en actividades bajo un enfoque probabilístico. Se realizaron estimaciones empíricas utilizando datos nacionales recogidos en 2015 en dos encuestas en Chile que se fusionaron estadísticamente para generar un conjunto de datos de uso del tiempo y gastos. Se obtuvieron resultados que muestran diferentes patrones para las restricciones de tiempo, que no pueden generalizarse en un único segmento y que pasar por alto la variabilidad en las restricciones de tiempo puede conducir a resultados sesgados sobre las preferencias y los valores del tiempo. En cuanto a los patrones de participación en actividades, los resultados muestran factores centrales basados en variables socioeconómicas que pueden afectar a la probabilidad de realizar un conjunto determinado de actividades en un ciclo de tiempo semanal. Ambos modelos muestran diferentes tasas de sustitución entre el tiempo comprometido (incluido el transporte) y el trabajo. Las conclusiones sugieren la importancia de realizar más estudios sobre las limitaciones de tiempo y su variación no sólo entre individuos, sino que también entre distintos momentos del tiempo. Además, resalta la relevancia de estudiar los factores que subyacen a los conjuntos de participación en actividades.

Palabras clave – uso del tiempo, valor del tiempo, modelos de clases latentes

Abstract

This work revisits microeconomic time-use models with committed time and committed activities, and address their current limitations regarding time constraints, which are often assumed to be common to all individuals, who participate on the same set of activities. The differences on preferences between individuals make the formulation of endogenous technical constraints for minimum time assigned to activities (committed activities) and the selection of activity participation sets a necessity to understand the whole picture of time allocation. A confirmatory latent classes approach is proposed for the endogenous segmentation of individuals to model different behaviours on committed time and activity participation under a probabilistic approach. Empirical estimations were conducted using nationwide data collected in 2015 in two surveys in Chile which were statistically merged to generate a time-use and expenditures dataset. Results were obtained showing different patterns for time constraints, which cannot be generalized into only one unique segment and that overlooking the variability on time constraints may lead to biased results on preferences and values of time. On the activity participation patterns, results show deciding factors based on socioeconomic variables that may affect the probability of performing a determined set of activities in a weekly time cycle. Both models show different rates of substitution between committed time (including transportation) and work. The conclusions suggest the importance of do further studies on time constraints and their variation not only between individuals but also moments of time and the relevance of studying the factors behind activity participation sets.

Keywords – time-use, value of time, latent class models.

Contents

List of tables

List of figures

1. Introduction

This section presents the antecedents on time-use models, the motivations that drive this work and the core advances that it proposes to achieve.

1.1. Motivations

Time-use allocation models have been thoroughly examined for many years. Most time-use literature is directly or indirectly based on consumer theory since Becker (1965) proposed the first formal integration of time into a microeconomic framework to estimate a consumption utility.

Years later, there are two main frameworks for time-use modelling (Jara-Díaz & Rosales-Salas, 2017): activity-based and microeconomic-based models. The first category treats time-use often using econometric techniques enabling the multiple interactions of time equations, such as structural equation modelling (SEM) (Chung et al., 2009; Golob, 2003; Koo et al., 2022; Lu & Pas, 1999); or the modelling of discrete-continuous decisions using models increasing in complexity until reaching Multiple Discrete-Continuous (MDC) models proposed by Bhat (2008) and further expanded to more specific applications to time-use by Castro et al. (2012) (explained later in the text). One drawback of these models is that the marginal utility of time allocation is always positive since they are usually applied for discretionary activities, which is not necessarily true for non-leisure activities like work or travel. Astroza et al. (2017) compared the results obtained using MDC and microeconomic models, finding that the first usually overestimated the value of time by assuming it was non-negative.

The second category of (microeconomic) models studies time from the consumer behaviour theory perspective, where time allocated to activities influences utility (Becker, 1965). These models derive implicit values for time in a framework where individuals balance time allocated to work -yielding economic benefits- with their available time for unpaid activities. To trace the evolution of these models, it is necessary to revisit Becker (1965). He proposed a consumer utility function in which time and goods were used to produce "final goods" that were a direct source of utility, requiring the expansion of budget restrictions to include a time constraint. This initial view was complemented by DeSerpa (1971), who incorporated time assigned to activities directly into the utility function and introduced a set of technical constraints to relate time and goods, with the consequent definition of three values of time: as a resource, as a commodity and the value of saving time in a particular activity. Time restrictions also allow to formulate a broader view for leisure activities as all activities to which is allocated more time than technically necessary, according to DeSerpa (1971). This restrictions were expanded by Evans (1972), who viewed goods as necessary materials for activities that are the source of utility, and later by Jara-Díaz et al. (2008) -expanding Jara-Diaz and Guevara (2003)- with the proposal of two families of technical restrictions: minimum goods consumption and minimum time allocation, from which the concepts of committed expenses and committed activities arise. This microeconomic model have received advances in the formulation of relations between activities and goods (Jara-Díaz et al., 2016), the study of the behaviour in households (Jara-Díaz & Candia, 2020), studying the replacement between committed time and work (Jara-Díaz & Contreras, 2024) and jointly estimate travel mode choice (Jokubauskaitė et al., 2019) among other advances (Jara-Díaz & Rosales-Salas, 2020; Jokubauskaitė et al., 2022; Schmid et al., 2021).

There are still some current limitations on the microeconomic models. To consider an activity in the model formulation, individuals must allocate a non-zero amount of time to that activity, which may be the case of travel time, work, and in-home activities. However, it is highly restrictive when a detailed (less broad) classification of activities is in play. Leisure activities are characterized by their variety since people have different incentives or limitations to engage in certain activities. For example, there might be budget barriers to practising certain hobbies, mobility limitations to attend certain places or different preferences when choosing an event. Traditional microeconomic time-use models must aggregate all these possibilities under the “leisure label”. While MDC resolves the previous issues, it comes with the disadvantage of the only positive values of time, thus not being a complete alternative to the microeconomic models.

Another limitation resides in determining the committed activities and expenditures, which is essential for the model as they limit the free allocations. To define whether an activity may be constrained, the authors usually consider if an activity cannot be delegated to a third and is necessary for a technical reason (Jara-Díaz & Rosales-Salas, 2017). Transportation (especially commuting) is often considered a constrained activity as it cannot be delegated, and there is a physical limitation behind its time allocation. Sleep and meals - a biological necessity- start a debate about them behaving as a constrained activity or a leisure activity since people tend to assign more than the minimum time necessary (Hössinger et al., 2019; Jara-Diaz, 2020; Jokubauskaitė et al., 2022). Recently, Jara-Diaz et al. (2023) and Jara-Diaz et al. (2025) explored a possible hierarchy for time assignment, helping to determine the set of committed activities. Moreover, the definition of the minimum time is also to be discussed. Are committed activities endogenous variables or exogenous variables? Do they behave similarly across individuals? This issue has been explored by Pellegrini and Rose (2024) incorporating Tobit models on MDC (but no microeconomic time-use models yet) for the endogenization of minimum consumption restrictions. Another view in this matter is to formulate conversion rates between time and goods explicitly. Jara-Díaz et al. (2016) proposed a model in which both restrictions are related in a unidirectional way and still assuming that all individuals in the sample share the same conversion rates.

Endogenous segmentation techniques, such as latent class models (or discrete mixtures), have emerged as a future for the endogenization of constraints in time-use models (Pellegrini et al., 2021). Latent class models allow the model to capture different behaviours (or taste preferences) inside a sample by jointly estimating a set of parameters depending on the segment membership and the corresponding membership probabilities (Kamakura & Russell, 1989). They have been widely used in transport research, especially in transport choice models (Bhat, 1997; Train, 2009), but also in time-use (Astroza et al., 2017). Latent class models also allow to put previous knowledge or taking different assumptions across classes, such as varying functional forms, decision forms or introducing systematic zeros; these types of models are referred to as confirmatory latent class models (Hess, 2014; Kim & Mokhtarian, 2023). While this is not the most popular approach, there are plenty of examples: Calastri et al. (2019) proposed a latent class model in which classes vary on transport mode availability, to account for unobserved real availability; Astroza et al. (2019) formulated classes with a different order for decision processes concerning to residential location, vehicle availability and share mobility choice; Kim and Mokhtarian (2021) studied long-distance domestic trip generation using the confirmative latent class models, formulating a model in which one class generate a non-zero number of trips while the other only generates zero, joint by a probabilistic class assignment; and more recently Kim and Mokhtarian (2024) follow a similar approach to study the purposes for using ride-hailing services, proposing segments that differ in the use (or not) of the service. The variety of examples shows that the use of the confirmatory approach allows the testing of different model structures bounded mainly by the creativity of the modeller in the proposition of different segments.

The advances and limitations about microeconomic time-use models, expose the existence of a research gap concerning the proposition of innovative formulations to allow corner solutions and the study of committed time in more detail.

1.2. Hypothesis

It is possible to extend current time-use models to incorporate latent classes, allowing corner solutions and endogenize activity restrictions.

1.3. General objective

Propose methodologies to deal with corner solutions and activity participation choices in the framework of microeconomic time-use models with committed time and expenses.

1.4. Specific objectives

- Explore a Chilean Time-use Database (ENUT)
- Prepare the database to estimate microeconomic time-use models.
- Identify the capabilities of the current state-of-the-art estimation methods for microeconomic time-use models.

- Estimate a microeconomic time-use model with latent class to study different activity participation patterns.
- Estimate a microeconomic time-use model to study committed expenses.

1.5. Document outline

The remainder of the document is organised as follows: Section 2 details the modelling approach followed in this work, detailing the basic microeconomic time-use model and the incorporation of latent classes; in section 3 the estimation method is developed, section 4 presents the generation of a Chilean weekly time-use data and expenditures data set from a time-use survey and an household budget survey, and the description of the resulting dataset; section 5 covers the results of the estimated models and section 6 discusses the obtained results and concludes the article.

2. Modelling approach

In this work, the model proposed by Jara-Díaz et al. (2008) is used as the platform to develop a time-use latent class model. This section explains the components of the model and the approaches to formulating endogenous activity constraints and corner solutions.

2.1. Single class time-use model

Given time allocated to work (T_w), time allocated to each activity i but work (T_i) and E_j the consumption of goods j during a period τ, the model displays a Cobb-Douglass utility function for which parameters θ_w, θ_i and η_j define the importance of T_w, T_i and E_j respectively and Ω is a non-negative parameter that scales the function. Restriction (2.2) defines the monetary budget, for which the total money spent on every good E_j must not surpass total income, composed of work income (ωT_w) and income from other sources (I_f), where ω is the wage rate. Equation (2.3) represents the time budget, where τ is the total available time (e.g. 24 hours). Inequalities (2.4) and (2.5) set the minimum time allocation to activities and minimum good consumption if there is a minimum for activity i or good j. The equation system may be solved using Lagrange multipliers, for which λ, μ, κ_i and ψ_j are defined. They represent the marginal utility of increasing money available, increasing available time, reducing minimum time constraints, and reducing minimum consumption constraints. Together, they form the following optimization problem:

$$Max\ U = \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j E_j^{\phi_j} \tag{2.1}$$

Subject to:

$$\sum_j E_j \leq \omega T_w + I_f \quad \rightarrow (\lambda) \tag{2.2}$$

$$\sum_i T_i + T_w = \tau \rightarrow (\mu) \tag{2.3}$$

$$T_i \geq T_i^{\min} \rightarrow (\kappa_i) \qquad \forall \boldsymbol{i} \in \boldsymbol{A^c} \tag{2.4}$$

$$E_j \geq E_j^{min} \rightarrow (\psi_j) \qquad \forall \boldsymbol{j} \in \boldsymbol{G^c} \tag{2.5}$$

The model differentiates between free activities (A^f) and committed activities (A^c). From the first is defined Θ, which is the summation of θ_i over all activities in A^f, and the second defines committed time ($T_c = \sum_{i \in A^c} T_i$). Analogously, there is a differentiation between free expenses (G^f) and committed expenses (G^c), defining Φ, the summation of ϕ_j across all expenses in G^f, and $\left(E_c = \sum_{j \in G^f} E_j\right)$. By essence committed activities and expenses are those for which the individuals would rather to assign less time and money, but are limited by a technical minimum (i.e. household maintenance), which turns to be their optimal allocation. From these definitions, the concepts of T_c and E_c are especially relevant. When replaced into equations (2.2) and (2.3) it is clear that they reduce the available time and money to be allocated and thus the correct definition of A^c and G^c crucial for the model. Another key aspect of the model is that time and expenditures to be allocated must be positive and non-zero, as an assignment of zero time or zero expenses nullifies the utility function. To avoid this issue, often researchers aggregate the activities and expenses until there are not any zero values, yielding a smaller set of activities A^f and goods G^f in which the individuals participate and spend money.

While the definitions of the activity and goods sets are usually given as exogenous and common for all individuals in the sample, nonetheless this is a practical solution instead of technical one. This work centres on endogenous definitions for both A^c and A^f by allowing individuals to statistically belong to different segments based on different alternatives for the sets, first focused on the choice of committed activities and second (on a separate model) on the disaggregation of leisure activities. This matter is further discussed in section 2.2.

The first order conditions (FOC) for the optimization problem describes by equations (2.1)-(2.5) are as follows:

$$\frac{\theta_w}{T_w} + \lambda\omega - \mu = 0 \tag{2.6}$$

$$\frac{\theta_i}{T_i} - \mu = 0 \qquad \forall \boldsymbol{i} \in \boldsymbol{A^f} \tag{2.7}$$

$$\frac{\phi_j}{E_j} - \lambda = 0 \qquad \forall \boldsymbol{j} \in \boldsymbol{G^f} \tag{2.8}$$

The values for μ and λ are found by solving the equation system presented by (2.6)-(2.8). Respectively, the interpretations of the Lagrange multipliers are the marginal utility of time (MUT), and the marginal utility of income (MUI) expressed in equations (2.9) and (2.10).

$$MUT = \frac{\partial U}{\partial T_i} = \mu = \frac{\Theta}{\tau - T_w - T_c} \tag{2.9}$$

$$MUI = \frac{\partial U}{\partial E_j} = \lambda = \frac{\Phi}{\omega T_w - E_c} \tag{2.10}$$

With the definition of the auxiliary variables $e_c = \frac{E_c}{\omega(\tau - T_c)}$ and $A = \sqrt{(\Phi + \theta_w + (\Theta + \theta_w)e_c)^2 - 4\theta_w(\Theta + \Phi + \theta_w)e_c}$, the optimal allocation of time and expenses is given by[1]:

$$T_w^* = \frac{(\tau - T_c)\,[\Phi + \theta_w + (\Theta + \theta_w)e_c + A]}{2(\Theta + \Phi + \theta_w)} \tag{2.11}$$

[1] The original author normalizes the parameters using a series of auxiliar variables. It was found more econometrically efficient to estimate the model without that normalization. The alternative specification is presented in section 8.1.

$$T_i^* = \frac{\theta_i}{\Theta}(\tau - T_w^* - T_c) \qquad \forall \boldsymbol{i} \in \boldsymbol{A^f} \tag{2.12}$$

$$E_j^* = \frac{\phi_j}{\Phi}(\omega T_w^* - E_c) \qquad \forall \boldsymbol{j} \in \boldsymbol{G^f} \tag{2.13}$$

In Jara-Díaz et al. (2008), the values of time are derived from the marginal utilities of time and income: the value of time as a resource or value of leisure (VoL) and the value of time assigned to work ($VTAW$), given by the following expressions:

$$VoL = \frac{\partial U/\partial T_i}{\partial U/\partial E_j} = \frac{\mu}{\lambda} = \frac{\Theta}{\Phi}\frac{wT_w - E_c}{\tau - T_w - T_c} \tag{2.14}$$

$$VTAW = \frac{\partial U/\partial T_w}{\partial U/\partial E_j} = \frac{\theta_w}{\Phi}\frac{wT_w - E_c}{T_w} = VoL - w \tag{2.15}$$

Let define $\boldsymbol{T}$ as the vector of times $T_w, T_1, T_2, \dots, T_N$ allocated to work and activities in A_f, where $N = \left|A^f\right|$ and $\boldsymbol{E}$ is the vector of expenditures $E_1, E_2, \dots, E_M$ allocated to the purchase of goods in G^f, where $M = \left|G^f\right|$. Following equations (2.11)-(2.13), it is clear that time and expenditure allocations q depend on a set of unobserved parameters $\Psi = \left\{\Theta, \Phi, \theta_w, \theta_i, \phi_j\right\}$ and a set of observed parameters $\Omega = \{T_c, E_c, \omega\}$. While parameters in Ψ are common to every individual in the sample, parameters in Ω vary across individuals. Thus, the set of observed parameters for individual q is given by $\Omega_\mathrm{q} = \left\{T_{c_q}, E_{c_q}, \omega_q\right\}$ and thus the probability of allocating time $\boldsymbol{T}$ to activities in A^f and expenditures $\boldsymbol{E}$ to goods in G^f may be formulated as:

$$P(\boldsymbol{T}, \boldsymbol{E}) = f\left(g\left(\Psi, \Omega_q\right)\right) \tag{2.16}$$

Where $g\left(\Psi, \Omega_q\right)$ denotes the equation system (2.11)-(2.13) and f is a valid probability density function (that will be discussed in detail in section 3 which is dependent on the sets of parameters Ψ and Ω_q.

2.2. Latent class time-use model

As mentioned in Section 1, this work explores endogenous segmentations within the model. This is achieved through the formulation of a latent class model.

Latent class models aim to explain the variability in preferences across markets, making each class a market segment (Kamakura & Russell, 1989). Thus, the first component of a latent class model is the probability that individual q belongs to segment s, also called membership probability (π_{qs}). The second component of a latent class model is the conditional allocation probabilities $P(\boldsymbol{T},\boldsymbol{E}|s)$ (in-class probability). The membership probability is formulated as a function of the socioeconomic characteristics of the individuals ($\boldsymbol{X_q}$), which can be structured as a logit model (Astroza et al., 2017; Bhat, 1997). In this way, the membership probability characterized by equation (2.17), in which $\boldsymbol{w_s}$ is the vector of parameters associated with each covariate.

$$\pi_{qs} = \frac{\exp\left(\boldsymbol{w_s'X_q}\right)}{\sum_{s'=1}^{S}\exp\left(\boldsymbol{w_{s'}'X_q}\right)} \tag{2.17}$$

The probability of an individual q allocating time $\boldsymbol{T}$ and expenditures $\boldsymbol{E}$ is derived from a modification in equation (2.16) now considering that the time-use parameters also vary with the segment, the set of unobserved parameters is given by: $\Psi_{qs} = \left\{\Theta_{\mathrm{s}}, \Phi_{\mathrm{s}}, \theta_{ws}, \theta_{is}, \phi_{js}\right\}$ and then $P(\boldsymbol{T},\boldsymbol{E}|s)$ is no more than:

$$P(\boldsymbol{T},\boldsymbol{E}|s) = f\left(g\left(\Psi_{\mathrm{s}},\Omega_q\right)\right) \tag{2.18}$$

Where f is a valid probability density function. Finally, the joint probability in the latent class case is denoted by:

$$\begin{aligned} P(\boldsymbol{T},\boldsymbol{E}) &= \sum_{s\in S}\pi_{qs}\cdot f\left(g\left(\Psi_{\mathrm{s}},\Omega_q\right)\right) \\ &= \sum_{s\in S}\frac{\exp\left(\boldsymbol{w_s'X_q}\right)}{\sum_{s'=1}^{S}\exp\left(\boldsymbol{w_{s'}'X_q}\right)}\cdot f\left(g\left(\Theta_{\mathrm{s}},\Phi_{\mathrm{s}},\theta_{ws},\theta_{is},\phi_{js},T_{c_q},E_{c_q},\omega_q\right)\right) \end{aligned} \tag{2.19}$$

2.2.1. Introducing endogeneity to committed activities

As mentioned in the introduction chapter, one of the main research objectives of this work is to introduce endogeneity in the selection of constrained activities. This work proposes that the set of activities A^c varies between classes to achieve the endogenous segmentation of restrictions. A strong argument for this modelling decision is that the actual restrictions are not observed, and thus introducing variability through latent classes might improve the quality of the models, as observed in Calastri et al. (2019) modelling different options for mode choice availability as they were unobserved in their dataset. In this way, the expectation is that individuals whose behaviour tends to treat certain activities as committed to have a higher probability of being a member of the corresponding class.

The two activities in which their behaviour is often discussed are meals and sleep. Thus, the experiments conducted in this work are focused on them. To unveil possible behaviours of these activities, the proposition is to formulate a series of model in which just one of the activity constraint is processed as endogenous at a time, in which the first class displays the disputed activity as committed and the second class as not committed, as shown in equation (2.20), where A_1^c is the set of constrained activities for the first segment (the baseline).

$$T_{c_{qs}} = \sum_{i \in A_s^c} T_i \,, \forall s \in \{1,2\} \qquad \boldsymbol{A_2^c} = \boldsymbol{A_1^c} - \left\{\boldsymbol{T_{c_{disputed}}}\right\} \tag{2.20}$$

The elasticity parameters of activities and expenditures are not required to estimate the values of time. Thus, in this approach, all free activities and free expenses are aggregated into just one "leisure" activity and "recreation" expense, respectively. From equations (2.19) and (2.20) the probability of allocating time $\boldsymbol{T}$ and expenditures $\boldsymbol{E}$ is characterized as:

$$P(\boldsymbol{T},\boldsymbol{E}) = \sum_{s \in S} \frac{\exp\left(\boldsymbol{w_s'X_q}\right)}{\sum_{s'=1}^{S} \exp\left(\boldsymbol{w_{s'}'X_q}\right)} \cdot f\left(g\left(\Theta_s, \Phi_s, \theta_{ws}, T_{c_{\boldsymbol{qs}}}, E_{c_q}, \omega_q\right)\right) \tag{2.21}$$

2.2.2. The inclusion of corner solutions

Conversely, the approach to include corner solutions is modelling activity participation as an endogenous decision. Latent classes are formulated according to the possible sets of activity participation observed in the sample. In this way, individuals are expected to have a higher probability of belonging to the class with their corresponding set of performed activities. This approach proposes to be in between models in which all activities must be performed and models in which activity participation is estimated individually. Thus, the classes vary in their equations (2.11)-(2.13), which may be denoted by g_s, the equation systems for each segment s. Allowing the model to vary equation systems along segments consequently makes the sets of free activities segment-dependent $\left(A_s^f\right)$ and their cardinality. Therefore, the parameters incumbent to g_s are not the same for all groups, making their utility functions essentially different. Thus, as extension of equation (2.19) the probability of allocating time $\boldsymbol{T}$ and expenditures $\boldsymbol{E}$ is characterized as:

$$\begin{aligned} P(\boldsymbol{T},\boldsymbol{E}) &= \sum_{s\in S} \pi_{qs} \cdot f\left(\boldsymbol{g_s}(\Psi_s,\Omega_q)\right) \\ &= \sum_{s\in S} \frac{\exp\left(\boldsymbol{w}_s'\boldsymbol{X_q}\right)}{\sum_{s'=1}^{S}\exp\left(\boldsymbol{w}_{s'}'\boldsymbol{X_q}\right)} \cdot f\left(\boldsymbol{g_s}\left(\Theta_s,\Phi_s,\theta_{ws},\theta_{is},\phi_{js},T_{c_q},E_{c_q},\omega_q\right)\right) \end{aligned} \quad \textbf{(2.22)}$$

3. Estimation method

Equations (2.11)-(2.13) denote a system of G nonlinear equations denoted as $g(\Psi,\Omega_\mathrm{q})$, from which the set of parameters Ω may be estimated. Note that as Θ and Φ are aggregates of θ_i and ϕ_j for all activities i and goods j in A^f and G^f it is possible to estimate only estimate $N-1$ elasticities for activities and $M-1$ for goods and construct the left-out parameter from the others as $\theta_N = \Theta - \sum\theta_i$ and $\phi_M = \Phi - \sum\phi_j$, reducing the number of equations in (2.12) and (2.13)[2]. Additionally, due to system identifiability Θ must be fixed to 1[3]. Let $\boldsymbol{Y_q}$ be the vector of observed times and expenditures $T_w, T_1, T_2, \dots, T_N - 1, E_1, E_2, \dots, E_{M-1}$ in A^f and G^f assigned by the individual q, then the following nonlinear system of equations is defined:

$$\boldsymbol{Y_q} = g\left(\Psi,\Omega_\mathrm{q}\right) + \boldsymbol{\eta}_q \quad \textbf{(3.1)}$$

---

[2] This work does not focus on expenditures and thus, omits equation (2.13), but it is included in the explanation of the methodology for completeness.

[3] This is equivalent to dividing all estimated parameters by Θ. Alternatively, the parameters could be normalized with respect to Φ, but with te purpose of being comparable with previous authors this work fixes Θ.

Where $\boldsymbol{\eta_q}$ is the vector of error terms of the individual q. Assuming that $\boldsymbol{\eta_q}$ follows a multivariate normal distribution with an unknown covariance matrix ($\boldsymbol{\eta} \sim MVN_{M+N-1}(0, \boldsymbol{\Sigma})$). Let us define $\mathcal{N}(.\,|\mu, \Sigma)$ as the probability density function of a multivariate normal distribution with mean $\boldsymbol{\mu}$, covariance $\boldsymbol{\Sigma}$. Then, joining equations (2.16) and (3.1), the probability of individual q assigning time $\boldsymbol{T_q}$ and expenditures $\boldsymbol{E_q}$ (assigning $\boldsymbol{Y_q}$) is as follows:

$$P(\boldsymbol{Y_q}) = \mathcal{N}\left(\boldsymbol{Y_q} | g(\Psi, \Omega_\mathrm{q}), \boldsymbol{\Sigma}\right) \tag{3.2}$$

Note that equation (3.2) is a probability density function for which maximum likelihood is a valid method to calibrate the unknown parameters in Ψ and the elements of Σ. The equation may be rewritten in function of the error terms. Let us define $\varphi(.\,|\boldsymbol{\Delta})$ as the standard multivariate normal distribution with correlation matrix $\boldsymbol{\Delta} = \boldsymbol{\sigma}^{-1}\boldsymbol{\Sigma}\boldsymbol{\sigma}^{-1}$ and $\boldsymbol{\sigma}$ the diagonal matrix of standard deviations. Then, the likelihood function is presented in equation (3.3). Section 8.3 introduces a brief simulation study of the parameter recover efficiency of the estimator proposed by this work.

$$L = \prod_q \left(\prod_{k=1}^{N+M-1} \sigma_k \right)^{-1} \varphi(\boldsymbol{\eta_q} | \boldsymbol{\Delta}) \tag{3.3}$$

The likelihood function for the latent class model requires including the membership function in equation (2.17) and assuming that the conditional probability expressed in equation (2.18) also follows a normal distribution with covariance matrix $\boldsymbol{\Sigma_s}$ then:

$$P(\boldsymbol{T}, \boldsymbol{E} | s) = \mathcal{N}\left(\boldsymbol{Y_{qs}} | g(\Psi_s, \Omega_\mathrm{q}), \boldsymbol{\Sigma_s}\right) \tag{3.4}$$

Analogously, let us define $\boldsymbol{\Delta_s} = \boldsymbol{\sigma_s}^{-1}\boldsymbol{\Sigma_s}\boldsymbol{\sigma_s}^{-1}$ and $\boldsymbol{\sigma_s}$ as the correlation matrix and the diagonal matrix of standard deviations of the segment s. Then the likelihood function is:

$$L = \prod_q \sum_{s \in S} \frac{\exp\left(\boldsymbol{w}_s' \boldsymbol{X_q}\right)}{\sum_{s'=1}^{S} \exp\left(\boldsymbol{w}_{s'}' \boldsymbol{X_q}\right)} \left(\prod_{k=1}^{N+M-1} \sigma_{sk} \right)^{-1} \varphi(\boldsymbol{\eta_{qs}} | \boldsymbol{\Delta_s}) \tag{3.5}$$

Note that for the latent class models with corner solutions, different sets of activities mean that the cardinality of the normal distributions of each segment may vary from each other. This may cause an artificially higher likelihood function due to assigning time to an omitted activity in the segment, which is not considered for the computation of the likelihood of the segment. To correct this, penalization term (ξ) is included depending on the deviation from a minimum time considered as participating into the activity. The value of the penalization was set to 1.5 after searching for the value with the best results and the less aggressive penalization. Let define $\bar{A}^f = \cup_s A_s^f$, the set of all activities performed across segments, then the penalized likelihood function is:

$$L = \prod_q \sum_{s \in S} \left[\frac{\exp(\boldsymbol{w}_s' \boldsymbol{X}_q)}{\sum_{s'=1}^{S} \exp(\boldsymbol{w}_{s'}' \boldsymbol{X}_q)} \left(\prod_{k=1}^{N+M-1} \sigma_{sk}^{-1} \right) \left(\prod_{l=1}^{\bar{A}^f \setminus A_s^f} \exp(-\xi T_l) \right) \varphi(\boldsymbol{\eta}_{qs} | \boldsymbol{\Delta}_s) \right] \quad \textbf{(3.6)}$$

The estimation method can be carried on with the maximum likelihood maximization method, as shown by Munizaga et al. (2008) and, more recently, by Jokubauskaitė et al. (2022). However, the formulation of latent classes makes the estimation process more complex and thus requires the additional use of the Expectation-Maximization (EM) algorithm for the initial values (see section 8.2), as discussed by previous authors (Bhat, 1997; Train, 2008; Train, 2009), as well as the use of multiple starting points (Hess, 2014).

The time-use models with endogenous committed activities were estimated considering covariates in their Φ_s and θ_w parameters. The idea is to allow the model to support internal variability inside both classes to avoid excessive noise in the segmentation probabilities. In this way, the model is expected to make segments according to their committed activity behaviour instead of only because of differences in general preferences between both segments. The final Φ_s (and θ_{ws}) parameters are as follows:

$$\Phi_s = \Phi_s^{(base)} + \Phi_s^{(female)} + \Phi_s^{(age \geq 45)} + \Phi_s^{(\#underage \geq 1)} + \Phi_s^{(only\ worker)} \quad \textbf{(3.7)}$$

The assessment of the models requires a comparison with a fair alternative. For the latent class models with endogenous committed time, the comparison is made against the models with the alternative modeller decisions that may be made with respect to including or not an activity in the committed activities. For the latent class models with corner solutions, the models with a priori segmentation according to the realization of activities are the alternative. The expectation for the first model to present significative improvements in model fit due to the proposal of a less restrictive model, which adjusts to individual characteristics. In contrast, the second model is expected to perform statistically worse in terms of likelihood -mainly because comparing with a priori segmentation would be comparing with a perfect alternative- but with the advantage of using the whole sample. The likelihood ratio test and Bayesian Information Criterion (BIC) will be computed for all comparisons, as previous authors showed them as valuable tools for assessing latent class models (Spurk et al., 2020). The formulas are as follows:

$$LL_{ratio} = -2[LL_R - LL_U] \tag{3.8}$$

$$BIC = -2 \cdot LL + p \cdot log(n) \tag{3.9}$$

Where LL_R and LL_U are the log-likelihoods of the restricted and unrestricted alternative models, p is the number of parameters and n is the number of individuals in the sample. A decrease in the BIC is associated with a significative increase in the model fit.

4. Data

Data on time-use allocation for this work comes from the National Time-Use Survey (ENUT) of Chile (INE, 2015), a list-based survey in which a sample of the population of the country declared their time allocation to 105 activities during one working day and one non-working day, while also collecting time assigned by domestic workers and caregivers outside the household (external providers of time), satisfaction with time allocation related to life aspects, and sociodemographic variables. Expenses were imputed from the Chilean Household Budget Survey (HBS) (INE, 2016) into the ENUT.

The time-use sample was processed to overcome over- and under-declarations of time allocation and abnormal day reports, such as respondents being on vacation or having medical rest. A weekly period was reconstructed based on it being the minimum span to capture a leisure-work cycle, according to Jara-Díaz and Rosales-Salas (2015). The weekly dataset was constructed by multiplying the time declared on weekdays by five and matching twins on the data based on sociodemographic characteristics (Munizaga et al., 2011) for the weekends (see section 8.4). A sample was selected consisting of people over 18 who declared to work and allocated time to it during the week due to the importance of time allocated to work and the wage rate for the basic model proposed by Jara-Díaz et al. (2008).

The imputation of expenses is similar to Hössinger et al. (2019): expenditure data from the EPF is used to train a linear model to predict savings and a logit model to predict the assignment of the remaining money (more details are provided in 8.5). The models are formulated with socioeconomic variables in both databases to extrapolate the expenditure behaviour from the EPF to each individual in the ENUT with the appropriate variation due to personal characteristics. After the process, individuals whose predicted expenses were more than five times their income were left out of the sample (3 individuals). The details of the models are available in section 8.5. Expenditures from EPF were classified into 10 items: food, recreation and culture, restaurants and hotels, clothing, communications and electronics, transport, health, home maintenance, education and bills. Based on previous researchers approaches (Astroza et al., 2017; Jara-Diaz et al., 2025; Jokubauskaitė et al., 2022) the later five items are aggregated and by subtracting the fixed income committed expenses (E_c) are calculated. Negative values of committed expenses restrict the feasible value of the elasticity of time assigned to work, not necessarily representing the whole sample, thus 27 individuals that exhibited negative values of E_c were also left out of the sample.

The final sample comprises 2253 workers, of which 53.57% are men, 12.61% receive external help in their household (paid or nonpaid) and 53.35% live with a partner in the household. The rest of the individual sociodemographic characteristics and household sociodemographic characteristics are described in Figure 4.1 and Figure 4.2 respectively. On the individual level, the key aspects are that most of the sampled individuals are young workers between 25 and 34 years old (29.21%), half of the sample (50.82%) have secondary school as its educational attainment (secondary school is the required minimum schooling according to Chilean law) and more frequently live in the Metropolitan Region and Central Valley (combined 50.47%). The sample individuals come from 2015 different households, mostly composed of between three and four people (50.82%), of which usually two are workers (44.52%) and at least one is a minor (59.45%).

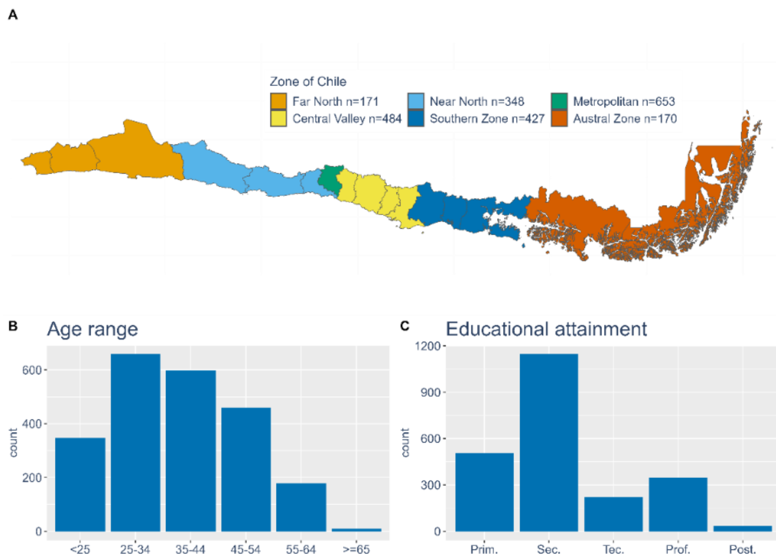

Figure 4.1: Individual sociodemographic characteristics of the ENUT sample

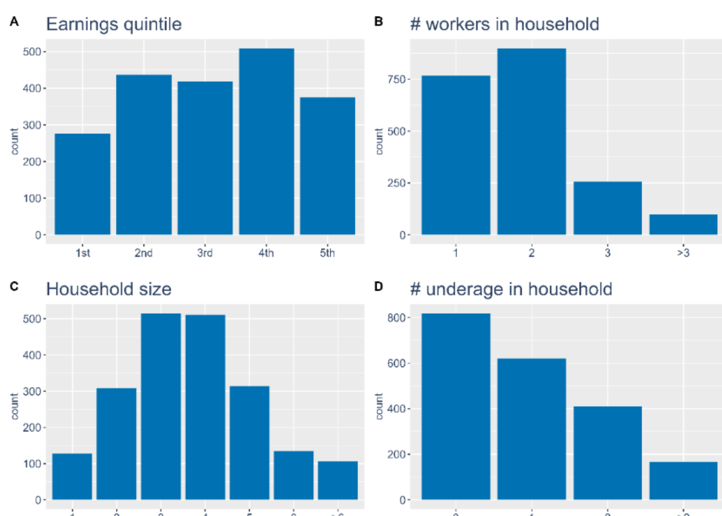

Figure 4.2: Household sociodemographic characteristics of the ENUT sample

Figure 4.3 shows the time assignment by sociodemographic segments: number of workers in the household *(# workers),* age, educational attainment, gender, geographic location and number of underage individuals in the household *(# underage).* Some of the most remarkable differences are present with the increase in the # *underage* leading to more activities, the increase in dedication to paid work with age, reaching its peak between 55 and 64 years old and reducing at 65 years old (age of retirement in Chile), differences in activities for the most extreme regions of Chile and finally, the most remarkable, the gap in work between men and women, men tend to spend more time on paid work. In contrast, women assign more time to unpaid work activities. The latter is not a new finding; it has been explored previously in more detail in the Chilean case (Jara-Diaz et al., 2025; Jara-Diaz et al., 2023).

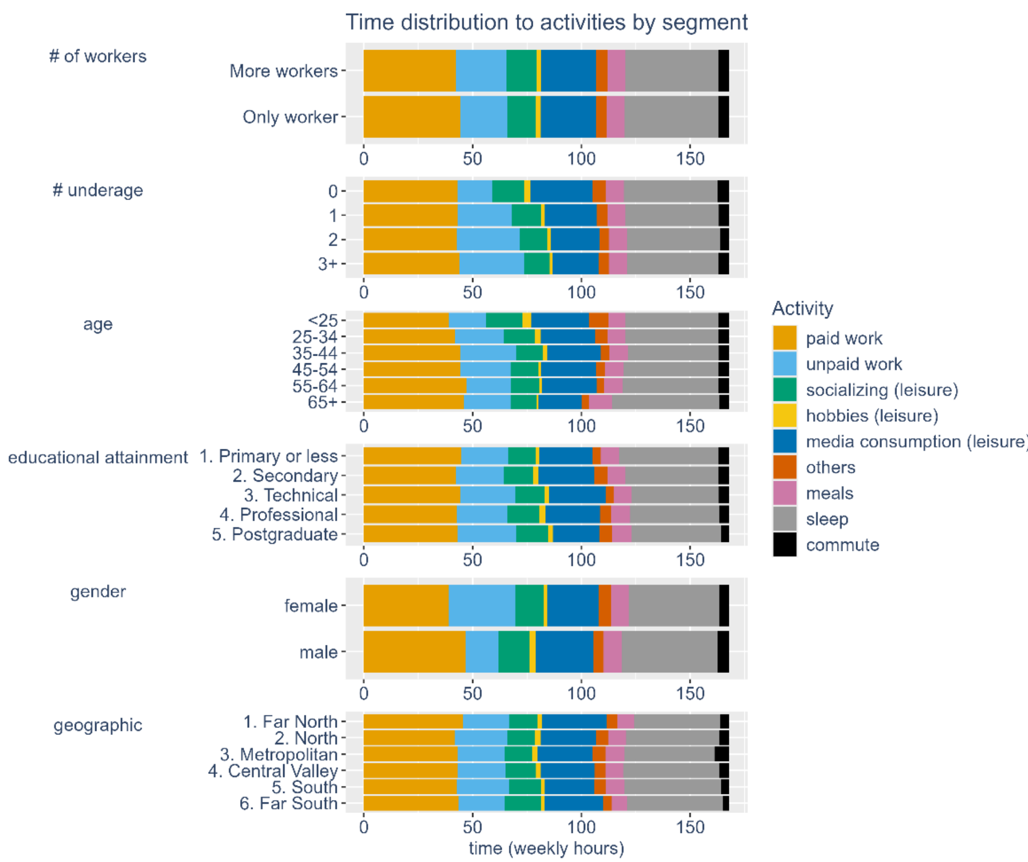

Figure 4.3: Distribution of time for different population segments

More detail into the subclassification of leisure activities makes it possible to unveil that some are not assigned any time (or assigned very little), making it an excellent example of corner solutions, as shown in Figure 4.4.

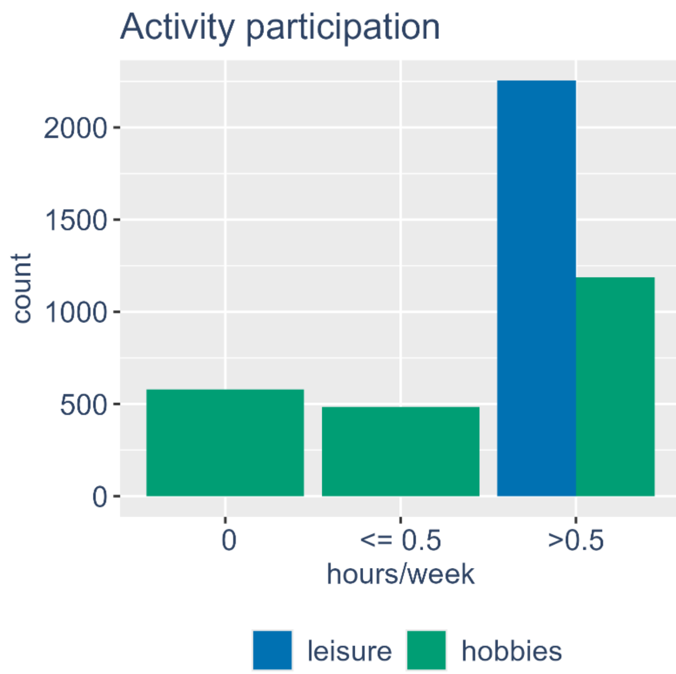

Figure 4.4: Participation in hobbies

Figure 4.5 describes the expenditure behaviour of the sample by segments. It is noticeable that individuals in households where they are the only worker spend more than households with more workers (+36.61%), as the same with men (+40.84% with respect to women). Individuals with higher educational attainment also tend to spend more, especially professionals and postgraduates (+149.70% and +251.63% with respect to secondary). The number of underage individuals also have the effect of increasing the spending in the case of households that live with minors.

A crucial part of latent class models is to make the class membership probabilities dependent on exogenous variables of the sample. From the available variables presented in Figure 4.1 and Figure 4.2, a set of binary variables was formulated to simplify the models identification. The initial set is confomed by age ($age \geq 45 \rightarrow 1$), number of underage individuals in household

($\#underage \geq 1 \rightarrow 1$), number of workers in household ($\#workers = 1 \rightarrow 1$, which implies the individual is the $only\ worker$), people that live with a partner ($lives\ with\ partner \rightarrow 1$), educational attainment ($university \rightarrow 1$), region of residence ($metropolitan \rightarrow 1$) and external helps received by the household the household ($receives\ help \rightarrow 1$).

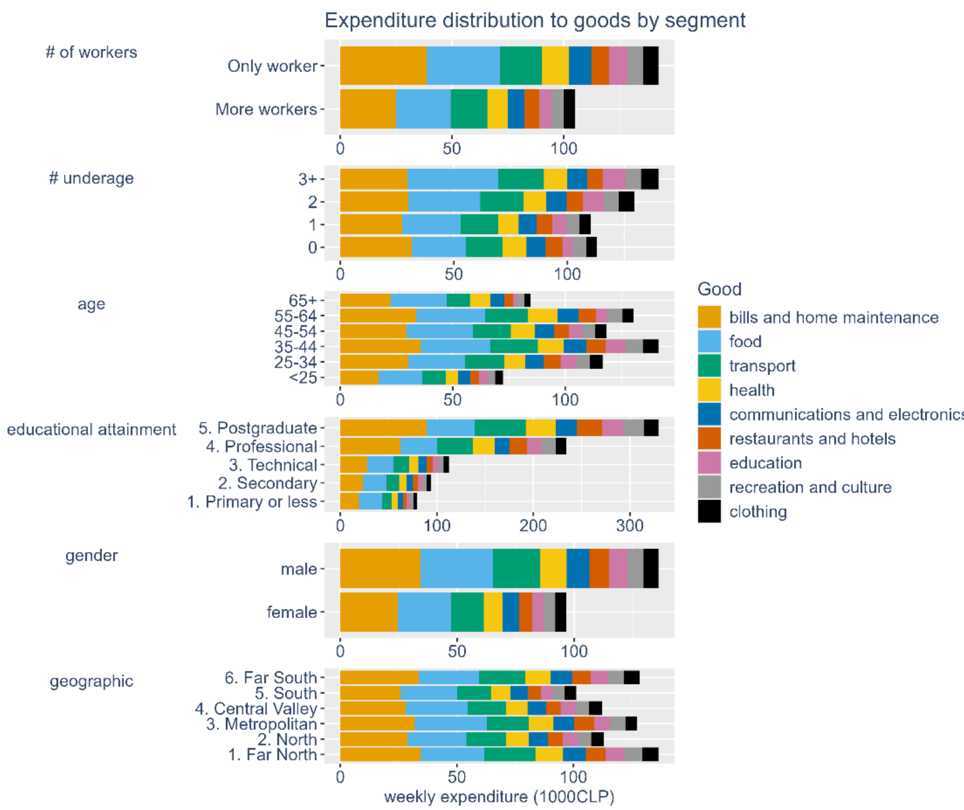

Figure 4.5: Distribution of expenditures for different population segments

Figure 4.6 describes the correlation between model variables and binary segmentation variables. In the case of binary variables the correlation between them was computed with tetrachorical correlation. Model variables are correlated mostly negatively between them, with the exception of meals and sleep, committed time is higly correlated with leisure and work, which is expected as activities are assumed to depend on the time restriction. Free expenses and committed expenses are higly correlated wich is expected as the first constrained by the latter. A closer inspection to the correlation between binary variables unveils that $lives\ with\ partner$ is highly correlated with

$\#underage \geq 1$ and $reeives\ external\ help$ with $university$. For this reason $lives\ with\ partner$ and $receives\ external\ help$ were left out of the membership function.

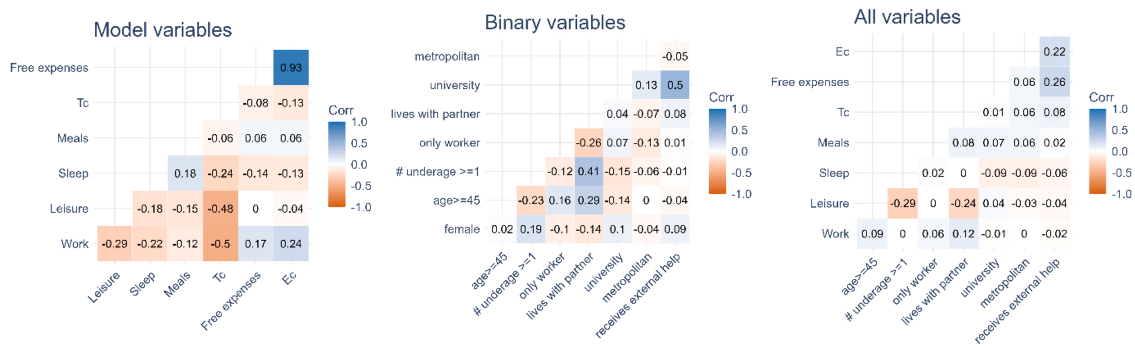

Figure 4.6: Correlations between variables.

5. Results

In this section the results of the models are presented. All models were implemented in R (Team, 2024) with the help of the Apollo package (Hess & Palma, 2024). The standard errors for the values of time in the tables were computed using the δ-method, as presented in section 8.6. For all experiments we estimated deterministic models with similar assumptions to the ones in the deterministic models; we compared the models with endogenous committed activities against deterministic models representing each specification of A^c represented by the latent classes, while for the models with selection of activity sets, we estimated models with a-priori segmentation based on the observed activity participation as a baseline for comparison.

5.1. Time-use models with endogenous time constraints

Three groups of models are presented in this section. The first and second groups of models explore whether Sleep and Meals, respectively, should be considered a committed activity or not. The third group of models tries to determine the nature of these two activities jointly. Each group is composed of one latent model allowing for endogenous definition of the activity's nature (committed or not committed), and by other models assuming this definition exogenously. All activities but work, leisure and the studied activity (sleep, meals or both) are considered as committed time for the purpose of this experiments. The comparison of models within each group allows determining which approach is better.

Table 5.1 presents the results of the latent class model with endogenous committed activities applied to sleep, and two corresponding models without latent classes, one assuming Sleep to be committed, and one assuming it is not committed. In the latent class model, one estimates the probability of an individual having a certain set of committed activities. In the deterministic model committed activities are defined a-priori. The latent class model shows an increase in the likelihood function. Besides this, there is a decrease in BIC against both deterministic models, and the loglikelihood ratio test (LL-ratio) rejects the null hypothesis of both models having equal likelihood. *Thus, it is concluded that the latent class model improves significantly in the model fit*.

In general, neither latent segment prevails over the other as their membership probabilities are similar (48.8% against 51.2%). The individual's characteristics associated with a greater probability of treating sleep as a committed activity are $age \geq 45$, $only\ worker$ and $university$. Figure 5.1 A shows the values across all socio-economic characteristics, weighted by the membership probabilities of the sample. This reveals that those individuals who belong to the higher income quintiles and have higher educational attainment are more likely to treat sleep as a committed activity; according to Table 5.1Table 5.1 they are also those with higher wages. Figure 5.1B presents the change in T_w and in the values of time (VoT) with respect to committed time (e.g. due to a reduction in commuting time) for both segments. When evaluating working time in the class with committed sleep time, there is a more abrupt increase in weekly hours assigned to work when committed time is decreased. The figure also shows that the average values of time are consistently higher in the segment with sleep as a committed activity.

Table 5.1: Models treating alternatives for time assigned to sleep

	Latent class				Modeller decision			
	Sleep committed		Sleep not committed		Sleep committed		Sleep not committed	
Model specification	value				value		value	
# individuals	2253				2253		2253	
# parameters	29				11		11	
LL	-7518.06				-7760.62		-7875.09	
LL-ratio					485.1 (χ^2_{18}=28.9)		714.1 (χ^2_{18}=28.9)	
BIC	15260.01				15606.17		15835.10	
Time-use parameters	est. (t.val)		est. (t.val)		est. (t.val)		est. (t.val)	
$\Phi^{(base)}$	0.732	**(5.522)**	2.979	**(47.648)**	1.834	**(6.494)**	0.834	**(9.962)**
$\Phi^{(female)}$	-0.109	(-0.933)	0.252	(0.492)	0.082	(0.285)	0.023	(0.300)
$\Phi^{(age\geq45)}$	0.004	(0.030)	-0.513	**(-2.601)**	-0.238	(-0.806)	-0.108	(-1.347)
$\Phi^{(\#underage\geq1)}$	0.022	(0.182)	-0.569	**(-1.513)**	-0.070	(-0.400)	-0.055	(-0.844)
$\Phi^{(only\ worker)}$	-0.392	**(-2.956)**	-1.466	**(-5.044)**	-1.038	**(-2.720)**	-0.479	**(-4.656)**
$\theta_w^{(base)}$	-0.541	**(-2.160)**	-4.590	**(-47.319)**	-2.342	**(-4.693)**	-0.996	**(-6.681)**
$\theta_w^{(female)}$	0.171	(0.713)	-0.518	(-0.570)	-0.245	(-0.447)	-0.103	(-0.675)
$\theta_w^{(age\geq45)}$	0.301	(1.263)	1.069	**(3.173)**	0.741	(1.329)	0.312	**(2.028)**
$\theta_w^{(\#underage\geq1)}$	0.178	(0.709)	0.622	(0.939)	0.146	(0.432)	0.063	(0.498)
$\theta_w^{(only\ worker)}$	0.610	**(2.363)**	2.398	**(4.644)**	1.634	**(2.327)**	0.760	**(3.960)**
σ	8.291	**(23.134)**	4.386	**(19.347)**	7.581	**(43.126)**	7.976	**(41.492)**
Weighted wage rate	3.582		2.353		2.953		2.953	
[95% C.I.]	[3.556 ; 3.608]		[2.342 ; 2.365]		[2.816 ; 3.090]		[2.816 ; 3.090]	
segment size (*)	0.488		0.512		-		-	
Segmentation prob.	est. (t.val)		est. (t.val)					
ASC	-		1.284	**(4.355)**				
female	-		0.091	(0.387)				
age >= 45	-		-0.486	**(-1.963)**				
#underage >= 1	-		-0.425	(-1.675)				
only worker	-		-1.387	**(-5.243)**				
university	-		-17.161	**(-3.945)**				
Metropolitan	-		0.248	(0.883)				

(*) probability evaluated in the class allocation function

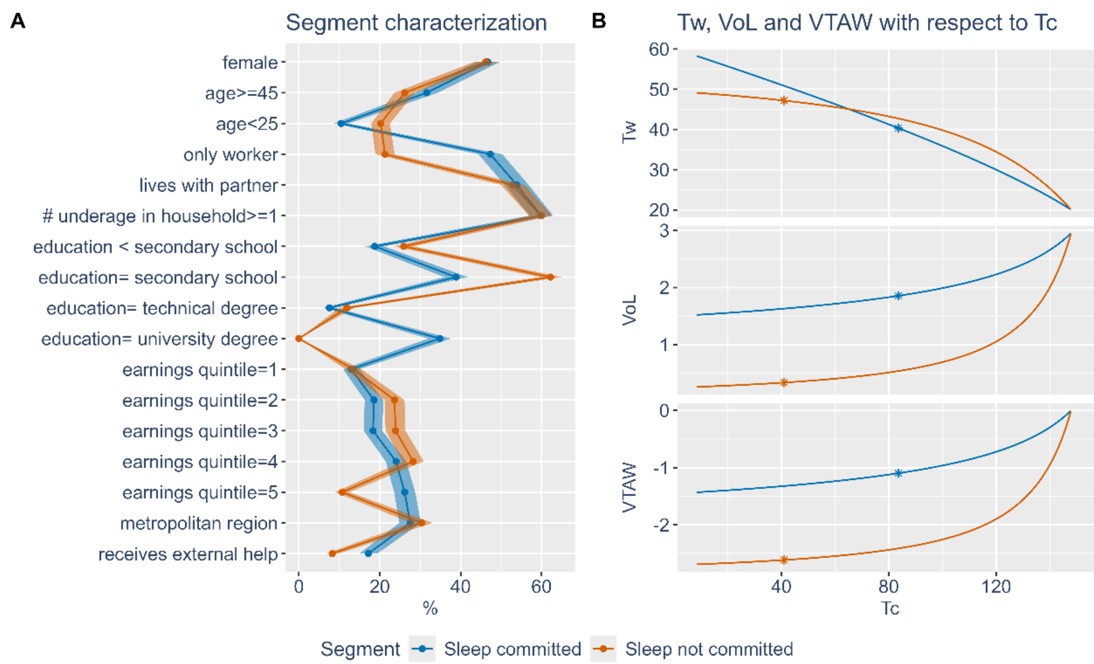

Figure 5.1: Standardized segment characteristics and predicted variables for sleep model

When sleep is treated as a committed activity, the estimated parameters of the latent class model exhibit a lower relative baseline consumption elasticity (Φ) and a less negative elasticity of work (θ_w) - but still negative -.This implies that for both cases, allocating time to work reduces utility, but when treating sleep as an uncommitted activity the reduction in utility is lower. In the same case, *only worker* is the only significative covariate, increasing the elasticity of work and decreasing the elasticity of consumption. In the case of treating sleep as not committed, $only\ worker$, $\#underage \geq 1$ and $age \geq 45$ decrease the elasticity of consumption and increase the elasticity of work ($\#underage \geq 1$ not significatively). Konduri et al. (2011) studied time-use and values of time in Santiago with the model proposed by Jara-Díaz et al. (2008). They estimated the model on deterministic segments based on gender, age and location, identifying lower (and negative) values of the elasticity of work[4] for men and elder

[4] The original work applied a different regularization for the elasticity parameters. We transformed the obtained values to our regularization.

individuals. In this work we identify the opposite behaviour for these socioeconomic variables.

Figure 5.2 shows the comparison of both modelling approaches. Figure 5.2A presents the distribution of the values of time for both deterministic (a-priori) models, and the expected values of time of the latent class model. All values of time are divided by the wage rate. We observe that the values of time are nearly equal across deterministic models (VoL is 0.520 and 0.471 respectively[5]), but they are both below the expected values of time obtained with the latent class model (VoL is 0.640) by 18.75% and 26.41%, which, as mentioned before, had been found to be a much reliable model statistically. This suggests that the imposition of an important activity, such as sleep, as committed or uncommitted in this case, does not affect time values comparatively. Still, the four VoT figures underestimate the VoT obtained with the best (unconstrained) model. Figure 5.2B shows the differences in estimated parameters; the latent class model displays broader error bars for the segment with sleep time as an uncommitted activity. It is interesting to note that while the baseline ($\phi,\ \theta_w$) for the elasticities between approaches is different, the estimates of the covariates are not significatively different, except for $only\ worker$. Nonetheless, as the baseline elasticities are statistically different, the combined effects are dissimilar across both segments.

Figure 5.2 centres in the differences along both modelling approaches, the left part presents the distribution of the values of time for all members of the sample. Here it is observed that the mean of the estimated values of time differ between approaches, being closer between segments with the latent class approach. The latter also shows a higher dispersion in values of time for people with committed sleep time and lower for the other segment. The right part of the figure shows the differences in estimated parameters; it is observed that the latent class model displays broader error bars for the segment with sleep time as a not committed activity. It is interesting that while there are differences in the confidence intervals for the covariates of the parameters between both

[5] VTAW is omitted as $VoL = VTAW + w$

approaches, they tend to be similar, meaning that even as the baseline for the elasticities of the time-use model vary along approaches, the effects of socioeconomic variables tend to maintain. Nonetheless, as the baseline elasticities are statistically different, the combined effects are dissimilar across both segments.

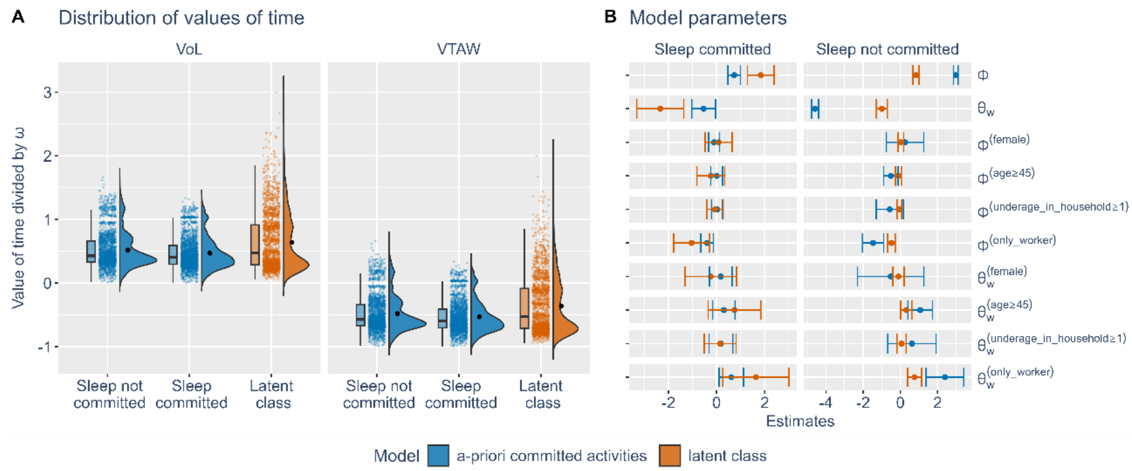

Figure 5.2: Distribution of values of time and parameters for sleep time models

Table 5.2 presents the results of models applied to meals time. The latent class model shows an increase in the likelihood function, and the evaluation metrics of the model allow us to reject the null hypothesis of both models having equal likelihood. *Then, the latent class model applied to meals also improves significantly in the model fit*.

The differences in membership probabilities for the latent class model applied to meals (43.1% and 56.9%), shows a slight prevalence of the probability of meals being an uncommitted activity. The socioeconomic variables associated with an increase in the probability of treating meals as a committed activity are $age \geq 45$, $only\ worker$ and $university$. Table 5.3A shows that those individuals who belong to the higher income quintiles, have higher educational attainment, are the only worker in their household, and who receive external help (voluntary or paid) for household chores, are more likely to treat meals as a committed activity. Table 5.3B presents T_w and VoT with respect to committed time for both segments. When evaluating in the class with committed meals time, there is a more abrupt increase (and quasi linear) in weekly hours assigned to work when committed time is decreased and have higher average values of time.

Table 5.2: Models treating alternatives for time assigned to meals

	Latent class				Modeller decision			
	Meals committed		Meals not committed		Meals committed		Meals not committed	
Model specification	value				value		value	
# individuals	2253				2253		2253	
# parameters	29				11		11	
LL	-7500.13				-7760.62		-7785.93	
LL-ratio					521.0 (χ^2_{18}=28.9)		571.6 (χ^2_{18}=28.9)	
BIC	15224.15				15606.17		15656.78	
Time-use parameters	est.	(t.val)	est.	(t.val)	est.	(t.val)	est.	(t.val)
$\Phi^{(base)}$	0.624	**(4.565)**	5.498	**(29.190)**	1.834	**(6.494)**	1.479	**(7.389)**
$\Phi^{(female)}$	-0.149	(-1.447)	0.494	(1.135)	0.082	(0.285)	0.028	(0.147)
$\Phi^{(age\geq45)}$	0.112	(0.966)	-1.073	**(-3.287)**	-0.238	(-0.806)	-0.153	(-0.761)
$\Phi^{(\#underage\geq1)}$	-0.077	(-0.688)	-1.537	**(-4.061)**	-0.070	(-0.400)	-0.075	(-0.550)
$\Phi^{(only\ worker)}$	-0.281	**(-2.393)**	-2.270	**(-8.188)**	-1.038	**(-2.720)**	-0.866	**(-3.337)**
$\theta_w^{(base)}$	-0.366	(-1.376)	-8.581	**(-27.895)**	-2.342	**(-4.693)**	-1.848	**(-5.198)**
$\theta_w^{(female)}$	0.269	(1.257)	-0.960	(-1.213)	-0.245	(-0.447)	-0.135	(-0.363)
$\theta_w^{(age\geq45)}$	0.125	(0.557)	2.121	**(3.582)**	0.741	(1.329)	0.507	(1.333)
$\theta_w^{(\#underage\geq1)}$	0.373	(1.526)	2.158	**(3.069)**	0.146	(0.432)	0.136	(0.512)
$\theta_w^{(only\ worker)}$	0.427	(1.817)	3.674	**(7.396)**	1.634	**(2.327)**	1.382	**(2.884)**
σ	8.391	**(22.553)**	4.664	**(17.238)**	7.581	**(43.126)**	7.667	**(41.972)**
Weighted wage rate	3.735		2.362		2.953		2.953	
[95% C.I.]	[3.486 ; 3.984]		[2.220 ; 2.504]		[2.816 ; 3.090]		[2.816 ; 3.090]	
segment size (*)	0.431		0.569		-		-	
Segmentation prob.	est.	(t.val)	est.	(t.val)				
ASC	-		1.575	**(4.841)**				
female	-		0.044	(0.175)				
age >= 45	-		-0.620	**(-2.418)**				
#underage >= 1	-		0.000	(-0.000)				
only worker	-		-1.663	**(-6.580)**				
university	-		-5.161	**(-4.441)**				
metropolitan	-		0.078	(0.268)				
(*) probability evaluated in the class allocation function								

The differences in membership probabilities for the latent class model applied to meals (43.1% and 56.9%), shows a slight prevalence of the probability of meals being an uncommitted activity. The socioeconomic variables associated with an increase in the probability of treating meals as a committed activity are $age \geq 45$, *only worker* and *university*. Figure 5.3A shows that those individuals who belong to the higher income quintiles, have higher educational attainment, are the only worker in their household, and who receive external help (voluntary or paid) for household chores, are more likely to treat meals as a committed activity. Figure 5.3B presents T_w and VoT with respect to committed time for both segments. When evaluating in the class with committed meals time, there is a more abrupt increase (and quasi linear) in weekly hours assigned to work when committed time is decreased and have higher average values of time.

Figure 5.3: Segment characteristics and predicted variables for meals time models

When meals are treated as a committed activity, the estimated parameters behave in a similar way as the model for sleep time, Φ is lower and θ_w is higher (but negative). In the same case *only worker* decreases the relative elasticity of consumption and is the only significative covariate. In the case of treating meals as an uncommitted activity all covariates, but *female* decrease the relative elasticity of consumption and increase the elasticity of work significatively. The estimated elasticity parameters and their covariates also follow opposite directions to the ones estimated by Konduri et al. (2011), and are similar to the parameters estimated in the model in Table 5.1. We observe that the variability in the effect of the covariates, allow for more specific groups of people to display much lower relative elasticities of consumption and much higher elasticities of work (even positive), as is the case of only workers when meals are treated as a committed activity.

Figure 5.4 shows the comparison of both modelling approaches. In Figure 5.4A we observe that, again, the values of time are nearly equal across deterministic models (VoL is 0.498 and 0.471, respectively). Still, they are both below the expected values of time obtained with the latent class model (VoL is 0.719) by 30.74% and 34.49%. Figure 5.4B shows the differences in estimated parameters. The baseline elasticities differ between both estimation approaches. At the same time, the effect of the covariates is similar for the segment with meals as a committed activity and statistically different for most covariates in the segment with meals as an uncommitted activity, with the only exception being *female*.

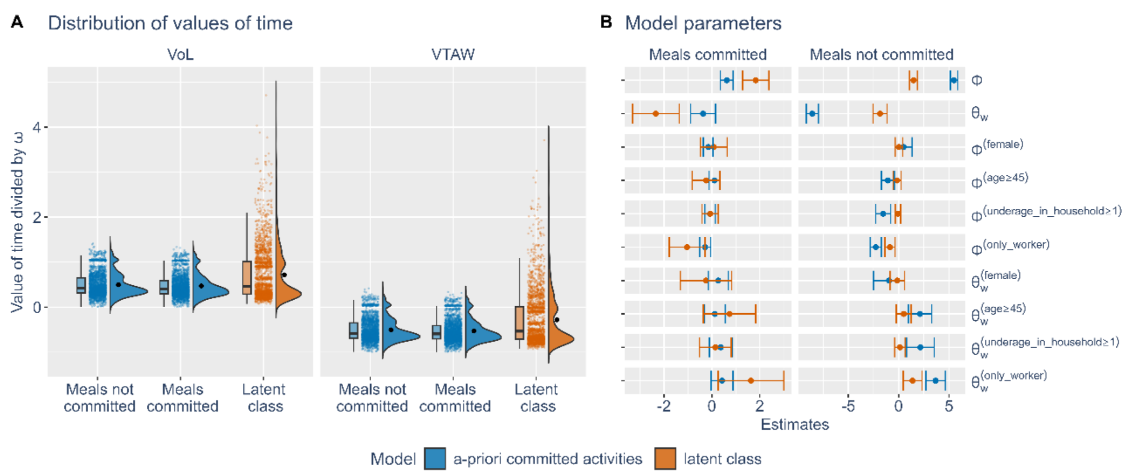

Figure 5.4: Distribution of values of time and model parameters for meals time models

In summary the latent class model applied to meals essentially reproduces the same observed behaviour as the model applied to sleep. Differences are found in a higher presence of *only workers* in the segment with meals as a committed activity, with respect to its equivalent for sleep time; and in the effect of covariates for the segment with meals as a not committed activity.

Figure 5.5 presents the relation between the probabilities of meals and sleep being committed according to the two latent class models, we observe a positive relation between both probabilities, whose correlation is 0.988. Then, the two latent class models suggest that the people who consider meals a committed activity tend to do the same with sleeping time. This is reinforced by the fact that the probabilities of treating both sleep and meals as committed activities are higher for the individuals with higher wage rates, educational attainment and are *only workers*.

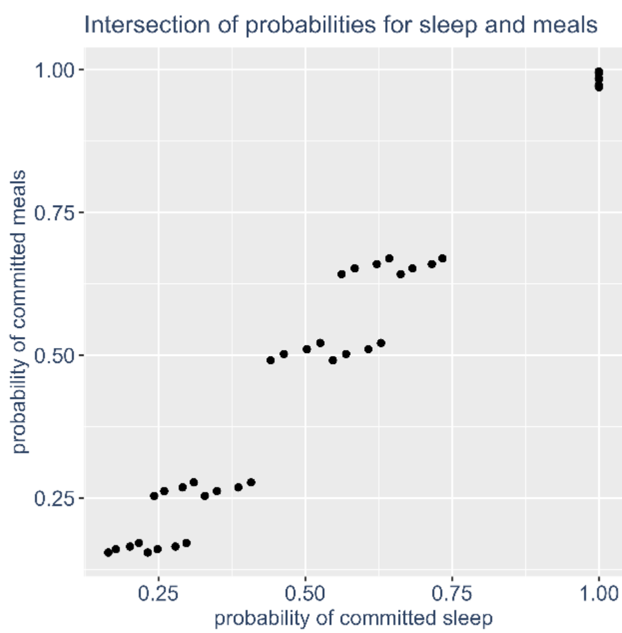

Figure 5.5: Intersection of probabilities for sleep or meals being a committed activity

We have treated both sleep and meals as committed activities, endogenously but one at a time. In the following model we formulate the same decisions jointly in a model with 4 latent classes (neither committed, only sleep committed, only meals committed, and both committed). In this case covariates were not considered, to avoid overloading the model with excessive number of parameters. Table 5.3 presents the resulting model. Although the loglikelihood increases with respect to the two previous latent class models by roughly 1%, the BIC increased by 4%, showing that the improvement in statistical fit was *not significant*.

Table 5.3: Model treating alternatives for time assigned to meals and sleep simultaneously

	Meals and sleep committed		Sleep committed		Meals committed		Neither committed	
Model specification	value							
# individuals	2253							
# parameters	33							
LL	-7413.88							
BIC	15834.53							
Time-use parameters	est. (t.val)		est. (t.val)		est. (t.val)		est. (t.val)	
Φ	0.279	**(2.870)**	2.926	**(6.315)**	3.581	**(6.125)**	0.068	**(3.265)**
θ_w	0.385	(1.512)	-4.628	**(-5.670)**	-5.284	**(-5.627)**	0.320	**(7.234)**
σ	10.094	**(8.221)**	3.786	**(18.053)**	5.033	**(13.091)**	3.826	**(8.125)**
Segmentation prob.	est. (t.val)		est. (t.val)		est. (t.val)		est. (t.val)	
ASC			3.764	**(5.082)**	3.100	**(3.720)**	2.453	**(3.146)**
female			-0.216	(-0.615)	-0.419	(-0.970)	-1.035	**(-2.176)**
age >= 45			-0.936	(-1.812)	0.143	(0.227)	0.097	(0.169)
#underage >= 1			-0.534	(-1.093)	-2.027	**(-3.614)**	-0.641	(-1.329)
only worker			-3.631	**(-6.910)**	-4.769	**(-7.575)**	-2.013	**(-2.504)**
university			-3.542	**(-3.159)**	-13.094	**(-13.311)**	-0.786	(-0.637)
metropolitan			0.070	(0.156)	1.330	**(2.762)**	-0.120	(-0.216)
Segment size (*)	0.206		0.409		0.155		0.230	
Values of time	value [95% C.I]		value [95% C.I]		value [95% C.I]		value [95% C.I]	
VoL	5.999		0.369		0.227		12.293	
[95% C.I.]	[1.902 ; 10.096]		[0.255 ; 0.483]		[0.154 ; 0.299]		[4.914 ; 19.672]	
VTAW	2.378		-2.100		-2.215		8.733	
[95% C.I.]	[-2.275 ; 7.032]		[-2.184 ; -2.016]		[-2.300 ; -2.130]		[1.206 ; 16.259]	
Weighted wage rate	3.621		2.469		2.441		3.561	
[95% C.I.]	[3.558 ; 3.646]		[2.453 ; 2.486]		[2.399 ; 2.484]		[3.506 ; 3.615]	

(*) probability evaluated in the class allocation function

In this model, the size of each segment, according to the membership probabilities, varies between 40.9% and 15.5%, where the predominant class is the one with only sleep as committed activity. The individual's characteristics associated with a lower probability of treating both activities as uncommitted are $female$ and $only\ worker$; $only\ worker$ and $university$ are both associated with a decrease in the probability of treating just one activity as committed; in particular for the case of treating only meals as a committed activity $\#underage\ >=\ 1$ also is associated to a decrease in its probability and $metropolitan$ with an increase.

Figure 5.6A shows the average weighted socioeconomic characteristics of the model segments. Individuals with higher probabilities of being members of the segment with sleep and meals treated as committed activities tend to belong in the higher income quintiles, have higher education and to be more frequently the only worker in the household. Individuals with a higher probability of treating neither activity as committed are similar but with a lower proportion of only workers and the lowest proportion of women across segments. Individuals with a higher probability of treating only meals as a committed activity are observed to belong more frequently in the fourth income quintile and have lower education, a lower proportion of people living with minors, and a higher proportion of people living in the metropolitan region. Individuals with a higher probability of treating sleep as a committed activity are observed to belong to intermediate income quintiles, be younger, and have a higher proportion of people living with minors.

Figure 5.6B presents T_w and VoT with respect to committed time for the four segments. The effect of a decrease in committed time is more abrupt when evaluating the class with sleep and meals as committed activities, which increases its working time in a quasi-linear manner. The second segment with higher change in time assigned to work is the one with neither activity as committed, though is constantly allocating less time to work. These segments are associated with increases in values of time due to a reduction in committed time, contrary to the segments with just one activity as committed. In these segments, the increase in time allocated to work when liberating time from T_C because that they have positive elasticities of work, meaning they have both an economic (wage rate, i.e. more money to spend) and a subjective incentive (enjoyment of the activity). To understand the changes in values of time due to a reduction in T_c we must recall the ratios Θ/Φ and θ_w/Φ present in equations (2.14) and (2.15) respectively. From the first, when Φ is lower than Θ we deduce a faster rate of increase/decrease in VoL; from the second, when θ_w is negative we deduce a decrease in $VTAW$. As VoL and $VTAW$ are directly related through the wage rate, the deductions from one apply also to the other. Then, the segments with negative θ_w and Φ higher than Θ (the segments with just one activity treated as committed) show a slow decrease in values of time when T_c is reduced and the segments; and the segments with positive θ_w and Φ lower than Θ show a faster increase in their values of time.

Figure 5.6: Segment characteristics and predicted variables for meals and sleep model

Concerning the model parameters, we observe that when both activities are treated as committed or uncommitted, the relative elasticity of consumption is lower, and the relative elasticity of work is higher and positive compared to the other two segments.

Figure 5.7 compares the distribution of the expected values of time of the latent class model and the distribution of the values of time of the four deterministic models equivalent to each segment of the latent class. All values of time are divided by the wage rate. We observe again that the values of time are nearly equal across deterministic models (0.460, 0.510, 0.513, and 0.485 for VoL, respectively) and are below the expected values of time obtained with the latent class model (1.18 for VoL, implying a positive VTAW) by 61.02%, 56.78%, 56.53% and 61.19%. It is relevant to note that this model is not as effective at the approach of one activity at a time and covariates. Still, it is more effective than the deterministic approach. Figure 5.8 shows the differences between the estimated parameters of the latent class model and the equivalent deterministic model for each segment; in general, all estimated parameters are statistically different. When Φ is overestimated by the deterministic models the contrary is done with θ_w, and vice versa.

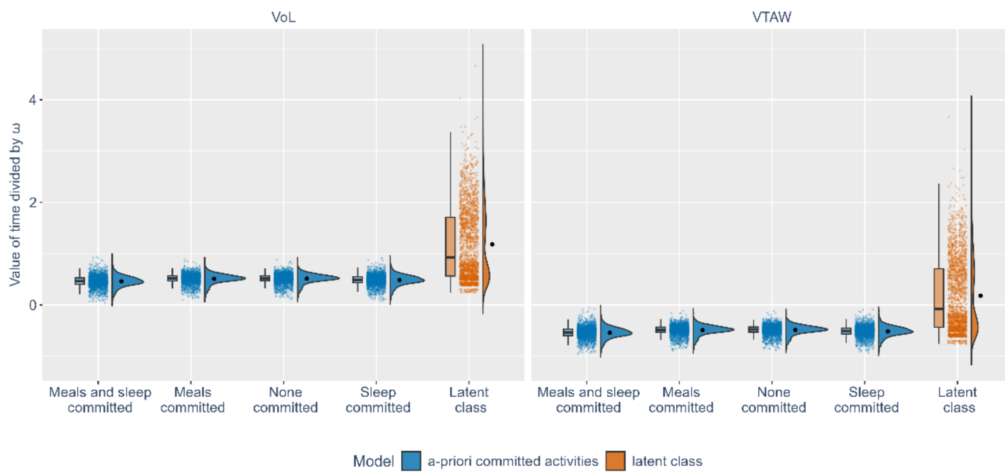

Figure 5.7: Distribution of values of time between models

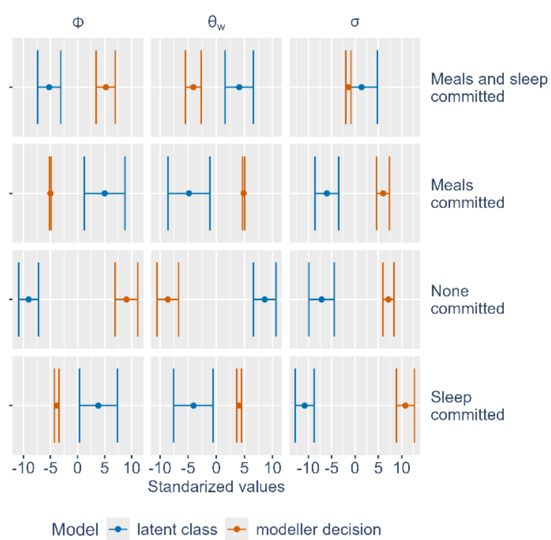

Figure 5.8: Differences in estimated parameters between models

5.2. Time-use model with activity mix choice

In the following section latent class models are formulated according to classes in which the set of activities (activity mix) differ between alternative segments. The chosen activity mixes for the study are: (i) a disaggregation of leisure time into socializing, hobbies and media consumption, generating two classes differing by the participation on hobbies[6] and (ii) the study of childcare as a separate activity from unpaid work, which is sometimes discussed as having both characteristics of leisure and care work (Dotti Sani, 2018). For the latter, a sample of workers who live in households with minors is selected. The "underage in household" socioeconomic variable is supressed for its relations with participating in childcare, and instead it was included a variable indicating if the individual "lives with partner". Both models also consider meals as a free activity. The a-priori segmentation considered that assigning less than 30 minutes to an activity in a week is not participating of it.

Table 5.4 presents the results of the latent class model with activity mix choice applied to hobbies activities and compares it with a deterministic alternative. In the latent class model, activity participation is model through the endogenous selection of segments with different alternatives for the mix of activities to be performed. In the deterministic models, individuals are segments a-priori according to their observed set of activities. The latent class model shows an increase in the likelihood of function, compared to the sum of the deterministic models (which together cover the whole sample). Also, the latent class model presents a decrease in BIC and the LL-ratio test rejects the null hypothesis of both models having equal likelihood. Thus, showing that *the latent class model improves significantly in the model fit*.

[6] one individual did not fit the definition of the segments and was left out

Table 5.4: Model time assigned to hobbies

	Latent class				A-priori segmentation			
	Does not participate in Hobbies		Participates in Hobbies		Does not participate in Hobbies		Participates in Hobbies	
Specification	value				value		value	
# individuals	2252				579		1673	
# parameters	32				10		15	
LL	-26125,93				-5620,46		-20935,04	
LL-ratio					859,1 (χ^2_7=14.1)			
BIC	52498,89				53296,55			
Time-use parameters	est. (t.val)		est. (t.val)		est. (t.val)		est. (t.val)	
Φ	0,760	**(6,676)**	0,883	**(13,692)**	0,820	**(6,176)**	0,834	**(17,283)**
θ_w	-0,461	**(-2,226)**	-0,949	**(-8,316)**	-0,565	**(-2,341)**	-0,781	**(-8,853)**
θ_1 $(socializing)$	0,240	**(28,754)**	0,303	**(43,148)**	0,265	**(39,102)**	0,278	**(73,144)**
θ_2 $(hobbies)$			0,080	**(18,067)**			0,061	**(25,129)**
θ_3 $(media\ consump.)$	0,578	**(64,599)**	0,472	**(58,864)**	0,565	**(73,468)**	0,506	**(113,627)**
σ_1	8,848	**(16,360)**	6,885	**(20,617)**	8,512	**(19,649)**	7,961	**(41,376)**
σ_2	5,841	**(17,074)**	8,560	**(31,289)**	7,423	**(24,777)**	7,709	**(49,317)**
σ_3			4,875	**(14,498)**			4,361	**(16,853)**
σ_4	9,692	**(22,205)**	9,128	**(25,522)**	9,783	**(33,296)**	9,415	**(49,752)**
ρ_{12}	-0,259	**(-6,519)**	-0,295	**(-9,358)**	-0,304	**(-7,455)**	-0,315	**(-14,436)**
ρ_{13}			-0,127	**(-3,532)**			-0,185	**(-7,276)**
ρ_{14}	-0,679	**(-19,293)**	-0,364	**(-11,322)**	-0,595	**(-17,628)**	-0,446	**(-19,856)**
ρ_{23}			-0,245	**(-6,665)**			-0,070	**(-2,521)**
ρ_{24}	-0,408	**(-9,869)**	-0,565	**(-20,339)**	-0,486	**(-11,683)**	-0,515	**(-25,306)**
ρ_{34}			-0,193	**(-4,539)**			-0,218	**(-8,281)**
Values of time	value [95% C.I]		value [95% C.I]		value [95% C.I]		value [95% C.I]	
VoL	1,678		1,39		1,591		1,521	
[95% C.I.]	[1,185 ; 2,170]		[1,191 ; 1,589]		[1,086 ; 2,095]		[1,349 ; 1,694]	
VTAW	-0,932		-1,894		-0,937		-1,58	
[95% C.I.]	[-1,481 ; -0,383]		[-2,075 ; -1,712]		[-1,427 ; -0,448]		[-1,754 ; -1,405]	
Weighted wage rate	2,61		3,284		2,528		3,101	
[95% C.I.]	[2,451 ; 2,769]		[3,064 ; 3,504]		[2,348 ; 2,708]		[2,928 ; 3,274]	
Segmentation prob.	est. (t.val)		est. (t.val.)		est. (t.val)		est. (t.val)	
ASC			0,975	**(5,167)**				
female			-0,953	**(-6,491)**				
age >= 45			-1,324	**(-7,457)**				
#underage >= 1			-0,536	**(-3,497)**				
only worker			-0,053	(-0,336)				
university			1,137	**(3,261)**				
metropolitan			0,085	(0,607)				
Segment size (*)	0,493		0,507					
Obs. proportion	0,257		0,743					

(*) probability evaluated in the class allocation function

The membership probability of the segment that participates in hobbies is 50.7%, in contrast to the observed 74.3% in the sample. According to the estimated influence of the socioeconomic variables, *men*, *people below 45 years old*, people who *does not live with minors* and who have *complete university studies* have a higher probability to be a member of the segment that participates from hobbies, which agrees with the intuitive effects of those variables.

Figure 5.9A shows that women, people over 45 years old, without university studies, from households belonging to lower income quintiles (especially those who do not belong to the 5th quintile) or who leave with minors are less likely to participate in hobbies. Figure 5.9B presents T_w and VoT with respect to committed time for both segments. In general, the replacement of working time and values of time due to changes in committed time share a similar slope and are quasi linear. But when evaluating in the class who does not participate in hobbies it is observed that time assigned to work, and values of time are higher.

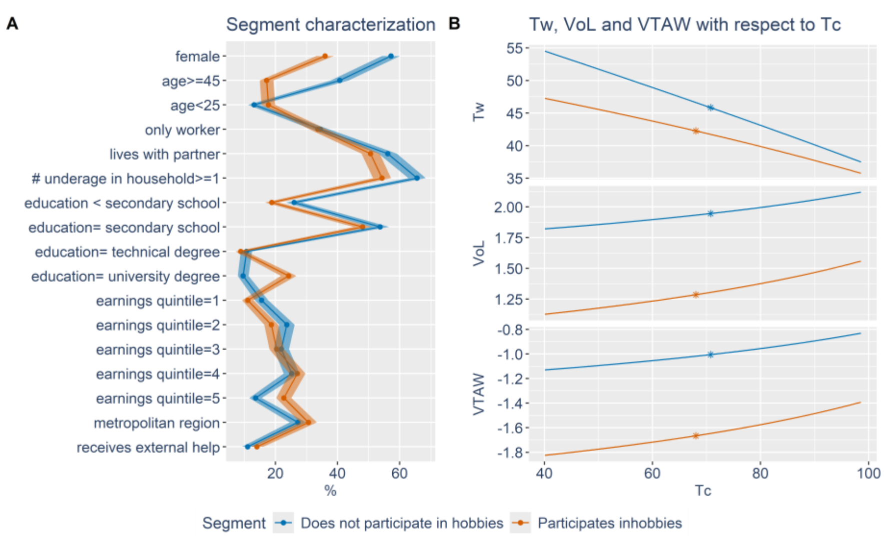

Figure 5.9: Segment characteristics and predicted variables for hobbies model

According to Table 5.4, when hobbies is not considered in the set of activities to be performed, the relative elasticity of consumption (Φ) is lower and the elasticity of work (θ_w) is higher. In both models θ_w is negative, which may be associated to the preference for not increasing time assigned to work, isolating the effect of the wage rate. Both classes consider elasticities for socializing ($\theta_{socializing}$)and media consumption (θ_{media}); and when including hobbies in the set of activities, a new elasticity is formulated for hobbies activities ($\theta_{hobbies}$). With the inclusion of another elasticity, it is expected to observe a decrease across all elasticities, as the same time must be assigned across more activities. The segment with hobbies in the activity mix (and thus, three elasticities for leisure activities) reduces its elasticity of media consumption and increase the elasticity of socializing, contrary to what was anticipated. This may be an indicator of complementary effects between hobbies and socializing. In comparison with the deterministic models, the latent classes estimate higher elasticities of work and hobbies (in the corresponding segments), indicating higher attraction for those activities.

Figure 5.10 presents the differences between the two modelling approaches. In Figure 5.10A the distribution of the values of time are presented for each deterministic model, for the combination of the deterministic models and for the latent class model. It is observed that while the a-priori segments capture different values of time (VoL is 0.513 and 0.635 respectively), the aggregated values of time of both segments (VoL is 0.544) are similar to the expected values of time of latent class model (VoL 0.564) with a difference of only 3.5% with respect to the latent class model. Figure 5.10B shows the difference between estimated parameters between approaches, with their respective confidence interval. No significative differences are observed.

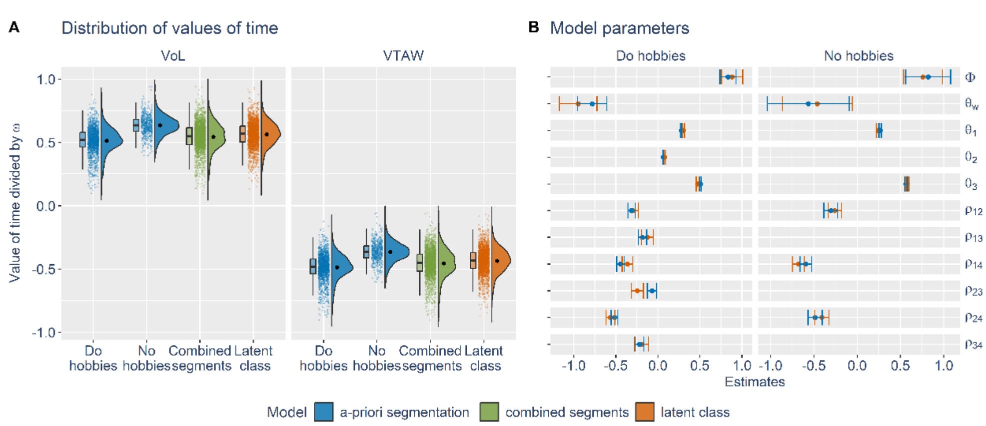

Figure 5.10: Distribution of VoT and model parameters for hobbies corner solutions

Table 5.5: Models for time assigned to childcare

	Latent class				A-priori segmentation			
	Does not participate in Childcare		Participates in Childcare		Does not participate in Childcare		Participates in Childcare	
Specification	value				value		value	
# individuals	1350				32		1246	
# parameters	23				6		10	
LL	-13299.37				-193.89		-13126.46	
LL-ratio					42.0 (χ^2_6 =12.59)			
BIC	26757.32				26755.64			
Time-use parameters	est. (t.val)		est. (t.val)		est. (t.val)		est. (t.val)	
Φ	0.787	**(1.672)**	0.760	**(16.185)**	1.258	**(2.975)**	0.748	**(13.382)**
θ_w	-0.439	(-0.530)	-0.786	**(-9.059)**	-1.624	**(-2.150)**	-0.736	**(-6.902)**
θ_1 $(leisure)$	0.866	**(66.735)**	0.666	**(120.520)**	0.835	**(65.264)**	0.674	**(137.251)**
θ_2 $(childcare)$			0.192	**(36.760)**			0.186	**(38.381)**
σ_1	12.822	**(19.974)**	8.372	**(26.724)**	6.248	**(8.521)**	8.913	**(26.841)**
σ_2	13.169	**(20.978)**	10.434	**(38.922)**	6.204	**(9.550)**	10.708	**(57.271)**
σ_3			9.548	**(26.339)**			9.613	**(33.679)**
ρ_{12}	-0.989	**(-359.027)**	-0.463	**(-14.892)**	-0.763	**(-11.457)**	-0.481	**(-16.491)**
ρ_{13}			-0.318	**(-12.821)**			-0.336	**(-14.925)**
ρ_{23}			-0.620	**(-21.095)**			-0.597	**(-22.632)**
Values of time	value [95% C.I]		value [95% C.I]		value [95% C.I]		value [95% C.I]	
VoL	1.706		1.313		0.813		1.365	
[95% C.I.]	[-0.294 ; 3.7068]		[1.1539 ; 1.4718]		[0.2774 ; 1.3483]		[1.1648 ; 1.5647]	
VTAW	-0.811		-1.498		-1.994		-1.429	
[95% C.I.]	[-2.874 ; 1.251]		[-1.6468 ; -1.3496]		[-2.537 ; -1.4508]		[-1.63 ; -1.2282]	
Weighted wage rate	2.518		2.811		2.807		2.794	
[95% C.I.]	[2.086 ; 2.949]		[2.658 ; 2.9641]		[1.6285 ; 3.985]		[2.6466 ; 2.9411]	
Segmentation prob.	est. (t.val)		est. (t.val.)		est. (t.val)		est. (t.val)	
ASC	-		2.361	**(3.086)**				
female	-		1.756	(1.601)				
age >= 45	-		-1.072	**(-2.054)**				
only worker	-		0.816	(0.647)				
university	-		1.969	(1.187)				
metropolitan	-		-0.420	(-0.605)				
Segment size (*)	0.058		0.942					
Obs. proportion	0.024		0.923					

(*) probability evaluated in the class allocation function

Table 5.5 presents the results of the latent class model with activity mix choice applied to childcare activities and compares it with an a-priori segmentation approach, analogous to the models showed in Table 5.4. The latent class model is observed to have a higher likelihood function than the sum of the deterministic models. The latent class model also presents an increase in BIC and rejects the null hypothesis of the LL-ratio test. Thus, it is concluded that *the latent class model improves significantly in the model fit*.

The membership probability of the segment that does not participate in childcare is 5.8%, close to the observed 2.4% in the sample. According to the estimated influence of the socioeconomic variables, people below 45 years and women have a higher probability of being a member of the segment who participates in childcare (the last not significatively), which agrees with the anticipated effects of those variables.

Figure 5.11A presents the differences between segments. The remarkable differences between segments are that women, people with university studies, are younger than 45 years old or are the only workers of their household are more likely to participate from childcare. Figure 5.11B shows T_w and VoT with respect to committed time for both segments based in activity sets. The relations between committed time and the mentioned variables are observed to follow a similar slope, slightly steeper for time assigned to work for the segment that does not consider childcare in their activity mix. That same segment is observed to have both higher times assigned to work and higher values of time than the segment that assign time to childcare. It is important to remark that all individuals in the sample, live with children.

According to Table 5.5, when childcare is considered in the set of activities to be performed, the elasticity of consumption does not present relevant differences, while the elasticity of work is decreased with respect to the segment who does not participate in childcare. This aligns with the lower dedication of time to work in Figure 5.11B. With respect to the elasticity of leisure and childcare, the introduction of childcare in the activity mix is associated with a decrease in the elasticity of leisure. This may be indicative of a substitutive effect between both activities. In comparison with the deterministic models, the elasticities of work and consumption of the segment that does not participate in childcare are closer to zero, while for the other segment they are similar across methods.

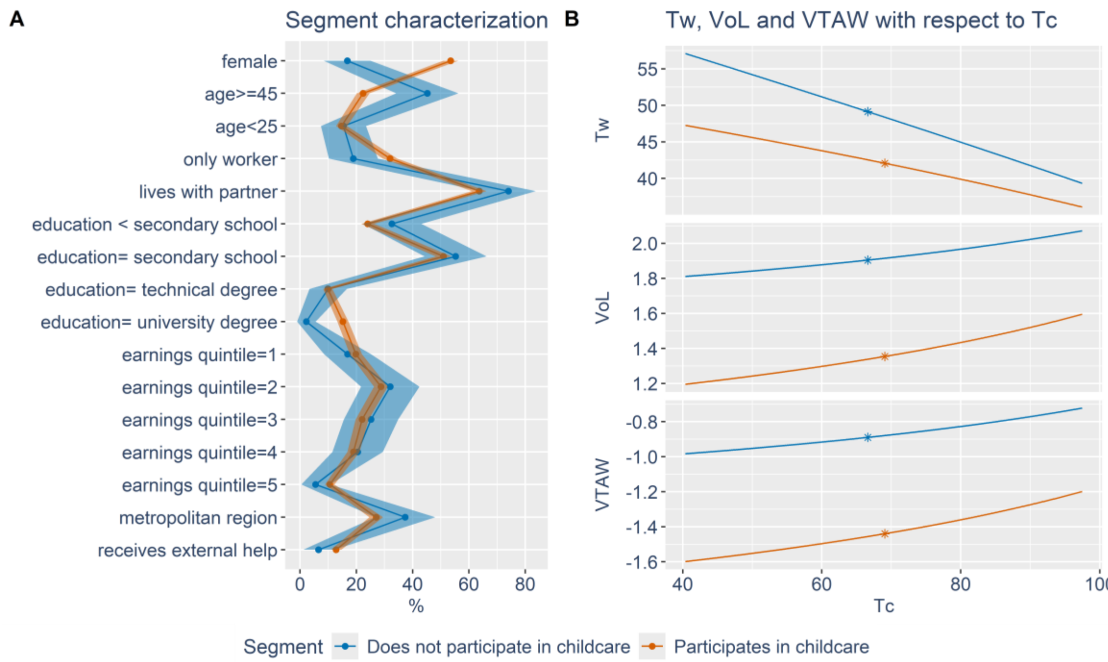

Figure 5.11: Segment characteristics and predicted variables for childcare models

Figure 5.12 shows the difference between modelling approaches in terms of values of time and estimated parameters. Figure 5.12A shows that while the distribution of values of time of both deterministic segments (VoL is 0.508 and 0.312 respectively) is different to the expected values of time of the latent class model (VoL is 0.498), when they are aggregated into one distribution (VoL is 0.503), they get closer to the latent class model, with a difference of 1%. Figure 5.12B shows that the differences in estimated parameters, despite being noticeable, are not significatively relevant.

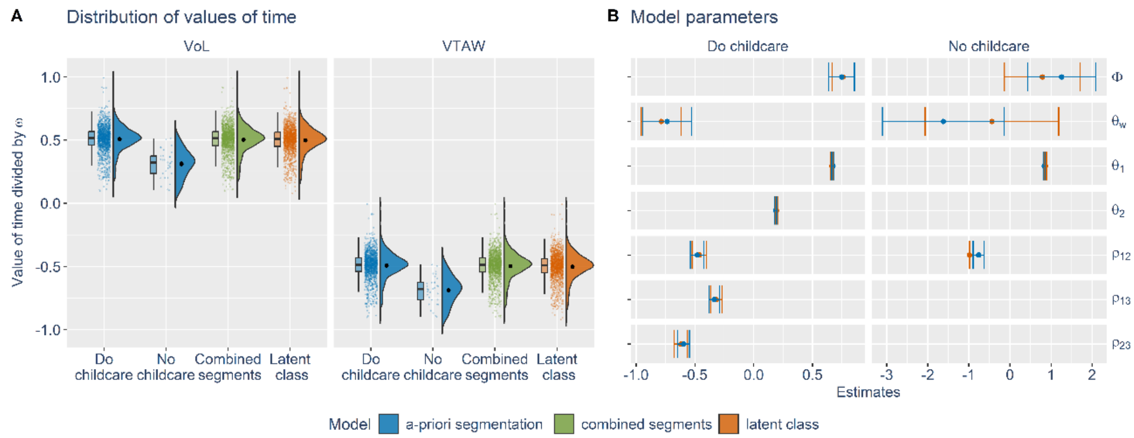

Figure 5.12: Distribution of values of time and model parameters for childcare model

6. Discussion and conclusions

This study introduces technical advances in the microeconomic time-use models that consider committed time and expenses. It formulates a latent class model approach to address two pending research points in this family of models: how to model technical constraints and what happens when someone assigns zero time to an activity. The estimations provide empirical indicators of the effectiveness of the approach.

The first part explores the opportunities of modelling technical time constraints as an endogenous variable in the model. The results show significant improvements when formulating the probability of an activity being constrained. Moreover, the probabilities are formulated to be influenced by socioeconomic variables, unveiling that for both meals and sleep, the probability of being constrained activities are affected by the same variables and in the same direction. In general, the individuals with higher wage rates and higher educational attainment are the ones with these activities constrained. In both models, a slight tendency for both activities to be not committed is noticeable, which should be considered when modelling with exogenous constraints. The interpretation of the segments with meals or sleep uncommitted according to DeSerpa (1971) is that these activities are considered leisure in those segments. In this work, we found that values of time are not heavily affected by the modeller's decision to impose one activity as committed or not. However, when the nature of an activity is treated as endogenous through latent classes, we obtain higher expected values of time, suggesting that the exogenous approach leads to a sub-estimation of the values of time. A model with two activities evaluated as committed or uncommitted simultaneously was also estimated. Still, due to the increased computational complexity, it had to be reduced to a more straightforward form, less effective than evaluating one activity at a time.

The second part of this work explores a method to model the choice of activities performed based on the selection of an activity set. The method requires the formulation of a penalization inside the likelihood function, which may be a matter of future discussion. The latent class technique shows to be a better approach according to the statistical criteria proposed in the methods of this work. The proposed procedure carries some empirical advantages, it can capture membership probabilities, which are influenced by socioeconomic variables, while it estimates the in-class model parameters. Moreover, those socioeconomic segmentation variables showed to be in line to what would be expected for each segment. The estimation of two empirical examples shows that the differences in estimated parameters are not statistically relevant, despite the existence of some differences. In terms of values of time it is also shown that the expected values of time obtained across both methods are similar. Finally, the proposed procedure has the advantage of using the whole sample, which might be helpful when data is scarce.

A limitation of this work arises in the interpretation of values of time between modelling approaches. As noted by (Hössinger et al., 2019), the values of time are latent variables, and thus it is not possible to know what the actual values of time of any population segment nor person is. Another limitation of this work is that the data used comes from two different sources that were statistically merged. The surge of higher quality data collection methods and the more extensive recent time-use datasets with up to four weeks of data and expenditure data as part of the survey (Winkler et al., 2024) open the opportunities to explore microeconomic time-use models without the need to impute expenditure data and explore with more detail disaggregation of activities and goods. These new data sources also present the opportunity to compare and improve the effectiveness of statistical merging time-use and expenditure data. A second line of research that might follow this work is to formulate a microeconomic time-use model for which activity components might be censored depending on the decision of the individual to participate in each activity instead of modelling based on activity sets.

7. References

Aschauer, F., Rösel, I., Hössinger, R., Kreis, H. B., & Gerike, R. (2019). Time use, mobility and expenditure: an innovative survey design for understanding individuals' trade-off processes. *Transportation*, *46*(2), 307-339. https://doi.org/10.1007/s11116-018-9961-9

Astroza, S., Garikapati, V. M., Pendyala, R. M., Bhat, C. R., & Mokhtarian, P. L. (2019). Representing heterogeneity in structural relationships among multiple choice variables using a latent segmentation approach. *Transportation*, *46*(5), 1755-1784. https://doi.org/10.1007/s11116-018-9882-7

Astroza, S., Pinjari, A. R., Bhat, C. R., & Jara-Díaz, S. R. (2017). A Microeconomic Theory–Based Latent Class Multiple Discrete–Continuous Choice Model of Time Use and Goods Consumption. *Transportation Research Record: Journal of the Transportation Research Board*, *2664*(1), 31-41. https://doi.org/10.3141/2664-04

Becker, G. S. (1965). A Theory of the Allocation of Time. *The Economic Journal*.

Bhat, C. R. (1997). An Endogenous Segmentation Mode Choice Model with an Application to Intercity Travel. *Transportation Science*, *31*(1), 34-48. https://doi.org/10.1287/trsc.31.1.34

Bhat, C. R. (2008). The multiple discrete-continuous extreme value (MDCEV) model: Role of utility function parameters, identification considerations, and model extensions. *Transportation Research Part B: Methodological*, *42*(3), 274-303. https://doi.org/10.1016/j.trb.2007.06.002

Bishop, Y. M., Fienberg, S. E., & Holland, P. W. (2007). *Discrete multivariate analysis: Theory and practice*. Springer Science & Business Media.

Calastri, C., Hess, S., Choudhury, C., Daly, A., & Gabrielli, L. (2019). Mode choice with latent availability and consideration: Theory and a case study. *Transportation Research Part B: Methodological*, *123*, 374-385. https://doi.org/10.1016/j.trb.2017.06.016

Castro, M., Bhat, C. R., Pendyala, R. M., & Jara-Díaz, S. R. (2012). Accommodating multiple constraints in the multiple discrete–continuous extreme value (MDCEV) choice model. *Transportation Research Part B: Methodological*, *46*(6), 729-743. https://doi.org/10.1016/j.trb.2012.02.005

Chung, J.-H., Kim, T., Baik, H., & Choi, Y.-S. (2009). A structural equation model of activity participation and travel behavior using longitudinal data. *Transportation Planning and Technology*, *32*(2), 163-185. https://doi.org/10.1080/03081060902861392

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the royal statistical society: series B (methodological)*, *39*(1), 1-22.

DeSerpa, A. C. (1971). A Theory of the Economics of Time. *The Economic Journal*.

Doob, J. L. (1935). The Limiting Distributions of Certain Statistics. *The Annals of Mathematical Statistics*, *6*(3), 160-169. https://doi.org/10.1214/aoms/1177732594

Dotti Sani, G. M. (2018). *Time Use in Domestic Settings Throughout the Life Course*. Springer.

Evans, A. W. (1972). On the Theory of the Valuation and Allocation of Time. *Scottish Journal of Political Economy*, *19*(1), 1-17. https://EconPapers.repec.org/RePEc:bla:scotjp:v:19:y:1972:i:1:p:1-17

Golob, T. (2003). Structural equation modeling for travel behavior research. *Transportation Research Part B 37*, 1-25. https://www.sciencedirect.com/science/article/pii/S0191261501000467

Hess, S. (2014). Latent class structures: taste heterogeneity and beyond. In *Handbook of choice modelling* (pp. 372-391). Edward Elgar Publishing.

Hess, S., & Palma, D. (2024). Apollo: a flexible, powerful and customisable freeware package for choice model estimation and application. http://www.ApolloChoiceModelling.com

Hössinger, R., Aschauer, F., Jara-Diaz, S., Jokubauskaite, S., Schmid, B., Peer, S., Axhausen, K. W., & Gerike, R. (2019). A joint time-assignment and expenditure-allocation model: value of leisure and value of time assigned to travel for specific population segments. *Transportation (Amst)*, *47*(3), 1439-1475. https://doi.org/10.1007/s11116-019-10022-w

INE (Instituto Nacional de Estadísticas). (2015). *Encuesta Nacional sobre Uso del Tiempo*. https://www.ine.gob.cl/estadisticas/sociales/genero/uso-del-tiempo

INE (Instituto Nacional de Estadísticas). (2016). *Encuesta de Presupuestos Familiares*. https://www.ine.gob.cl/estadisticas/sociales/ingresos-y-gastos/encuesta-de-presupuestos-familiares

Jara-Diaz, S. (2020). Transport and time use: The values of leisure, work and travel. *Transport Policy*, *86*, A7-A13. https://doi.org/10.1016/j.tranpol.2019.12.001

Jara-Diaz, S., Astroza, S., Candia, D., & Morales, M. (2025). Reallocation of time after an exogenous reduction in mandatory travel: transport, work, and leisure in Chilean two-worker two-gender households. *Travel Behaviour and Society*, *38*. https://doi.org/10.1016/j.tbs.2024.100925

Jara-Díaz, S., & Candia, D. (2020). A new look at the value of leisure in two-worker households. *Economics of Transportation*, *24*. https://doi.org/10.1016/j.ecotra.2020.100193

Jara-Díaz, S., & Contreras, R. (2024). Reassigning travel time savings under demanding income and time constraints: Work or leisure? *Travel Behaviour and Society*, *35*. https://doi.org/10.1016/j.tbs.2024.100749

Jara-Diaz, S., & Guevara, C. A. (2003). Behind the Subjective Value of Travel Time Savings: The Perception of Work, Leisure, and Travel from a Joint Mode Choice Activity Model. *Journal of Transport Economics and Policy*.

Jara-Diaz, S., Morales, M., & Astroza, S. (2023). Transcending the gender distribution of work: committed time, activity substitution, and leisure. *Transportation*. https://doi.org/10.1007/s11116-023-10438-5

Jara-Díaz, S., & Rosales-Salas, J. (2015). Understanding time use: Daily or weekly data? *Transportation Research Part A: Policy and Practice*, *76*, 38-57. https://doi.org/10.1016/j.tra.2014.07.009

Jara-Díaz, S., & Rosales-Salas, J. (2017). Beyond transport time: A review of time use modeling. *Transportation Research Part A: Policy and Practice*, *97*, 209-230. https://doi.org/10.1016/j.tra.2017.01.022

Jara-Díaz, S. R., Astroza, S., Bhat, C. R., & Castro, M. (2016). Introducing relations between activities and goods consumption in microeconomic time use models. *Transportation Research Part B: Methodological*, *93*, 162-180. https://doi.org/10.1016/j.trb.2016.07.014

Jara-Díaz, S. R., Munizaga, M. A., Greeven, P., Guerra, R., & Axhausen, K. (2008). Estimating the value of leisure from a time allocation model. *Transportation Research Part B: Methodological*, *42*(10), 946-957. https://doi.org/10.1016/j.trb.2008.03.001

Jara-Díaz, S. R., & Rosales-Salas, J. (2020). Time use: The role of sleep. *Transportation Research Part A: Policy and Practice*, *136*, 1-20. https://doi.org/10.1016/j.tra.2020.03.025

Jokubauskaitė, S., Hössinger, R., Aschauer, F., Gerike, R., Jara-Diaz, S., Peer, S., Schmid, B., Axhausen, K. W., & Leisch, F. (2019). Advanced continuous-discrete model for joint time-use expenditure and mode choice estimation [Article]. *Transportation Research Part B-Methodological*, *129*, 397-421. https://doi.org/10.1016/j.trb.2019.09.010

Jokubauskaitė, S., Hössinger, R., Jara-Díaz, S., Peer, S., Schneebaum, A., Schmid, B., Aschauer, F., Gerike, R., Axhausen, K. W., & Leisch, F. (2022).

The role of unpaid domestic work in explaining the gender gap in the (monetary) value of leisure. *Transportation*, *49*(6), 1599-1625. https://doi.org/10.1007/s11116-021-10221-4

Kamakura, W. A., & Russell, G. J. (1989). A Probabilistic Choice Model for Market Segmentation and Elasticity Structure. *Journal of Marketing Research*, *26*(4), 379-390. https://doi.org/10.1177/002224378902600401

Kim, S. H., & Mokhtarian, P. L. (2021). Who (never) makes overnight leisure trips? Disentangling structurally zero trips from usual trip generation processes. *Travel Behaviour and Society*, *25*, 78-91. https://doi.org/10.1016/j.tbs.2021.05.011

Kim, S. H., & Mokhtarian, P. L. (2023). Finite mixture (or latent class) modeling in transportation: Trends, usage, potential, and future directions. *Transportation Research Part B: Methodological*, *172*, 134-173. https://doi.org/10.1016/j.trb.2023.03.001

Kim, S. H., & Mokhtarian, P. L. (2024). Latent class choice models with an error structure: Investigating potential unobserved associations between latent segmentation and behavior generation. *Journal of Choice Modelling*, *53*. https://doi.org/10.1016/j.jocm.2024.100519

Koo, J., Kim, J., Choi, S., & Choo, S. (2022). Identifying the Causal Relationship between Travel and Activity Times: A Structural Equation Modeling Approach. *Sustainability*, *14*(8), 4615. https://doi.org/10.3390/su14084615

Lu, X., & Pas, E. I. (1999). Socio-demographics, activity participation and travel behavior. *Transportation Research Part A*, *33*.

Munizaga, M., Jara-Díaz, S., Greeven, P., & Bhat, C. (2008). Econometric Calibration of the Joint Time Assignment–Mode Choice Model. *Transportation Science*, *42*(2), 208-219. https://doi.org/10.1287/trsc.1080.0231

Munizaga, M., Jara-Díaz, S., Olguín, J., & Rivera, J. (2011). Generating twins to build weekly time use data from multiple single day OD surveys. *Transportation*, *38*(3), 511-524. https://doi.org/10.1007/s11116-010-9311-z

Pellegrini, A., Pinjari, A. R., & Maggi, R. (2021). A multiple discrete continuous model of time use that accommodates non-additively separable utility functions along with time and monetary budget constraints. *Transportation Research Part A: Policy and Practice*, *144*, 37-53. https://doi.org/10.1016/j.tra.2020.11.009

Pellegrini, A., & Rose, J. M. (2024). *On allowing endogenous minimum consumption bounds in the Multiple Discrete Continuous Choice Model:*

An application to expenditure patterns. https://hdl.handle.net/2123/32175

Schmid, B., Molloy, J., Peer, S., Jokubauskaite, S., Aschauer, F., Hössinger, R., Gerike, R., Jara-Diaz, S. R., & Axhausen, K. W. (2021). The value of travel time savings and the value of leisure in Zurich: Estimation, decomposition and policy implications. *Transportation Research Part A: Policy and Practice*, *150*, 186-215. https://doi.org/10.1016/j.tra.2021.06.015

Spurk, D., Hirschi, A., Wang, M., Valero, D., & Kauffeld, S. (2020). Latent profile analysis: A review and "how to" guide of its application within vocational behavior research. *Journal of Vocational Behavior*, *120*. https://doi.org/10.1016/j.jvb.2020.103445

Team, R. C. (2024). R: A Language and Environment for Statistical Computing. https://www.R-project.org/

Train, K. E. (2008). EM Algorithms for nonparametric estimation of mixing distributions. *Journal of Choice Modelling*, *1*(1), 40-69. https://doi.org/https://doi.org/10.1016/S1755-5345(13)70022-8

Train, K. E. (2009). *Discrete choice methods with simulation*. Cambridge university press.

Winkler, C., Meister, A., & Axhausen, K. W. (2024). The TimeUse+ data set: 4 weeks of time use and expenditure data based on GPS tracks. *Transportation*. https://doi.org/10.1007/s11116-024-10517-1

8. Appendix

8.1. Alternative parametrization

An alternative of fixing Θ for the correct identification of the equation system formed by (2.11)-(2.13) is to normalize utility parameters by defining the following auxiliary variables:

$$\alpha = \frac{\Theta + \theta_w}{2(\Theta + \Phi + \theta_w)} \tag{8.1}$$

$$\beta = \frac{\Phi + \theta_w}{2(\Theta + \Phi + \theta_w)} \tag{8.2}$$

$$\gamma_i = \frac{\theta_i}{\Theta + \Phi + \theta_w} \tag{8.3}$$

$$\delta_i = \frac{\phi_i}{\Theta + \Phi + \theta_w} \tag{8.4}$$

Then the system of equations for the optimal time and expenditure allocations may be rewritten as follows:

$$T_w^* = (\tau - T_c)\left[\beta + \alpha e_c + \sqrt{(\beta + \alpha e_c)^2 - (2\alpha + 2\beta - 1)e_c}\right] \quad \textbf{(8.5)}$$

$$T_i^* = \frac{\gamma_i}{1 - 2\beta}(\tau - T_w^* - T_c) \qquad \forall \boldsymbol{i} \in \boldsymbol{A^f} \quad \textbf{(8.6)}$$

$$X_j^* = \frac{\delta_j}{1 - 2\alpha}(\omega T_w^* - E_c) \qquad \forall \boldsymbol{j} \in \boldsymbol{G^f} \quad \textbf{(8.7)}$$

And the values of time may be computed as:

$$VoL = \frac{\partial U / \partial T_i}{\partial U / \partial E_j} = \frac{\mu}{\lambda} = \left(\frac{1 - 2\beta}{1 - 2\alpha}\right)\frac{wT_w - E_c}{\tau - T_w - T_c} \quad \textbf{(8.8)}$$

$$VTAW = \frac{\partial U / \partial T_w}{\partial U / \partial E_j} = \left(\frac{2\alpha + 2\beta - 1}{1 - 2\alpha}\right)\frac{wT_w - E_c}{T_w} = VoL - w \quad \textbf{(8.9)}$$

8.2. EM algorithm

The EM algorithm is a procedure originally develop to deal with missing data (Dempster et al., 1977), but later applied to the estimation of mixed logit and discrete mixture models (Train, 2008). The algorithm alternates iteratively between an expectation (E) step and a maximization (M). The core of the method is to express the likelihood function as the integral of the conditional probability over the missing data:

$$P(y|\theta) = \int P(y|z,\theta) f(z|\theta) dz$$

Where y is the observed data, θ is the vector of model paraemeters and z is the missing data. For latent class models, the method considers the class membership as the missing data.

Let define $L_{qs}(\psi_s)$ in-class probability of the individual q given that is a member of class s, a function of the parameters ψ_s; define π_{qs} as the membership probability of individual q to class s, modelled as a multinomial logit model, with individuals' covariates Z_q and their parameter vector associated γ_s. The steps of the algorithm as described in (Hess & Palma, 2024; Train, 2009) are described bellow:

1. Set the starting values for ψ_s^0 and γ_s^0.
2. Calculate the likelihood of the model in the starting values. Define $\gamma^0 = \langle \gamma_1^0, \ldots, \gamma_S^0 \rangle$ and $\psi^0 = \langle \psi_1^0, \ldots, \psi_S^0 \rangle$ and store the likelihood as L_0 and the membership probabilities as $\pi_q^0 = \langle \pi_{q1}^0, \ldots, \pi_{qS}^0 \rangle$
3. Calculate the membership probabilities conditional on the observed in-class probabilities for each class in iteration t as:

$$h_{qs}^t = \frac{\pi_{qs}^t(\gamma^t)L_{ns}(\psi_s^t)}{\sum_{s'} \pi_{qs'}^t(\gamma^t)L_{ns'}(\psi_{s'}^t)} \quad \textbf{(8.10)}$$

4. Maximize the membership probabilities weighted by h_{qs}^t and update parameters γ:

$$\gamma^{t+1} = \max \sum_q \sum_s h_{qs}^t \log(\pi_{qs}) \quad \textbf{(8.11)}$$

5. Update parameters ψ_s by maximizing the in-class probability for each class s, weighted by h_{qs}^t:

$$\psi^{t+1} = \max \sum_q \sum_s h_{qs}^t \log\left(L_{ns}(\psi_s^t)\right), \forall s \quad \textbf{(8.12)}$$

6. Calculate the likelihood of the model for $t+1$. Repeat steps 3-6 setting $t+1$ as the current iteration until convergence.

8.3. Simulations

A simulation experiment explored the adequacy of the chosen estimation method. Databases were sampled assuming the time-use parameters were known. Then, the recovery of parameters was tested for the estimation method. The main parameters for the individual characteristics; committed time, (normalized) committed expenses and wage rate; were adjusted to be similar to the MAED dataset (Aschauer et al., 2019). The parameters were fitted to suitable statistical distribution testing Gamma, Normal, Weibull, Lognormal and Logistic distributions for the case of committed time and wage and for the committed expenses, the alternatives were Normal, Uniform and Logistic distributions. The reason for testing with different distributions is that committed time and wage rate are both always positive quantities, while committed expenses may be negative when an individual's fixed income is greater than the sum of all its minimum expenses. For committed time, the Gamma distribution was selected as it approximates adequately to the observed data, as shown in Figure 8.1. For (normalized) committed expenses, the logistic distribution was selected as it approximated better to the actual data distribution than the Normal and Exponential distribution, as observed in Figure 8.2. The Lognormal distribution was selected to fit the wage rate, as it achieves a better approximation for wage rates in the data, as presented in Figure 8.3.

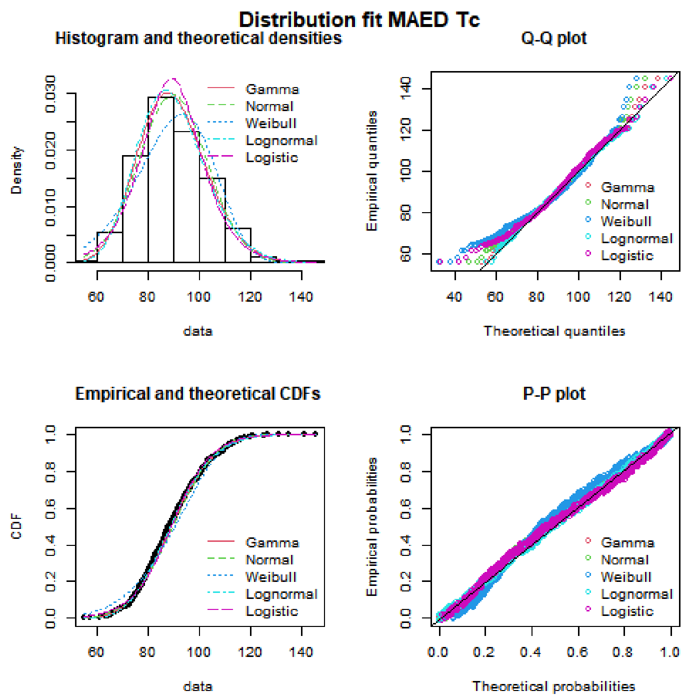

Figure 8.1: Distribution analysis for committed time

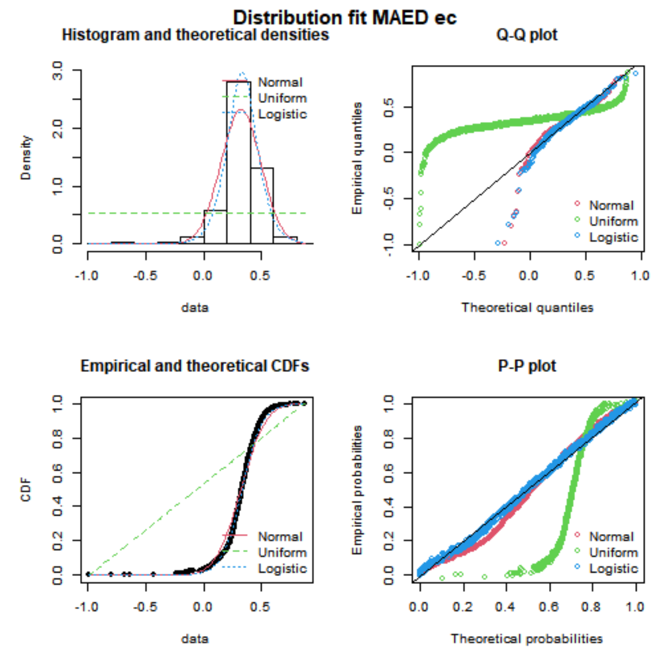

Figure 8.2: Distribution analysis for committed expenses

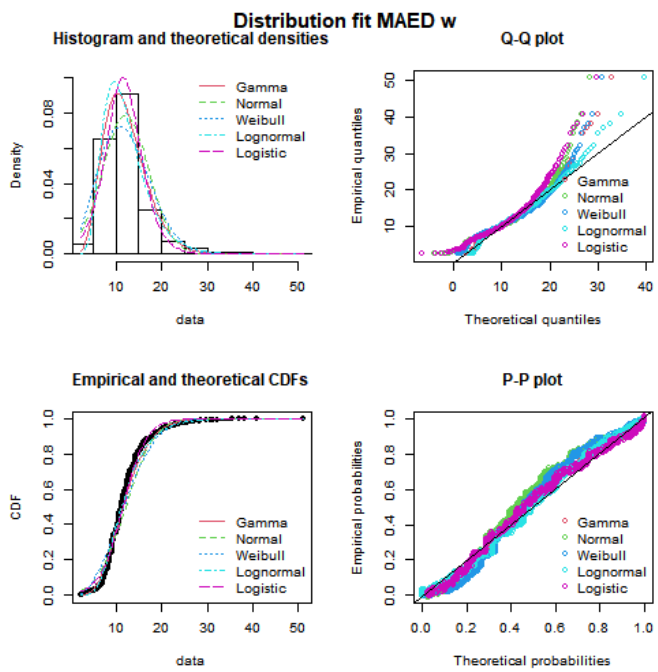

Figure 8.3: Distribution analysis for the wage rate

50 sets of model parameters (θ_w, θ_i, ϕ_j) were generated, for which there were sampled 50, 75, 100, 200, 500, 1000 and 2000 individuals, whose values for

committed time, committed expenses and wage rates were sampled from the fitted distributions discussed before. The optimal time allocations and expenditures of each individual were generated according to their elasticities and equations (2.11)-(2.13), then the observed values were generated by sampling observation errors from a random covariance matrix with variance and correlations common for all individuals. Finally, Table 8.1 summarises the distributions and the fixed values used for the setting of parameters for the simulation experiment.

Table 8.1: Parameter settings for the simulation experiments.

Parameter	Type	Distribution
Model		
# Activities	Fixed	3 + work
# Goods	Fixed	3
Θ_w	Random	$Uniform(-1,1)$
θ_i	Random	$Uniform(0,1)$
ϕ_j	Random	$Uniform(0,1)$
Individuals		
# individuals	Fixed	{50,75,100,200,500,1000,2000}
τ	Fixed	168 hours
T_c	Random	$Gamma(shape = 45.43, rate = 0.51)$
e_c	Random	$Logistic(lcoation = 0.33, scale = 0.08)$
ω	Random	$lognormal(mean = 2.42, sd = 0.39)$
Covariance matrix	($\mathbf{\Sigma}$)	
σ_i	Random	$Uniform(1.5,2.5)$
ρ_{ij}	Random	$Uniform(-1,1)$, corrected to ensure positive definite $\mathbf{\Sigma}$

The parameters and values of time for each database were estimated using Maximum likelihood estimation and 50 starting values for the optimization algorithm. Additionally, the parameters were estimated using 1 equation (Φ, θ_w), 3 equations ($\Phi, \theta_w, \theta_1, \phi_1$) and 5 equations ($\Phi, \theta_w, \theta_1, \theta_2, \phi_1, \phi_j$). Alternatively, the parameters were also estimated using the alternative parametrization normalizing with α, β, γ and δ. Three metrics were used to assess the capacities of recovering parameters of the estimation method: the average of the RMSE for all parameters, the average of the standard errors for the values of time (VoT), and the proportion of successful estimations made. Table 8.2 summarises the average metrics grouped by the number of individuals and number of equations for the results estimated fixing $\Theta = 1$. Table 8.3 presents the results estimated parametrizing the utility parameters of the model by $\alpha\beta$. In general, for both parametrizations it is observed that the rate of success decreases with the increase of the number of equations; RMSE increases with the number of equations, but there is not a trend with respect to the sample size; and for both RMSE and standards errors of the values of time the increase in sample size and number of equations is associated with a decrease in the metrics.

Table 8.2: Summary of simulation results estimated with ΘΦ parametrization

# individuals	# equations	RMSE	RMSE VoT	SE VoT	rate of success
50	1	0.214	1.627	1.085	1.00
	3	0.087	0.225	0.300	0.98
	5	0.082	0.086	0.161	0.92
75	1	0.131	0.965	1.352	1.00
	3	0.060	0.163	0.246	0.98
	5	0.061	0.064	0.116	0.94
100	1	0.107	0.788	1.148	1.00
	3	0.051	0.169	0.213	1.00
	5	0.050	0.052	0.100	0.90
200	1	0.081	0.593	0.849	1.00
	3	0.040	0.115	0.154	0.98
	5	0.037	0.035	0.071	0.94
500	1	0.054	0.427	0.535	1.00
	3	2.075	0.058	0.093	0.96
	5	0.027	0.032	0.042	0.90
1000	1	0.037	0.286	0.384	1.00
	3	0.019	0.049	0.070	0.94
	5	0.018	0.021	0.031	0.94
2000	1	0.025	0.161	0.274	1.00
	3	0.015	0.038	0.049	0.94
	5	0.014	0.014	0.022	0.94
Mean		0.157	0.284	0.347	0.965

Table 8.3: Summary of simulation results estimated with $\alpha\beta$ parametrization

# individuals	# equations	RMSE	RMSE VoT	SE VoT	rate of success
50	1	1.955	1.548	1.549	1.00
	3	0.121	0.231	0.290	0.92
	5	0.687	0.945	0.194	0.90
75	1	0.134	0.965	1.352	1.00
	3	0.757	0.635	0.343	0.96
	5	0.212	0.355	0.161	0.88
100	1	2.035	0.763	1.139	1.00
	3	0.170	0.442	0.239	0.96
	5	0.474	0.654	0.143	0.88
200	1	1.869	0.588	0.842	1.00
	3	0.306	0.269	0.198	0.98
	5	0.143	0.297	0.084	0.88
500	1	1.861	0.423	0.531	1.00
	3	0.032	0.058	0.096	0.96
	5	0.138	0.314	0.065	0.88
1000	1	0.984	0.286	0.383	1.00
	3	0.025	0.048	0.070	0.98
	5	0.123	0.294	0.039	0.88
2000	1	0.044	0.161	0.274	1.00
	3	0.020	0.039	0.050	0.92
	5	0.121	0.288	0.027	0.88
Mean		0.581	0.457	0.384	0.945

A Wilcoxon test was conducted to compare the RMSE of the values of time estimated with both parametrizations, the p-value of the test was 0.7 and thus it is not possible to reject the null hypothesis of both methods being equivalent. The same was done with the success rate; in this case the p-value obtained was 0.04, and thus we can conclude the results achieves by both parametrizations are not equivalent on the success rate, being the $\Theta\Phi$ parametrization a better alternative.

A series of linear models were estimated to associate the explained metrics with sample size and number of equations. Table 8.4 summarises the result of the linear models. The sample size helps to reduce both the RMSE and the SE for the values of time at a steady but significative rate, the number of equations have the same effect for the values of time metrics and for overall RMSE, and estimating the results with the $\alpha\beta$ parametrization increases the RMSE for the overall parameters and for the values of time. Thus, it is suggested to use the $\Theta\Phi$ parametrization.

Table 8.4: Summary of linear models

Explained variable	Estimates (t-val)			
	Intercept	# individuals	# equations	Parametrization=$\alpha\beta$
RMSE	0.663 **(2.58)**	-0.000211 (-1.34)	-0.132 **(-2.04)**	0.440 **(2.10)**
RMSE VoT	0.763 **(11.47)**	-0.000250 **(-6.09)**	-0.112 **(-6.72)**	0.165 **(3.04)**
SE VoT	1.05 **(34.18)**	-0.000241 **(-12.078)**	-0.189 **(-24.46)**	0.0362 (1.44)

Figure 8.4 presents a comparison between the estimated values and real values for a database with 1000 individuals estimated with 3 equations. The rest of the plots are available as a supplementary material of this article.

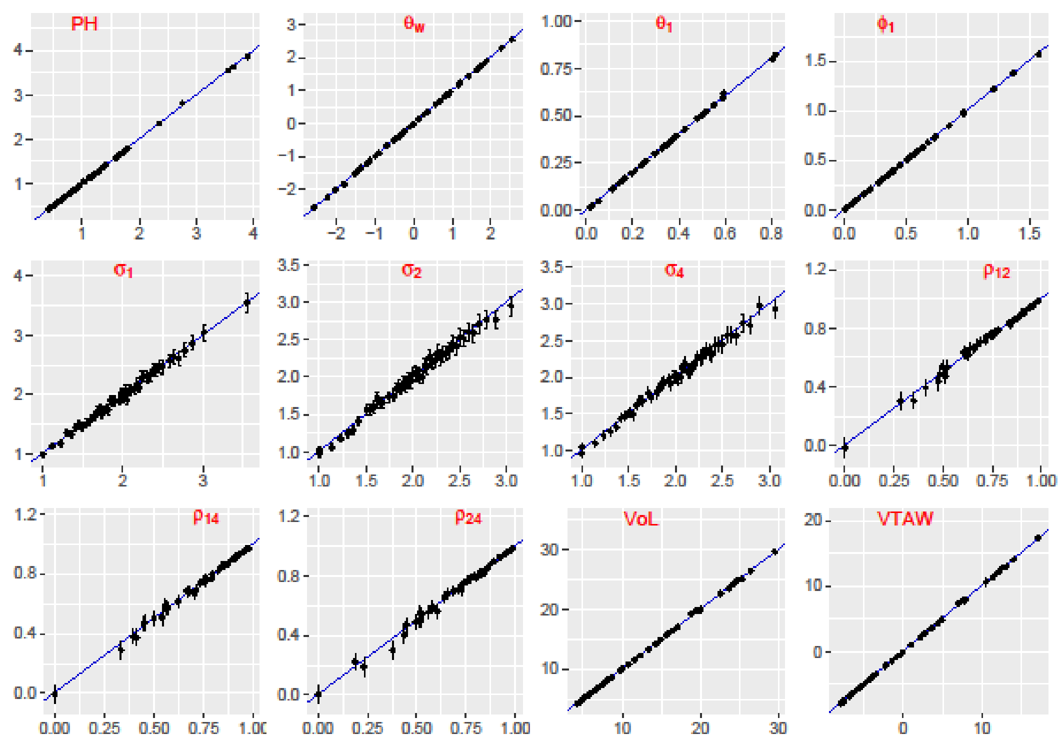

Figure 8.4: Parameter recover in a database with 1000 observations

8.4. Imputation of weekend time-use

As part of the generation of weekly observations a method was followed for the generation of twins inside the sample, which allows to construct the day of an individual from the declared time allocations of its twin. Munizaga et al. (2011) proposed an optimization problem that allows to create twins as a convex combination of the closest individuals to each person.

Let define the variable μ_{ij} with domine between zero and one, which indicates the proportion in which each individual j apports to the generation of the twin for individual i; define K_i as the vector of socioeconomic characteristics of each individual i; and define S_i as the set of individuals that declared time on the day that the individual i is missing. The latter is composed by gender, age, income quintiles, educational attainment, current study and work situation, regional location, contractual work hours, the number of persons in the household and the number of minors. Then for each individual i, the optimization problem is as follows:

$$\min \left\| K_i - \sum_{j \in S_i} \mu_{ij} K_j \right\| + \sum_{j \in S_i} \mu_{ij} \| K_i - K_j \| \qquad \textbf{(8.13)}$$

Subject to:

$$\sum_{j \in S} \mu_{ij} = 1 \qquad \textbf{(8.14)}$$

$$\mu_{ij} \geq 0 \qquad \textbf{(8.15)}$$

Then the time assigned by the individual i for each activity k on the day s ($T_{i,s}^{k}$) may be constructed as follows:

$$T_{i,s}^{k} = \sum_{j \in S_i} \mu_{ij} T_{j,s}^{k} \qquad \textbf{(8.16)}$$

8.5. Imputation of weekly expenditures

To impute data from the Chilean Household Budget Survey (EPF). First, a linear model was estimated to predict the difference between the income of the household and their total expenditures, i.e. the savings. The linear model is based on income variables, household composition, average age and location. The linear model is presented in Table 8.5. Non significative parameters are retained in the model, as its objective is to predict instead of searching or structural relations. Second, a fractional logit model is used to predict the shares of expenses distributed among food, clothing, bills, household maintenance, health, transportation, communications, recreation, education and restaurants. Table 8.6 presents the values of the estimated model for the distribution of expenditures among items. The item with the higher share of expenses corresponds to food, with approximately 28%, and the items with the lower proportion are clothing, recreation and education, all with approximately 4% of the total expenses. The clothing item had its parameters fixed to zero as the baseline alternative. The rest of the procedure for imputation of expenses is describes in the following steps:

1. Predict household savings using the linear model on the ENUT.
2. Subtract savings from total income to obtain total household expenses.
3. Distribute total expenses proportionally to the income of the household members.
4. Impute the specific expenses to each expenditure item using the fractional logit model.

Table 8.5: Linear model for predicted savings

Parameter	est. (t.val)	
(Intercept)	-70.73	**(-11.461)**
Household income	0.69	**(141.252)**
Earnings quintile=2	-31.63	**(-8.432)**
Earnings quintile=3	-62.47	**(-15.317)**
Earnings quintile=4	-103.4	**(-23.454)**
Earnings quintile=5	-212.5	**(-36.455)**
# persons	-10.10	**(-8.608)**
# kids (age< 6)	10.82	(1.697)
# children (6<=age<12)	-11.26	**(-2.161)**
# teens (12<=age<18)	-9.60	**(-4.507)**
# workers	9.69	**(5.601)**
# professionals	-44.90	**(-26.582)**
Average age	0.75	**(8.624)**
Metropolitan region	-0.50	(-0.205)
$\boldsymbol{R^2 = 0.6515; R^2_{adj} = 0.6512}$		

Table 8.6: Fractional logit model for expenditure shares

	food		bills		house maintenance		health		transportation	
Parameters	est.	(t.val)	est.	(t.val)	est.	(t.val)	est.	(t.val)	est.	(t.val)
(Intercept)	0.28	**(6.380)**	1.37	**(29.377)**	-1.11	**(-17.834)**	-0.87	**(-14.612)**	0.22	**(4.019)**
#persons	0.22	**(26.380)**	-0.07	**(-7.577)**	0.01	(0.943)	0.10	**(10.051)**	0.06	**(6.044)**
#kids ($age <$ 6)	-0.04	(-0.923)	0.06	(1.378)	0.08	(1.461)	-0.04	(-0.802)	-0.21	**(-3.879)**
#children ($6 \leq age < 12$)	-0.08	**(-2.450)**	-0.12	**(-3.167)**	-0.03	(-0.739)	-0.12	**(-2.708)**	0.04	(0.873)
#teens ($12 \leq age < 18$)	-0.02	(-1.234)	-0.05	**(-3.674)**	-0.08	**(-5.133)**	-0.08	**(-5.437)**	0.04	**(2.666)**
#professionals	-0.10	**(-8.518)**	0.00	(0.016)	0.09	**(6.410)**	0.07	**(4.913)**	0.02	(1.608)
Average age	0.03	**(38.873)**	0.01	**(19.290)**	0.03	**(24.978)**	0.03	**(32.499)**	0.01	**(13.111)**
Earnings quintile=2	-0.17	**(-6.085)**	-0.07	**(-2.416)**	0.16	**(4.048)**	0.16	**(4.073)**	0.18	**(5.068)**
Earnings quintile=3	-0.30	**(-9.913)**	-0.06	(-1.907)	0.27	**(6.121)**	0.25	**(6.013)**	0.27	**(7.051)**
Earnings quintile=4	-0.45	**(-14.370)**	-0.11	**(-3.137)**	0.38	**(8.665)**	0.31	**(7.651)**	0.34	**(8.588)**
Earnings quintile=5	-0.70	**(-19.032)**	-0.14	**(-3.673)**	0.64	**(13.222)**	0.28	**(6.076)**	0.46	**(9.898)**
Metropolitan region	0.35	**(18.707)**	0.19	**(9.476)**	0.12	**(4.734)**	0.14	**(5.744)**	0.08	**(3.413)**
	communications		recreation		education		restaurants		clothing	
Parameters	est.	(t.val)	est.	(t.val)	est.	(t.val)	est.	(t.val)	est.	(t.val)
(Intercept)	-0.30	**(-6.250)**	-0.58	**(-10.042)**	0.28	**(2.863)**	-0.28	**(-4.522)**		
#persons	0.04	**(4.877)**	0.00	(0.044)	0.17	**(11.246)**	-0.04	**(-3.731)**		
#kids ($age <$ 6)	-0.05	(-0.937)	-0.07	(-1.385)	-0.52	**(-5.549)**	-0.18	**(-3.020)**		
#children ($6 \leq age < 12$)	-0.11	**(-2.678)**	0.01	(0.283)	0.03	(0.464)	-0.04	(-0.898)		
#teens ($12 \leq age < 18$)	-0.01	(-0.616)	-0.05	**(-2.850)**	-0.26	**(-10.636)**	0.03	(1.931)		
#professionals	-0.01	(-1.127)	0.11	**(6.992)**	0.04	**(1.964)**	0.07	**(5.215)**		
Average age	0.02	**(22.106)**	0.01	**(13.955)**	-0.02	**(-17.199)**	0.01	**(6.151)**		
Earnings quintile=2	0.09	**(2.755)**	0.14	**(3.601)**	0.27	**(3.980)**	0.16	**(3.658)**		
Earnings quintile=3	0.15	**(4.341)**	0.16	**(3.985)**	0.49	**(7.444)**	0.31	**(7.209)**		
Earnings quintile=4	0.14	**(3.969)**	0.27	**(6.405)**	0.58	**(8.908)**	0.45	**(10.277)**		
Earnings quintile=5	0.01	(0.139)	0.41	**(8.416)**	0.86	**(12.216)**	0.57	**(11.933)**		
Metropolitan region	0.13	**(6.549)**	0.13	**(5.240)**	0.06	(1.704)	0.28	**(11.281)**		

8.6. Standard errors for the estimator of values of time

The standard error of the estimators for the values of time are computed following the δ-method (Bishop et al., 2007; Doob, 1935). The method proposes that, given a parameter vector ψ, its estimator $\hat{\psi}$ asymptotically converges to a normal distribution:

$$\hat{\psi} \rightarrow N\left(\psi, \frac{\mathbf{\Sigma}(\psi)}{n}\right) \quad \textbf{(8.1)}$$

Where $\mathbf{\Sigma}(\psi)$ is the asymptotic covariance matrix and n the sample size. Let define h as a differentiable function that takes the values from ψ. Then $h(\hat{\psi})$ converges to:

$$h(\hat{\psi}) \rightarrow N\left(h(\psi), \frac{\left(\frac{\partial h}{\partial \psi}\right) \mathbf{\Sigma}(\psi) \left(\frac{\partial h}{\partial \psi}\right)'}{n}\right) \quad \textbf{(8.2)}$$

As the values of time are essentially functions of the parameters Θ, Φ and θ_w, then the δ-method may be used for estimating their standard errors, setting confidence intervals and for hypothesis testing that $H_0: \psi = \hat{\psi}$. The latter is done by defining the critical value as:

$$Z_n = \frac{\mathrm{g}(\psi)}{\sqrt{\frac{J_g(\psi)^T \Sigma(\psi) J_{\mathrm{g}}(\psi)}{n}}} \sim N(0,1) \quad \textbf{(8.3)}$$

To obtain the standard errors for the estimators of the values of time it is necessary to replace h by the equations for the calculation of the values of time. Replace $\Sigma(\Phi)$ by the asymptotic covariance matrix of the estimator and compute the partial derivate elements as described hereafter for both parametrizations of the model.

8.6.1. *ΘΦ* parametrization

Recalling equations (2.14) and (2.15) the value of leisure (VoL) and the value of time assigned to work ($VTAW$) are a function of the parameter vector $\psi = [\Phi,\ \theta_w]'$. Then the elements of the Jacobian matrixes for the values of time are as follows:

$$J_{VTAW}(\psi) = \begin{bmatrix} \frac{\partial VTAW}{\partial \Phi} \\ \frac{\partial VTAW}{\partial \theta_w} \end{bmatrix} = \begin{bmatrix} -\frac{\theta_w}{\Phi^2} \frac{wT_w - E_c}{T_w} \\ \frac{1}{\Phi} \frac{wT_w - E_c}{T_w} \end{bmatrix} \quad \textbf{(8.4)}$$

$$J_{VoL}(\psi) = \begin{bmatrix} \frac{\partial VTAW}{\partial \Phi} \\ \frac{\partial VTAW}{\partial \theta_w} \end{bmatrix} = \begin{bmatrix} -\frac{\Theta}{\Phi^2} \cdot \frac{wT_w^* - E_c}{\tau - T_w^* - T_c} \\ 0 \end{bmatrix} \tag{8.5}$$

8.6.2. $\boldsymbol{\alpha\beta}$ parametrization

Recalling equations (8.8) and (8.9) the values of time may be also expressed in function to the parameter vector $\psi = [\alpha, \beta]'$. Then the Jacobian matrixes for the values of time are as follows:

$$J_{VTAW}(\psi) = \begin{bmatrix} \frac{\partial VTAW}{\partial \alpha} \\ \frac{\partial VTAW}{\partial \beta} \end{bmatrix} = \frac{wT_w - E_c}{T_w} \cdot \begin{bmatrix} \frac{4\beta}{(1-2\alpha)^2} \\ \frac{1}{\Phi} \frac{wT_w - E_c}{T_w} \end{bmatrix} \tag{8.6}$$

$$J_{VoL}(\psi) = \begin{bmatrix} \frac{\partial VTAW}{\partial \alpha} \\ \frac{\partial VTAW}{\partial \beta} \end{bmatrix} = \frac{wT_w^* - E_c}{\tau - T_w^* - T_c} \cdot \begin{bmatrix} \frac{-2(1-2\beta)}{(1-2\alpha)^2} \\ \frac{-2}{1-2\alpha} \end{bmatrix} \tag{8.7}$$